CHAPTER: Square and Square roots

Grade: **8**
Board: **CBSE**
Book: **NCERT**

1.

Chapter Summary and Placement

<Read through the following to understand how this chapter is connected with the previous and next grades and also the important concepts covered in it.>

1. This chapter connects indirectly to exponents chapter of grade 7
2. Here we learn to find squares and square roots of numbers
3. It connects to most of the arithmetic chapters of future grades

2.

LEARNING OUTCOMES

<On completion of the chapter, students will be able to:>

1. Recap of square of a number
2. Acquire the knowledge of properties of squares
3. Examine interesting patterns of numbers
4. Find square of a number
5. Examine pythagorean triplets
6. Find square root of a number[by repeated subtraction, by prime factorisation, long division]
7. Finds the square root of decimal

3. TEACHING PLAN

Follow this section for getting ideas on teaching the content covered in the chapter. The subtopic given in the first column with the matching learning outcome can be explained using instructions and resources listed subsequently

Subtopic and Learning Objective #1

Subtopic:Square of a number
LO: Recap of square of a number
1.To make the student recollect square of a number

Teacher Activity

Recap square of a number with examples

Check for understanding:

a) In a^2 a is called _____, 2 is called _____
b) Square of 123 is _____
c) 729 is square of_____

Resources Suggested

Blackboard, pen and paper
Resource 1 can be used for explanation

Resource 1

Square: Number obtained when a number is multiplied by itself. It is the number raised to the power 2.

$2^2 = 2 \times 2 = 4$(square of 2 is 4).

If a natural number m can be expressed as n^2, where n is also a natural number, then m is a square number.

Perfect Square or Square number: It is the square of some natural number.

If $m = n^2$, then m is a perfect square number where m and n are natural numbers.

Example: $1 = 1 \times 1 = 1^2$, $4 = 2 \times 2 = 2^2$.

Properties of Square number:

1) A number ending in 2, 3, 7 or 8 is never a perfect square. Example: 152, 1028, 6593 etc
2) All square numbers end with 0, 1, 4, 5, 6 or 9 at unit's place but A number ending in 0, 1, 4, 5, 6 or 9 may not necessarily be a square number. Example: 20, 31, 24, etc.
3) Square of even numbers are even. Example: $2^2 = 4$, $4^2 = 16$
4) Square of odd numbers are odd. Example: $5^2 = 25$, $9^2 = 81$, etc.
5) A number ending in an odd number of zeroes cannot be a perfect square that is square numbers can only have even number of zeros at the end. Example: 10, 1000, 900000, etc. can not be perfect squares

Resource 1

1) The difference of squares of two consecutive natural number is equal to their sum.
   $(n + 1)^2 – n^2 = n+1+n$. Example: $4^2 – 3^2=4 + 3=7$. $12^2 – 11^2=12+11 =23$, etc.
2) A triplet (m, n, p) of three natural numbers m, n and p is called Pythagorean triplet, if $m^2 + n^2 = p^2$: $3^2 + 4^2 = 25 = 5^2$

   An interesting fact: a Pythagorean Triple always consists of:
   1. All even numbers, or two odd numbers and an even number.
   2. A Pythagorean Triple can never be made up of all odd numbers or two even numbers and one odd number. This is true because:

The square of an odd number is an odd number and the square of an even number is an even number.
The sum of two even numbers is an even number and the sum of an odd number and an even number is in odd number.

So, when both a and b are even, c is even too. Similarly when one of a and b is odd and the other is even, c has to be odd!

Constructing Pythagorean Triples: It is easy to construct sets of Pythagorean Triples.

When **m** and **n** are any two positive integers (m > n):

$a = m^2 - n^2$

$b = 2nm$

$c = m^2 + n^2$ Then a, b, and c form a Pythagorean Triplet.

Resource 1

Note : To check whether the given number is a member of pythagorean triplet it must satisfy one of the above conditions and can find the other two numbers

1. Check whether 25 is pythagorean triplet or not?

25= 3^2+4^2 So, pythagorean triplets can be 4^2-3^2, 2x3x4. That is 7, 24, 25 can be pythagorean triples

25= $13^2 - 12^2$ so other members are 13^2+12^2=313, 2 x 13 x 12=312. That is 25, 312, 313 are pythagorean triples

2. Check whether 12 is a member of pythagorean triplet or not.

12= 2 x 3 x 2. So the members are 3^2+2^2 and $3^2- 2^2$ that is 13 and 5. That is 5,12,13 are pythagorean triples.

