NCERT SOLUTIONS

CLASS - 9th

Class : 9th
Subject : Maths
Chapter : 10
Chapter Name : CIRCLE

Exercise 10.1

Q1 Fill in the blanks:
(i) The centre of a circle lies in_ of the circle. (exterior/ interior)
(ii) A point, whose distance from the centre of a circle is greater than its radius lies in _of the circle. (exterior/ interior)
(iii) The longest chord of a circle is a _ of the circle.
(iv) An arc is a _ when its ends are the ends of a diameter.
(v) Segment of a circle is the region between an arc and _ of the circle.
(vi) A circle divides the plane, on which it lies, in _ parts.

Answer. (i) The centre of a circle lies in interior of the circle.
(ii) A point, whose distance from the centre Of a circle is greater than its radius lies In exterior of the circle.
(iii) The longest chord of a circle is a diameter of the circle.
(iv) An arc is a semi-circle when its ends are the ends of a diameter.
(v) Segment of a circle is the region between an arc and chord of the circle.
(vi) A circle divides the plane, on which it lies, in three parts.

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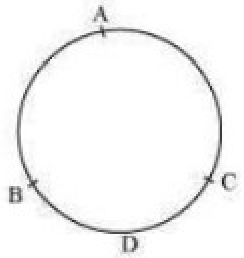
Q2 Write True or False: Give reasons for your answers.
(i) Line segment joining the centre to any point on the circle is a radius of the circle.
(ii) A circle has only finite number of equal chords.
(iii) If a circle is divided into three equal arcs, each is a major arc.
(iv) A chord of a circle, which is twice as long as its radius, is a diameter of the circle.
(v) Sector is the region between the chord and its corresponding arc.
(vi) A circle is a plane figure

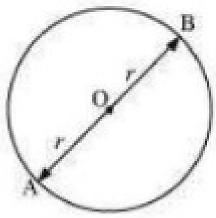
Answer. (i) True. All the points on the circle are at equal distances from the centre of the circle, and this equal distance is called as radius of the circle.

(ii) False. There are infinite points on a circle. Therefore, we can draw infinite number of chords of given length. Hence, a circle has infinite number of equal chords.

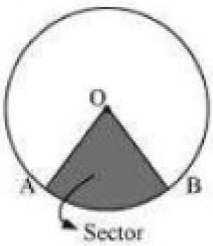
(iii) false. Consider three arcs of same length as AB, BC, and CA. It can be observed that for minor arc BOC, CAB is a major arc. Therefore, A3, BC, and CA are minor arcs of the circle.



(iv) True. Let AB be a chord which is twice as long as its radius. It can be observed that in this situation, our chord will be passing through the centre of the circle. Therefore, it will be the diameter of the circle.



(v) False. Sector is the region between an arc and two radii joining the centre to the end points of the arc. For example, in the given figure, OAB is the sector of the circle.



(vi) True. A circle is a two-dimensional figure and it can also be referred to as a plane figure.

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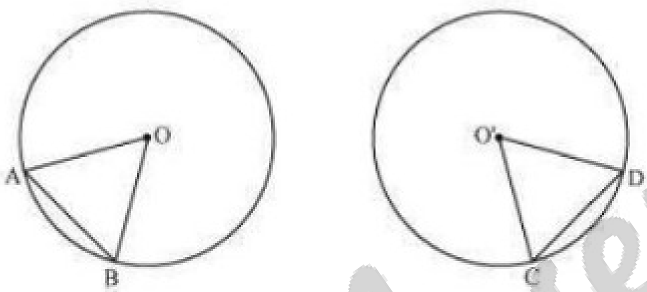
Exercise 10.2

Q1 Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.

Answer. A circle is a collection of points which are equidistant from a fixed point. This fixed point is called as the centre of the circle and this equal distance is called as radius of the circle. And thus, the shape of a circle depends on its radius. Therefore, it can be observed that if we try to superimpose two circles of equal radius, then both circles will cover each other. Therefore, two circles are congruent if they have equal radius.
Consider two congruent circles having centre O and O' and two chords AB and CD of equal lengths.
In $\triangle \mathrm{AOB}$ and $\triangle \mathrm{CO'D}$
AB = CD (Chords of same length)
OA = O'C (Radii of congruent circles)
0B = O'D (Radii of congruent circles)
$\triangle \mathrm{AOB}$ $\triangle \mathrm{CO'D}$ (SSS congruence rule)
$\angle AOB = \angle CO'D$ (By CPCT)
Hence, equal chords of congruent circles subtend equal angles at their centres.



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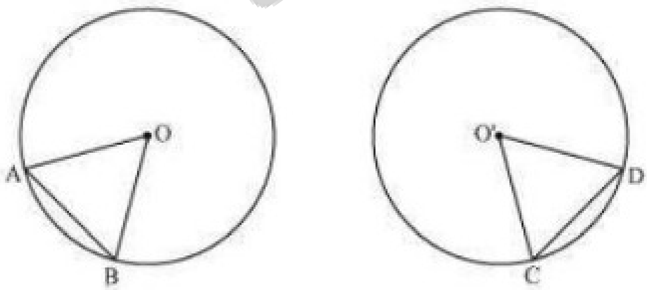
Q2 Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.



Answer.

In $\triangle \mathrm{AOB}$ and $\triangle \mathrm{CO'D}$
$\angle AOB = \angle CO'D$ (given)
OB = O'D (Chords of same length)
OA = O'C (Radii of congruent circles)
$\triangle \mathrm{AOB}$ $\triangle \mathrm{CO'D}$ (SSS congruence rule)
AB=CD (By CPCT)

Hence , if the chord of congruent circle equal angles at their centers, then the chords are equals.

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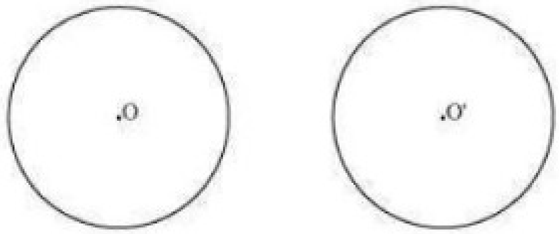
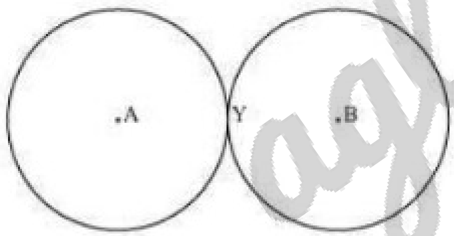
Exercise 10.3

Q1 Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common points?

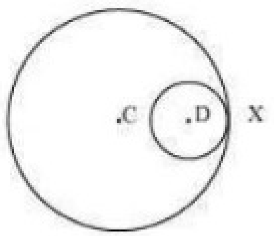
Answer. Consider of the following pair of circles.



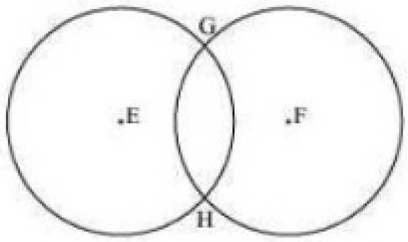
The above circles do not intersect each other at any point . therefore, they do not have any point in common.



The above circles touch each other only at one point Y. therefore, there is 1 point in common.



The above circles touch each other at 1 point X only. Therefore the circles have one point in common.



These circles intersect each other at two points G and H. Therefore, the circles have two points in common. It can be observed that there can be a maximum of 2 points in common. Consider the situation in which two congruent circles are superimposed on each other. This situation can be referred to as if we are drawing the circle two times.

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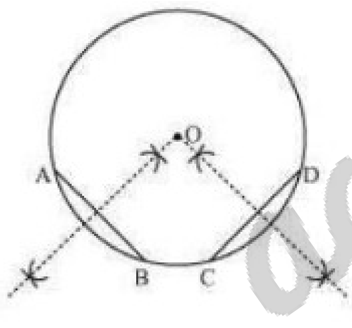
Q2 Suppose you are given a circle. Give a construction to find its centre.

Answer. The below given steps will be followed to find the centre of the given circle.
Step1. Take the given circle.
Step2. Take any two different chords Ad and CD of this circle and draw perpendicular bisectors of these chords.
Step3. Let these perpendicular bisectors meet at point O. Hence, O is the centre of the given circle.



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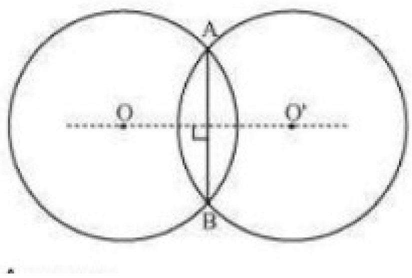
Q3 If two circles intersect at two points, prove that their centres lie on the perpendicular bisector of the common chord.



Answer. Consider two circles centered at point O and O', intersecting each other at point A and B

respectively.
Join AB. AB is the chord of the circle centered at O. Therefore, perpendicular bisector of AB will pass through O.
Again, AB is also the chord of the circle centered at O'. Therefore, perpendicular bisector of AB will also pass through O'.
Clearly, the centres of these circles lie on the perpendicular bisector of the common chord.

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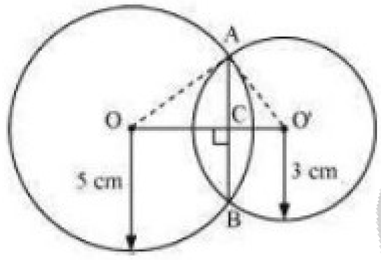
Exercise 10.4

Q1 Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.

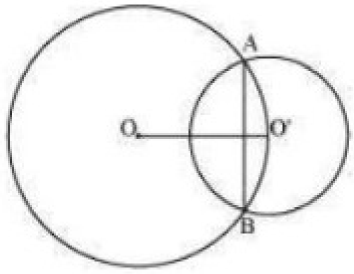
Answer.



Let the radius of the circle centered at O and O' be 5 cm and 3 cm respectively.
OA = OB=5 cm
O'A= O'B = 3 cm

00' will be the perpendicular bisector of chord AB.
AC=CB
It is given that, 00' = 4 cm
Let OC be x. Therefore, O'C will be 4 - x .
In$\triangle \mathrm{OAC}_1$
$\mathrm{OA}^2=\mathrm{AC}^2+\mathrm{OC}^2$
$5^2=\mathrm{AC}^2+\mathrm{x}^2$
$\Rightarrow 25-x^2=\mathrm{AC}^2$.........(1)
In $\triangle \mathrm{O}'\mathrm{AC}_1$
$\mathrm{O}'\mathrm{A}^2=\mathrm{AC}^2+\mathrm{O}'\mathrm{C}^2$
$3^2=\mathrm{AC}^2+(4-\mathrm{x})^2$
$9=AC^2+16+x^2-8x$
$AC^2=-x^2-7+8x$.......(2)
From equations (1) and (2) , we obtain

$25-x^{2}=-x^{2}-7+8x$
8X=32
x=4
Therefore, the common chord will pass through the centre of the smaller circle i.e., O' and hence, it will be the diameter of the smaller circle.



$AC^2 = 25 - x^2 = 25 - 4^2 = 25 - 16 = 9$
AC= 3m
Length of the common chord AB = 2AC =(2x3)m = 6m

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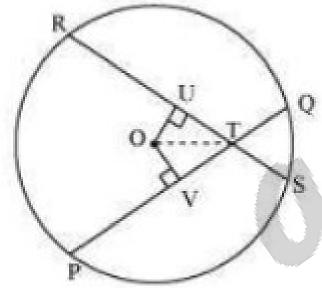
Q2 If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

Answer. Let PQ and RS be two equal chords of a given circle and they are intersecting each other at point T.