Subtopic and Learning Objective #2

Sub topic:Properties of squares
LO:
Acquire the knowledge of properties of squares
1. To make student understand the properties of squares

Teacher Activity

Introduce properties of squares

Check for understanding:

a) Square of any number ends with_____

b) If 9 is the unit's digit of a number then its square number ends with_____

c) If a number ends with 2 zeros then square number ends with _____number of zeros

d) If n-1 and n+1 are two consecutive odd numbers then product of them is _____

Resources Suggested

Blackboard, pen and paper
Resource 1 can be used for explanation

Subtopic and Learning Objective #3

Sub topic: Number patterns
LO:
Examine interesting patterns of numbers
1. To make student know about the patterns in numbers

Teacher Activity

Introduce number patterns with examples

Check for understanding:

a) Complete next

b) four steps of following pattern

$4^2 = 16$

$34^2 = 1156$

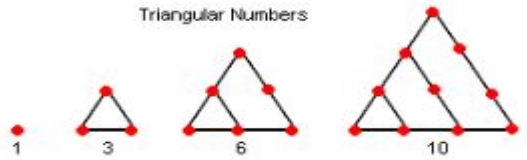
$334^2 = 111556$

Search for some more patterns and make students practice

Resource 2

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1.Triangular numbers are those whose dot patterns can be arranged as triangles. We can find an interesting pattern in them.



$1+3=4=2^2$

$3+6=9=3^2$

$6+10=16=4^2$

We can observe that sum of two consecutive triangular numbers is a perfect square.

2. The sum of first n odd natural numbers is n^2. In other words if a natural number cannot be expressed as a sum of successive odd natural numbers starting with 1, then it is not a perfect square.

Example: $1=1=1^2$

$1+3=4=2^2$

$1+3+5=9=3^2$

$1+3+5+7=16=4^2$

$1+3+5+7+11+13+.....+n$ odd numbers$= n^2$

Resource 2

3.Difference of squares of two consecutive numbers is sum of the numbers

Example: $2^2 - 1^2 = 4-1 = 3$[which is sum of 2 and 1]

$3^2 - 2^2 = 9-4 = 5$ [Which is sum of 3 and 2]

$4^2 - 3^2 = 16-9 = 7$ [Which is sum of 4 and 3]

Similarly $(n+1)^2 - n^2 = n+1+n = 2n+1$

4. The square of any odd number is the sum of two consecutive positive integers.

Example: $3^2 = 9 = 4+5$

$5^2 = 25 = 12+13$

$7^2 = 49 = 24+25$

Resource 2

1. Product of two consecutive even or odd natural numbers is difference of squares of between number and 1

   Example: 11 x 13 = 143 = $12^2 - 1^2$

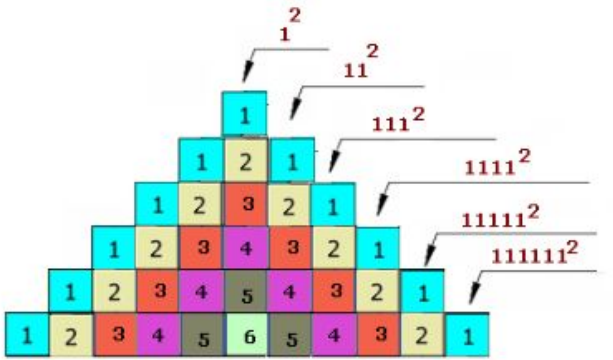
   12 x 14 = 168 = $13^2 - 1^2$

   24 x 26 = 624 = $25^2 - 1^2$

   In general $(a+1)(a-1) = a^2 - 1^2$

2. **Let us look into some square patterns:**

   Interesting pattern of squares of 1, 11, 111, 1111, ……



Resources Suggested

Blackboard, pen and paper
Resource 2 can be used for explanation

Subtopic and Learning Objective #4

Sub topic:Pythagorean triples
LO:
Examine pythagorean triplets
1. To make student
Understand concept of pythagorean triples

Teacher Activity

Introduce pythagorean triples with good number of examples

Check for understanding:

a) Check whether 36 is a member of pythagorean triples or not.
b) Check whether 20,99,101 form pythagorean triples or not

[Give more number of examples to give more

Resources Suggested

Blackboard, pen and paper
Resource 2 can be used for explanation

Subtopic and Learning Objective #5

Sub topic:Square root of a number
LO: **Finds square root of a number[by repeated subtraction, by prime factorisation, long division]**
1. To make student to find the square root by repeated subtraction

2. To make the student to find square root by prime factorisation

3. To make the student to find square root by long division

Teacher Activity

Introduce method of finding square root by repeated subtraction

Introduce method of finding square root by Prime factorisation

Introduce method of finding square root by long division

Check for understanding:

a) Find the square root of 1225 by prime factorisation
b) Find the square root of 121 by repeated subtraction
c) Find the square root of 1500625 by long division

Resources Suggested

Blackboard, pen and paper
Resource 3 can be used for explanation

Resource 3

Square roots: Square root is the inverse operation of square. It is denoted by the symbol

There are two integral square roots of a perfect square number.