Draw perpendiculars OV and OU on these chords.
In ΔOVT and $DeltaOUT$
OV =OU (Equal chords of a circle are equidistant from the centre)
$\angle \mathrm{OVT} = \angle \mathrm{OUT}$ (each 90 degree)
OT=OT (common)
$\Delta OVT \equiv \Delta OUT$ (RHS congruence rule)
VT=UT (By CPCT)……(1)
It is given that,
PQ=RS……..(2)
$\frac{1}{2}\mathrm{PQ} = \frac{1}{2}\mathrm{RS}$
PV=RU……..(3)
On adding equation (1) and (3) , we obtain
PV + VT = RU + UT

PT= RT......(4)
On subtracting equation (4) from equation (2),we obtain
PQ - PT = RU + UT
QT = ST......(5)
Equations (4) and (5) indicate that the corresponding segments Of chords PQ and RS are congruent to each other.

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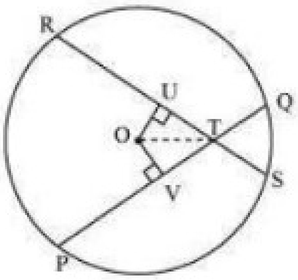
Q3 If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

Answer.



Let PQ and RS are two equal chords of a given circle and they are intersecting each other at point T.
Draw perpendiculars OV and OU on these chords.
In ΔOVT and $Delta OUT$
OV = OU(Equal chords of a circle are equidistant from the centre)
$\angle \mathrm{OVT} = \angle \mathrm{OUT}$ (each 90 degree)
OT=OT (common)
$\Delta OVT = \Delta OUT$ (RHS congruence rule)
$\square OTV = \square OTU$
Therefore, it is proved that the line joining the point Of intersection to the centre makes equal angles with the chords.

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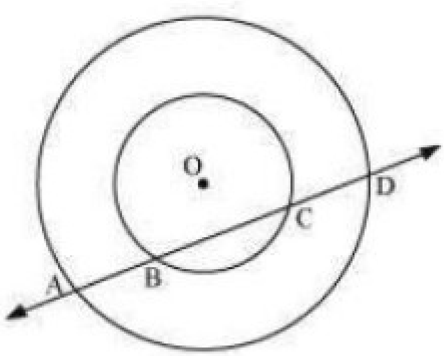
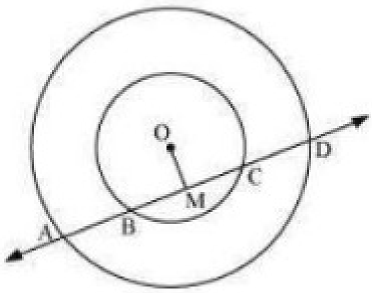
Q4 If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see Fig. 10.25).



Answer. Let us draw a perpendicular OM on line AD.



It can be observed that BC is the chord Of the smaller circle and AD is the chord Of the bigger circle.
We know that perpendicular drawn from the centre of the circle bisects the chord.
$\square \mathrm{BM} = \mathrm{MC} \ldots (1)$
$\text{And, AM} = \mathrm{MD} \ldots (2)$
On subtracting equation (2) from (1), we obtain
AM - BM = MD - MC
$\square AB = CD$

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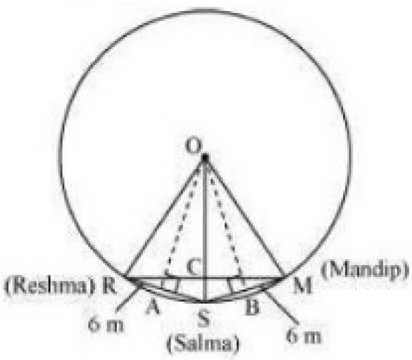
Q5 Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6m each, what is the distance between Reshma and Mandip?

Answer. Draw perpendiculars OA and 03 on RS and SM respectively.



$\mathrm{AR} = \mathrm{AS} = \frac{6}{2} = 3\mathrm{m}$
OR =OS OM =5m (radi of the circle)
In ΔOAR_1
$\mathrm{OA}^2 + \mathrm{AR}^2 = \mathrm{OR}^2$
$\mathrm{OA}^2 + (3\mathrm{m})^2 = (5\mathrm{m})^2$
$\mathrm{OA}^2 = (25 - 9)\mathrm{m}^2 = 16\mathrm{m}^2$
OA =4m
{ ORSM will be a kite }(\mathrm{OR}=\mathrm{OM} \text { and } \mathrm{RS}=\mathrm{SM}\ ).
We know that the diagonals of a kite are perpendicular and the diagonal common to both the isosceles triangles is bisected by another diagonal.
□□RCS will be of 90° and $RC = CM$
Area of $\Delta\mathrm{ORS} = \frac{1}{2} \times \mathrm{OA} \times \mathrm{RS}$
$\frac{1}{2} \times \mathrm{RC} \times \mathrm{OS} = \frac{1}{2} \times 4 \times 6$
RC x 5 = 24
RC = 4.8

RM = 2 RC = 2(4.8) = 9.6

Therefore, the distance between Reshma and Mandip is 9.6 m.

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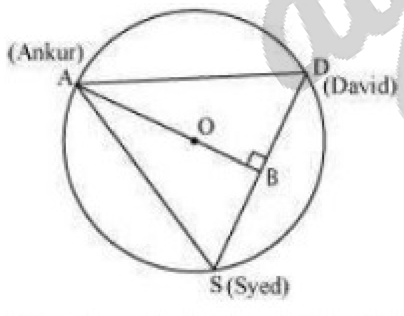
Q6 A circular park of radius 20m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Answer.



It iS Given that AS = SD - DA
Therefore, $\triangle\mathrm{ASD}$ is an equilateral triangle.
OA (radius) = 20 m
Medians of equilateral triangle pass through the circum entre (O) of the equilateral triangle ASD. We also know that medians intersect each other in the ratio 2: 1. As AB is the median of equilateral triangle ASD, we can write
$\Rightarrow \frac{\mathrm{OA}}{\mathrm{OB}} = \frac{2}{1}$
$\Rightarrow \frac{20\mathrm{m}}{\mathrm{OB}} = \frac{2}{1}$

$\Rightarrow \mathrm{OB} = \left(\frac{20}{2}\right) \mathrm{m} = 10\mathrm{m}$
$\square AB = OA + OB = (20 + 10)m = 30\mathrm{m}$
$\triangle\mathrm{ABD}$,
$\mathrm{AD}^2 = \mathrm{AB}^2 + \mathrm{BD}^2$
$AD^2 = (30)^2 + \left(\frac{AD}{2}\right)^2$
$\mathrm{AD}^2 = 900 + \frac{1}{4}\mathrm{AD}^2$
$\frac{3}{4}\mathrm{AD}^2 = 900$
$\mathrm{AD}^2 = 1200$
$\mathrm{AD} = 20\sqrt{3}$
Therefore , the length of the string of each phone will be $20\sqrt{3}\mathrm{m}$

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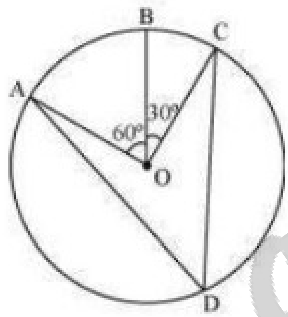
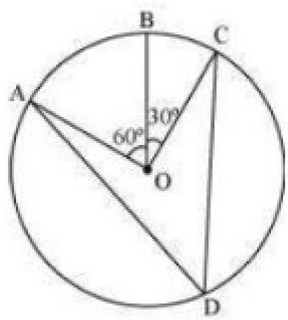
Exercise 10.5

Q1 In Fig. 10.36, A,B and C are three points on a circle with centre O such that ∠ BOC = 30° and ∠ AOB = 60°. If D is a point on the circle other than the arc ABC, find ∠ADC.



Answer.



It can observed that
$\square AOC = \square AOB + \square BOC$
$b \; = 60^\circ + 30^\circ$
$= 90^\circ$
We know that angle subtended by an arc at the centre is double the angle subtended

by it any point on the remaining part of the circle.
$\angle \mathrm{ADC} = \frac{1}{2} \angle \mathrm{AOC} = \frac{1}{2} \times 90^{\circ} = 45^{\circ}$

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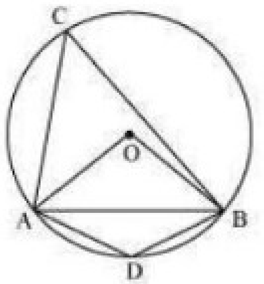
Q2 A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

Answer.



In $\triangle OAB$
OA =OB =AB (radius)
□$\triangle$OAB is an equilateral triangles.
□□$AOB = 60^{\circ}$
$\angle \mathrm{ACB} = \frac{1}{2} \angle \mathrm{AOB} = \frac{1}{2}(60^{\circ}) = 30^{\circ}$
In cyclic equilateral ABCD,
□$ACB +$ □$ADB = 180^{\circ}$ (opposite angle in cyclic quadrilateral)
□□$ADB = 180^{\circ} - 30^{\circ} = 150^{\circ}$
Therefore , angle subtended by this chord at a point on the major arc and the minor arc are \(30^{\circ} \text { and } 150^{\circ}) respectively

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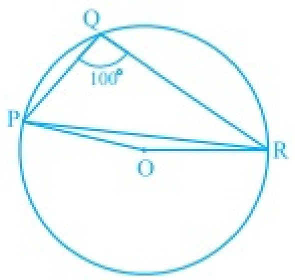
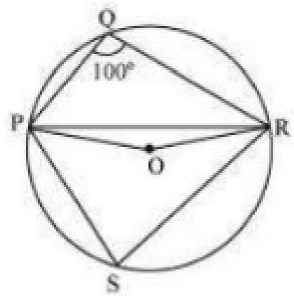
Q3 In Fig. 10.37, ∠ PQR = 100°, where P, Q and R are points on a circle with centre O. Find ∠ OPR.



Answer.



Consider PR as a chord of the circle.
Take any point S on the major arc of the circle.
PQRS is a cyclic quadrilateral.

$\square \mathrm{PQR} + \square \mathrm{PSR} = 180^\circ$
$\square\square \mathrm{PSR} = 180^\circ - 100^\circ$ =80

We know that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$\square\square POR = 2\square PSR = 2\left(80^\circ\right) = 160^\circ$

In $\triangle \mathrm{POR}$

OP=OR (radii of the same circle)

$\square\square OPR = \square ORP$ (Angles opposite to equal sides of a triangle)

$2\square OPR + 160^\circ = 180^\circ$

$2\square OPR = 180^\circ - 160^\circ = 20^\circ$

$\square \mathrm{OPR} = 10^\circ$

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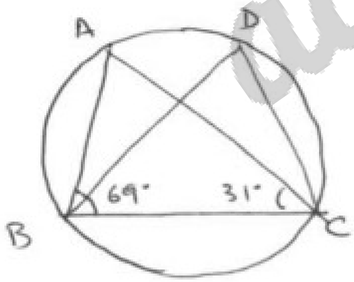
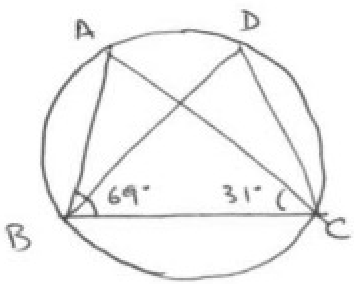
Q4 In Fig. 10.38, ∠ ABC = 69°, ∠ ACB = 31°, find ∠ BDC.