Example: $\sqrt{9}$ =+3 or -3. Since $(+3)^2=9$ and $(-3)^2=9$

Positive square root of a number is known as **principal square root**

Square roots can be found by 3 methods.

1. Repeated subtraction
2. Prime factorisation
3. Long division

Repeated subtraction: Square root of a number can be found by **repeatedly subtracting** successive odd numbers(starting from 1) from the given **square** number, till you get zero.

Example: 81-1=80; 80-3=77; 77-5=72; 72-7=65; 65-9=56; 56-11=45; 45-13=32; 32-15=17; 17-17=0

Resource 3

So, $81 = 1+3+5+7+9+11+13+15+17 = 81 = 9^2 \Rightarrow$

Or in much simpler way how many consecutive odd numbers that are being subtracted to get zero, that number itself will be the square root

$$\sqrt{81} = \sqrt{9^2} = 9$$

Prime factorisation method:

1. Resolve the given number into prime factors.
2. Make pairs of similar factors.
3. Take the product of prime factors, choosing one factor out of every pair.

Example: The square root of 324 by prime factorization, we get

$324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$

$\sqrt{324} = \sqrt{(\underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3})}$

$= 2 \times 3 \times 3$

2	324
2	162
3	81
3	27
3	9
3	3
	1

Resource 3

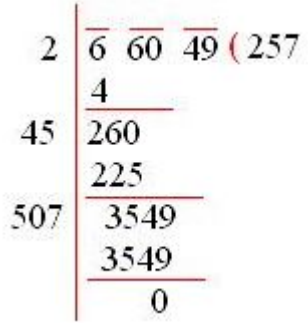
Long division Method:

1. Group the digits in pairs, starting with the digit in the units place. Each pair and the remaining digit (if any) is called a period.
2. Think of the largest number whose square is equal to or just less than the first period. Take this number as the divisor and also as the quotient.
3. Subtract the product of the divisor and the quotient from the first period and bring down the next period to the right of the remainder. This becomes the new dividend.
4. Now, the new divisor is obtained by taking two times the quotient and annexing with it a suitable digit which is also taken as the next digit of the quotient, chosen in such a way that the product of the new divisor and this digit is equal to or just less than the new dividend.
5. Repeat steps (2), (3) and (4) till all the periods have been taken up. Now, the quotient so obtained is the required square root of the given number.

Resource 3

Example: Evaluate √66049.

Therefore, √66049 = 257

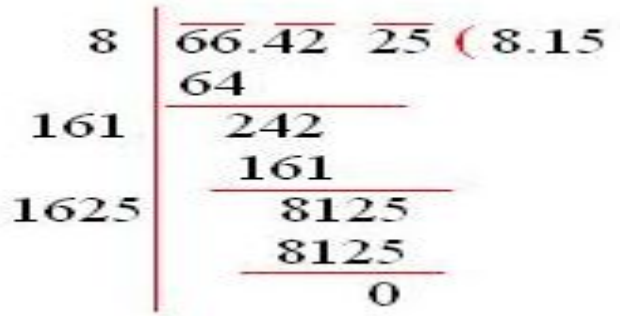


Square root of decimals:

1. Make the number of decimal places even by affixing a zero on the extreme right of the decimal part (if required).
2. In the integral part, mark the periods as done while finding the square root of a perfect square of some natural number.
3. In the decimal part, mark the periods on every pair of digits beginning with the first decimal place.
4. Now, find the square root by long division method.
5. Put the decimal point in the square root as soon as the integral part is exhausted.

Resource 3

Example: Evaluate: √66.4225



Therefore, √66.4225 = 8.15

Subtopic and Learning Objective #5

LO
Introduce the method of finding square root of decimal

Check for understanding:

a) Find the square root of 158.76
b) Evaluate $\sqrt{0.4225}$

Teacher Activity

Introduce the method of finding square root of decimal

Check for understanding:

a) Find the square root of 158.76
b) Evaluate $\sqrt{0.4225}$

Resources Suggested

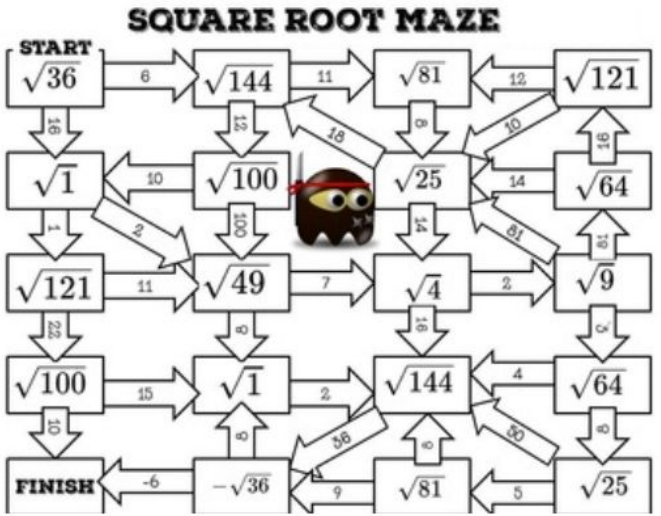
Blackboard, pen and paper

Resource 3 for reference

Teacher Activity

Name of the activity- Solve the puzzle
Type of activity – Individual
Use it for – Practice
Materials – Puzzle paper
Procedure:

1. Ask the students to solve the puzzle by reaching finish point from starting point
2. Need to follow the square root to reach next number



4. WORKSHEETS

Print or copy the worksheet questions on the blackboard and ask the students to attempt them. They can be given as homework or a class test as well.

Worksheet 1

Use it like: test after the chapter is complete, homework

Answer the following:

1. Express 64 as the sum of eight odd numbers.
2. Express 121 as the sum of eleven odd numbers.
3. Which of the following are the squares of odd numbers?
   (i) 121 (ii) 225 (iii) 196 (iv) 484 (v) 529
4. Which of the following are the squares of even numbers?
   (i) 196 (ii) 441 (iii) 400 (iv) 324 (v) 625 (vi) 169
5. Without any calculation check which of the following is perfect square give reasons?
   (i)62 (ii)143 (iii) 217 (iv) 168 (v) 90 (vi) 4000

Worksheet 2

Use it like: test after the chapter is complete, homework

Answer the following:

1. Fill the next two steps of the following patterns

$7^2 = 49$

$67^2 = 4489$

$667^2 = 444889$

2 $121 = \dfrac{22^2}{1+2+1}$

$12321 = \dfrac{(333)^2}{1+2+3+2+1}$

3.. $9^2 = 81$

$99^2 = 9801$

$999^2 = 998001$

Worksheet 3

Use it like: test after the chapter is complete, homework

Answer the following:

1. Which of the following is a Pythagorean triplet?
   (a) (2, 3, 5) (b) (5, 7, 9) (c) (6, 9, 11) (d) (8, 15, 17)
2. Write a Pythagorean triplet whose smallest member is
   (i) 6 (ii) 14 (iii) 16 (iv) 20

Worksheet 4

Use it like: test after the chapter is complete, homework

Answer the following:

1. Evaluate √11025 by prime factorisation method
2. Find √121 by repeated subtraction
3. Evaluate: √10609. By long division method
4. Find the smallest number by which 396 must be multiplied so that the product becomes a perfect square.
5. Find the smallest number by which 220 must be divided so that the quotient is a perfect square.
6. Find the least number that must be added to 6412 to make it a perfect square.
7. Evaluate√0.00059049
8. Evaluate: √0.4225

5. SOLUTIONS

This section gives you sample answers for activities and questions listed above.

Worksheet 1

1. $64 = 8^2 = (1 + 3 + 5 + 7 + 9 + 11 + 13 + 15)$
2. $121 = (11)^2 = (1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21)$
3. I, ii and v
4. I , iii and iv
5.
   (i) 2 in units place so can not be perfect square
   (ii) 3 in units place so can not be perfect square
   (iii) 7 in units place so can not be perfect square
   (iv) 8 in units place so can not be perfect square
   (v) odd number of zeros so can not be perfect square
   (vi) odd number of zeros so can not be perfect square

Worksheet 2

1.

$$6667^2 = 44448889$$
$$66667^2 = 4444488889$$

2.

$$1234321 = \frac{(4444)^2}{1+2+3+4+3+2+1}$$

$$123454321 = \frac{(55555)^2}{1+2+3+4+5+4+3+2+1}$$

3.

$$9999^2 = 99980001$$
$$99999^2 = 999980001$$

Worksheet 3

1. (8,15,17)
2. (i) (6, 8, 10) (ii) (14, 48, 50) (iii) (16, 63, 65) (iv) (20, 99, 101)

Worksheet 4

1. 105
2. 11
3. 103
4. 11
5. 55
6. 149
7. 0.0243
8. 0.65

6. EXTRA RESOURCES

Carry out the following suggested activities/ tasks if you have the time and resources to do them

Teacher can use these videos if time and facilities permit:

1. https://www.youtube.com/watch?v=FnecJm7Ea5c pythagorean triplets