Answer.



In $\Delta\mathrm{ABC}, \angle\mathrm{ABC} = 69^\circ, \angle\mathrm{ACB} = 31^\circ$
So, $\angle BAC = 180^\circ - (69^\circ + 31^\circ)$
$= 180^\circ - 100^\circ$
$= 80^\circ$
$\angle BDC = \angle BAC = 80^\circ$ (at same are BC)

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Q5 In Fig. 10.39, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that ∠ BEC = 130° and ∠ ECD = 20°. Find ∠ BAC.

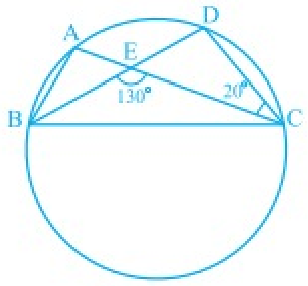

Fig. 10.39

Answer. In $\triangle\mathrm{CDE}$
$\square\mathrm{CDE} + \square\mathrm{DCE} = \square\mathrm{CEB}$(exterior angle
$\square\square CDE + 20^\circ = 130^\circ$
$\square\square CDE = 110^\circ$
However , $\square BAC = \square CDE$ (angles in the same segment of a circle)
$\square\square BAC = 110^\circ$

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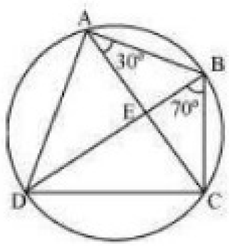
Q6 ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If ∠ DBC = 70°, ∠ BAC is 30°, find ∠ BCD. Further, if AB = BC, find ∠ ECD.

Answer.



$\square\mathrm{CBD} = \square\mathrm{CAD}$
$\square CAD = 70^\circ$
$\square BAD = DBAC + \square CAD = 30^\circ + 70^\circ = 100^\circ$
$\square BCD + DBAD = 180^\circ$
$\square BCD + 100^\circ = 180^\circ$

$\square BCD = 80^\circ$

In $\triangle$ABC
$AB = BC$(Given)
$\square\square BCA = \square CAB$
$\square\square BCA = 30^\circ$
We have $\square BCD = 80^\circ$
$\square\square BCA + \square ACD = 80^\circ$
$30^\circ + \square ACD = 80^\circ$
$\square\square ACD = 50^\circ$
$\square\square ECD = 50^\circ$

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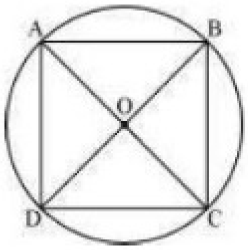
Q7 If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

Answer.

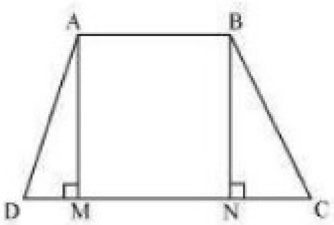


Let ABCD be a cyclic quadrilateral having diagonals BD and AC, intersecting each other at point O.
$\angle BAD = \frac{1}{2}\angle BOD = \frac{180^\circ}{2} = 90^\circ$
(consider AC is chord)
$\square ADC + \square ABC = 180^\circ$ (cyclic quadrilateral)
$90^\circ + \square ABC = 180^\circ$
$\square ABC = 90^\circ$
Each interior angle of a cyclic quadrilateral is of 90 degree. Hence, it is a rectangle.

Page : 185 , Block Name : Exercise 10.5

Q8 If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

Answer.



Consider a trapezium ABCD with AB I ICD and BC AD.
□CD and BN□CD.
AM =BM (perpendicular distance between two parallel lines is same)
$\square\triangle AMD \square \triangle BNC$ (RHS congruence rules)
$\square\square ADC = \square BCD$ (CPCT)..........(1)
$\square BAD$ and $\square ADC$ are on the same side of transversal AD
$\square BAD + \square ADC = 180^{\circ}$........(2)
$\square BAD + \square BCD = 180^{\circ}$ [using equation .....(1)]
This equation shows that the opposite angles are supplementary.
Therefore, ABCD is a cyclic quadrilateral.

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Q9 Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see Fig. 10.40). Prove that ∠ACP = ∠ QCD.

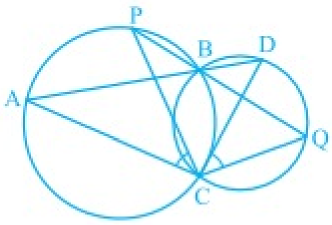
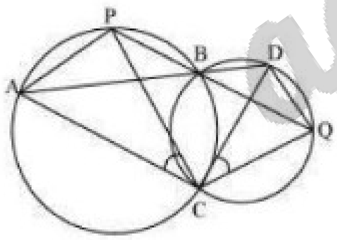


Fig. 10.40

Answer.



Join chords AP and DQ.
For chord AP,
$\square PBA = \square ACP$ (Angles in the same segment) ....(1)
For chord DQ,
$\square DBQ = \square QCD$ (Angles in the same segment) ....(2)
ABD and PBQ are line segments intersecting at B.
$\square PBA = \square ACP$ (Angles in the same segment) ....(3)
From equation (1) ,(2) ,and (3),we obtain
$\square ACP = \square QCD$

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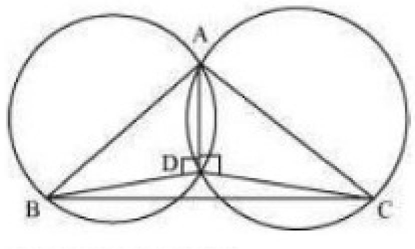
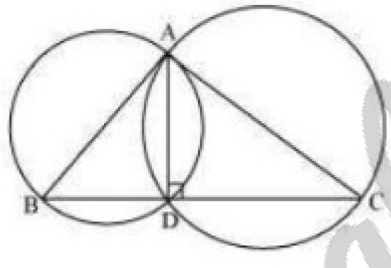
Q10 If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

Answer.



consider a $\triangle$ABC.
Two circles are drawn while taking AB and AC as the diameter.
Let they intersect each other at D and let D not lie on BC.
join AD.
$\square ADB = 90^\circ$ (angle subtended by semi - circle)
$\square ADC = 90^\circ$ (angle subtended by semi - circle)
$\square BDC = \square ADB + \square ADC = 90^\circ + 90^\circ = 180^\circ$
Therefore, BDC is a straight line and hence, our assumption was wrong.
Thus, Point D lies on third side BC of $\triangle ABC$.



Page : 186 , Block Name : Exercise 10.5

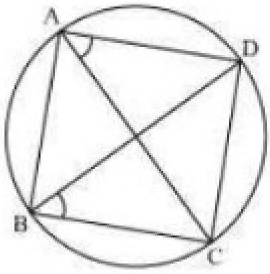
Q11 ABC and ADC are two right triangles with common hypotenuse AC. Prove that
∠ CAD = ∠ CBD.

Answer.
$\triangle$ABC
$\square ABC + \square BCA + \square CAB = 180^\circ$ (Angle sum property of a triangle)

$\square 90^\circ + \square BCA + \square CAB = 180^\circ$
$\square\square BCA + \square CAB = 90^\circ$........(1)
$\triangle$ADC
$\square CDA + \square ACD + \square DAC = 180^\circ$(Angle sum property of a trinangle)

$\square 90^\circ + \square ACD + \square DAC = 180^\circ$
$\square\square ACD + \square DAC = 90^\circ$........(2)

Adding equation (1) and (2) ,we obtain
$\square BCA + \square CAB + \square ACD + \square DAC = 180^\circ$
$\square(\square BCA + \square ACD) + (\square CAB + \square DAC) = 180^\circ$
$\square BCD + \square DAB = 180^\circ$.......(3)
However , it is given that
$\square B + \square D = 90^\circ + 90^\circ = 180^\circ$.......(4)
From equations (3) and (4), it can be observed that the sum of the measures of opposite angles Of quadrilateral A3CD is 1800. Therefore, it is a cyclic quadrilateral.
Consider chord CD.
$\square CAD = \square CBD$ (angle in the same segment)



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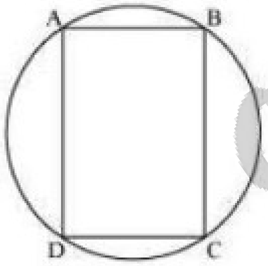
Q12 Prove that a cyclic parallelogram is a rectangle.

Answer.



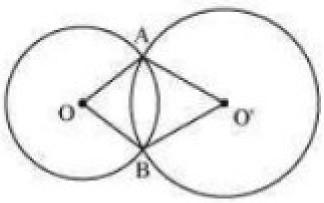
Let ABCD be a cyclic parallelogram.
$\square A + \square C = 180^\circ$
We know that opposite angles Of a parallelogram are equal.
$\square\square A = \square C$ and $\square B = \square D$
From equation (1),
$\square A + \square C = 180^\circ$
$\square\square A + \square A = 180^\circ$
$\square 2\square A = 180^\circ$
$\square\square A = 90^\circ$
Parallelogram ABCD has one of its interior angles as 900. Therefore, it is a rectangle.

Page : 186 , Block Name : Exercise 10.5

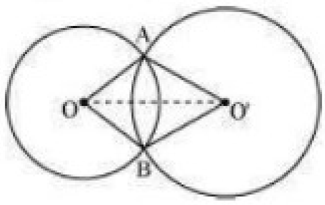
Exercise 10.6

Q1 Prove that the line of centres of two intersecting circles subtends equal angles at the two points of intersection.

Answer.



Let two circles having their centres as O and O' intersect each other at point A and B respectively. Let us join O O'

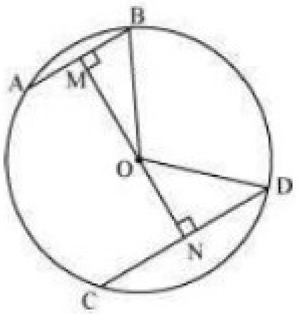


In \Delta A O O^{\prime} \text { and } B O^{\prime}
OA =OB
O'A = O'B
OO' = OO' (common)
$\triangle AO^{O'}\square\Delta BO^{O'}$ (by CPCT)
Therefore, line of centres of two intersecting circles subtends equal angles at the two points of intersection.

Page : 186 , Block Name : Exercise 10.6 (Optional)

Q2 Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6 cm, find the radius of the circle.

Answer. Draw OM \square AB and ON \square CD .
join OB and OD

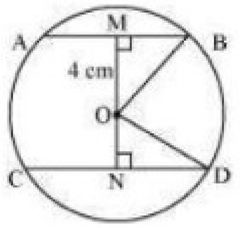


$\mathrm{BM} = \frac{\mathrm{AB}}{2} = \frac{5}{2}$
$\mathrm{ND} = \frac{\mathrm{CD}}{2} = \frac{11}{2}$
Let ON be x. Therefore, 0M will be 6 - x.
In $\triangle \mathrm{MOB}$
$\mathrm{OM}^2 + \mathrm{MB}^2 = \mathrm{OB}^2$
$(6-x)^2 + \left(\frac{5}{2}\right)^2 = \mathrm{OB}^2$
$36 + x^2 - 12x + \frac{25}{4} = \mathrm{OB}^2$................(1)
In $\triangle \mathrm{NOD}$
$\mathrm{ON}^2 + \mathrm{ND}^2 = \mathrm{OD}^2$
$x^2 + \left(\frac{11}{2}\right)^2 = \mathrm{OD}^2$
$x^2 + \frac{121}{4} = \mathrm{OD}^2$..............(2)
We have 0B = OD (Radii of the same circle)
Therefore, from equation (1) and (2),
$36 + x^2 - 12x + \frac{25}{4} = x^2 + \frac{121}{4}$
$12x = 36 + \frac{25}{4} - \frac{121}{4}$
$= \frac{144+25-121}{4} = \frac{48}{4} = 12$
x=1
From equation (2),
$(1)^2 + \left(\frac{121}{4}\right) = \mathrm{OD}^2$
$\mathrm{OD}^2 = 1 + \frac{121}{4} = \frac{125}{4}$
$\mathrm{OD} = \frac{5}{2}\sqrt{5}$
Therefore , the radius of circle is (\frac{5}{2} \sqrt{5}\) cm.

Page : 186 , Block Name : Exercise 10.6 (Optional)

Q3 The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at distance 4 cm from the centre, what is the distance of the other chord from the centre?

Answer.



Let AB and CD be two parallel chords in a circle centered at O. Join 03 and 00.
Distance of smaller chord AB from the centre of the circle = 4 cm ,0M = 4 cm

$\mathrm{MB}=\frac{\mathrm{AB}}{2}=\frac{6}{2}=3\mathrm{cm}$

In $\triangle OMB$

$\mathrm{OM}^2+\mathrm{MB}^2=\mathrm{OB}^2$

$(4)^2+(3)^2=\mathrm{OB}^2$

$16+9=\mathrm{OB}^2$

$\mathrm{OB}=\sqrt{25}$

OB= 5 cm

In In $\triangle OND$

$\mathrm{OD}=\mathrm{OB}=5\mathrm{cm}$

$\mathrm{ND}=\frac{\mathrm{CD}}{2}=\frac{8}{2}=4\mathrm{cm}$

$\mathrm{ON}^2+\mathrm{ND}^2=\mathrm{OD}^2$

$\mathrm{ON}^2+(4)^2=(5)^2$

$\mathrm{ON}^2=25-16=9$

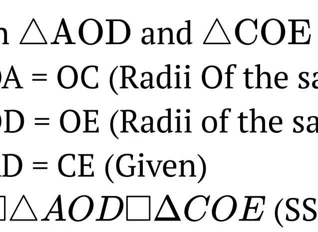
ON = 3 cm

Therefore, the distance of the bigger chord from the centre is 3 cm.

Page : 186 , Block Name : Exercise 10.6 (Optional)

Q4 Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect equal chords AD and CE with the circle. Prove that ∠ABC is equal to half the difference of the angles subtended by the chords AC and DE at the centre.

Answer.



In $\triangle\mathrm{AOD}$ and $\triangle\mathrm{COE}$

OA = OC (Radii Of the same circle)

OD = OE (Radii of the same circle)

AD = CE (Given)

$\square\triangle AOD\square\Delta COE$ (SSS congruence rule)

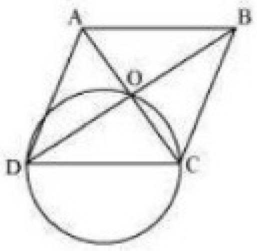
$\square OAD=\square OCE$ (by CPCT)............(1)

$\square ODA = \square OEC$ (by CPCT).............(2)
Also,
$\square OAD = \square ODA (ASOA = OD)$.............(3)
From equations (1), (2), and (3), we obtain
$\square OAD = \square OCE = \square ODA = \square OEC$
Let $\square OAD = \square OCE = \square ODA = \square OEC = x$
In $\triangle$OAC
OA=OC
$\square\square OCA = \square OAC$ (Let a)
In $\triangle$ODE
OD=OE
$\square OED = \square ODE$ (Let y)
ADEC is a cyclic quadrilateral,
$\square\square CAD + \square DEC = 180^\circ$
$x + a + x + y = 180^\circ$
$2x + a + y = 180^\circ$
$y = 180^\circ - 2x - a \ldots (4)$.................(4)
However $\square AOC = 180^\circ - 2a$
And $\square AOC = 180^\circ - 2a$
$\square DOE - \square AOC = 2a - 2y = 2a - 2\left(180^\circ - 2x - a\right)$
$= 4a + 4x - 360^\circ$....................(5)
$\square BAC + \square CAD = 180^\circ$
$\square\square BAC = 180^\circ - \square CAD = 180^\circ - (a + x)$
Similarly $\square ACB = 180^\circ - (a + x)$
In $\triangle$ABC
$\square ABC + DBAC + DACB = 180^\circ$ (Angle sum property of a triangle)
$\square ABC = 180^\circ - \square BAC - \square ACB$
$= 180^\circ - (180^\circ - a - x) - (180^\circ - a - x)$
$= 2a + 2x - 180^\circ$
$= \frac{1}{2}[4a + 4x - 360^\circ]$
$\square ABC = \frac{1}{2}[\square DOE - \square AOC][U\text{sing equation } (5)]$

Page : 186 , Block Name : Exercise 10.6 (Optional)

Q5 Prove that the circle drawn with any side of a rhombus as diameter, passes through the point of intersection of its diagonals.

Answer.

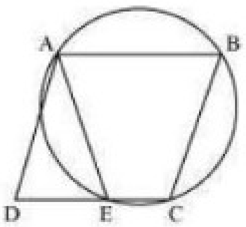


Let } A B C D be a rhombus in which diagonals are intersecting at point O and a circle is drawn while taking side CD as its diameter. We know that a diameter subtends 90^{\circ} on the arc.
$\square\square COD = 90^\circ$
Also, in rhombus, the diagonals intersect each other at 90^{\circ}
$\square AOB = \square BOC = \square COD = \square DOA = 90^\circ$
Clearly ,point O has to lie on the circle.

Page : 186 , Block Name : Exercise 10.5 (Optional)

Q6 ABCD is a parallelogram. The circle through A, B and C intersect CD (produced if necessary) at E. Prove that AE = AD.

Answer.

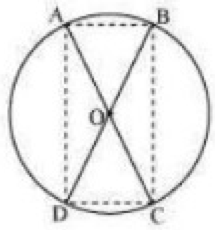


It can be observed that ABCE is a cyclic quadrilateral and in a cyclic quadrilateral, the sum of the opposite angles is 180^{\circ}
$\square AEC + \square CBA = 180^\circ$
$\square AEC + \square AED = 180^\circ$
$\square AED = \square CBA$...............(1)
For a parallelogram, opposite angles are equal.
$\square ADE = \square CBA$......................(2)
From (1) and (2)
$\square AED = \square ADE$
AD = AE (Angles opposite to equal sides of a triangle)

Page : 186 , Block Name : Exercise 10.6 (Optional)

Q7 AC and BD are chords of a circle which bisect each other. Prove that (i) AC and BD are diameters, (ii) ABCD is a rectangle.

Answer.

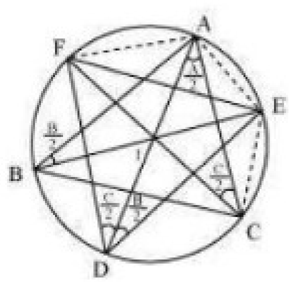


Let two choms Aa and CD are intersecting each other at point O.
In$\triangle AOB$ and ΔCOD,
OA = OC (Given)
OB = OD (Given)
$\square AOB=\square COD$
$\triangle$AOB□ΔCOD (SAS congruence rules)
AB=CD (by CPCT)
Similarly , it can be proved that $\triangle AOD$□ΔCOB
AD= CB (by CPCT)
Since in quadrilateral ACBD, opposite sides are equal in length,
ACBD is a parallelogram.
We know that opposite angles of a parallelogram are equal.
OA=OC
However OA+ OC=180
OA+OA=180
2 OA =180
OA =90
As ACBD is a parallelogram and one of its interior angles is 900, therefore, it is a rectangle.
{ \squareA }is the angle subtended by chord BD. And as \square A=90^{\circ}, therefore, B Dshould be the diameter of the circle. Similarly, A C \text is the diameter of the circle. end

Page : 186 , Block Name : Exercise 10.6 (Optional)

Q8 Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively. Prove that the angles of the triangle DEF are 90° – 1 2 A, 90° – 1 2 B and 90° – 1 2 C.

Answer.



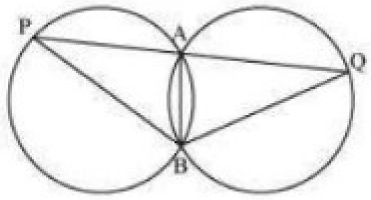
It is given that 3t is the bisector of OB.
$\square\square ABE=\frac{\angle B}{2}$
However $\square ADE=\square ABE$

$\square\square ADE = \frac{\angle B}{2}$
Similarly $\mathrm{DACF} = \square ADF = \frac{\angle C}{2}$
$\square D = \square ADE + \square ADF$
$= \frac{1}{2}(\angle B + \angle C)$
$= \frac{1}{2}(180^\circ - \angle A)$
$= 90^\circ - \frac{1}{2}\angle \mathrm{A}$
Similarly it can be proved that
$\angle \mathrm{E} = 90^\circ - \frac{1}{2}\angle \mathrm{B}$
$\angle \mathrm{F} = 90^\circ - \frac{1}{2}\angle \mathrm{C}$

Page : 186 , Block Name : Exercise 10.5 (Optional)

Q9 Two congruent circles intersect each other at points A and B. Through A any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that BP = BQ.

Answer.

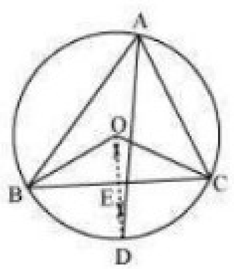


AB is the common chord in both the congruent circles.
$\square\square APB = \square AQB$
In ΔBPQ
$\square APB = \square AQB$
$\square \mathrm{BQ} = \mathrm{BP}$\{ Angles opposite to equal sides of a triangle) \}

Page : 186 , Block Name : Exercise 10.6 (Optional)

Q10 In any triangle ABC, if the angle bisector of ∠A and perpendicular bisector of BC intersect, prove that they intersect on the circumcircle of the triangle ABC.

Answer.



Let perpendicular bisector of side 3C and angle bisector of OA meet at point D. Let the perpendicular bisector of side 3C intersect it at E.
Perpendicular bisector of side BC will pass through circumcentre O of the circle.

delta\BOC and delta\BAC are the angles subtended by arc BC at the centre and a point A on the remaining part of the circle respectively. We also know that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$\square BOC = 2\square BAC = 2\square A$........(1)

In ΔBOE and ΔCOE

OE=OE (common)

OB=OC

$\square OEB = \square OEC$

$\square\triangle BOE\square\square COE$

However $\square BOE + \square COE = \square BOC$

$\square\square BOE + \square BOE = 2\square A$

$\square 2\square BOE = 2\square A$

$\square\square BOE = \square A$

$\square\square BOE = \square COE = \square A$

The perpendicular bisector Of side BC and angle bisector Of ÜA meet at point D.

$\square\square BOD = \square BOE = \square A$...........(3)

Since AD is the bisector of angle OA

$\square 2\square BAD = \square A$...........(4)

From equations (3) and (4), we obtain

$\square BOD = 2\square BAD$

This can be possible only when point BD Will be a chord Of the circle. For this, the point D lies on the circum circle.

Therefore, the perpendicular bisector of side BC and the angle bisector of OA meet on the circum circle of triangle ABC.

Page : 186 , Block Name : Exercise 10.6 (Optional)