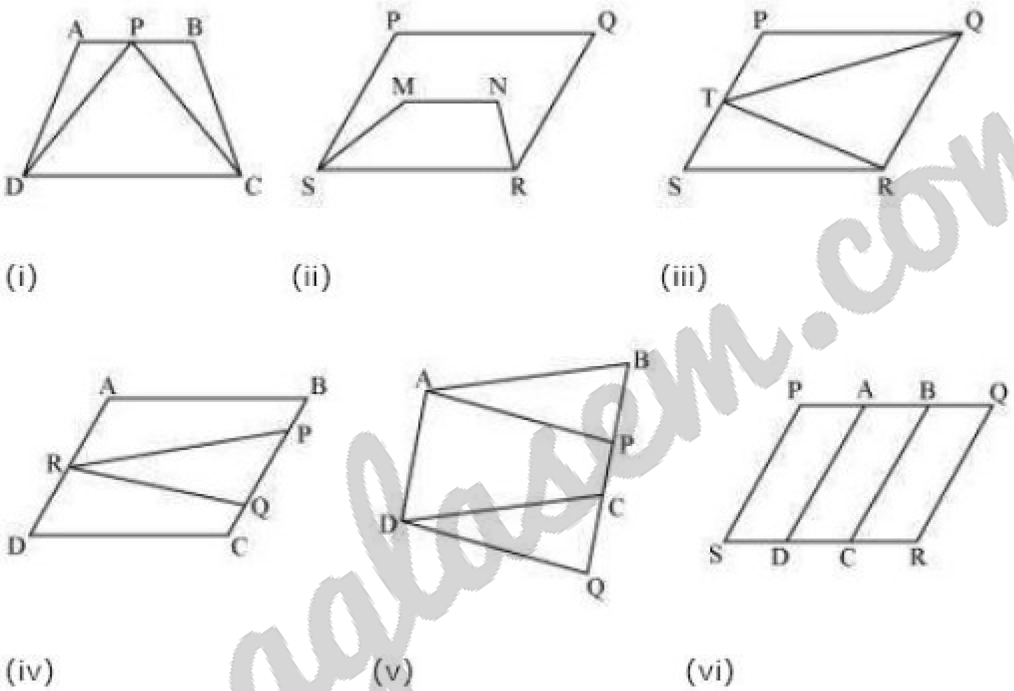
NCERT SOLUTIONS

CLASS - 9th

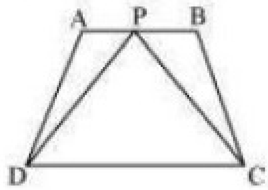
Class : 9th
Subject : Maths
Chapter : 9
Chapter Name : Areas Of Parallelograms And Triangles

Exercise 9.1

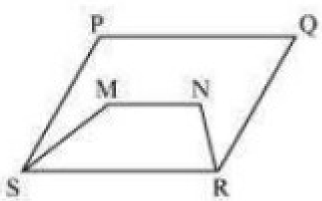
Q1 Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and the two parallels.



Answer. (i)

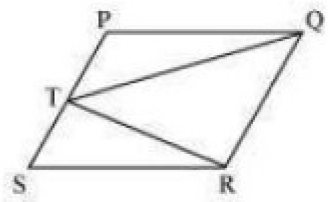


Yes. It can be observed that trapezium ABCD and triangle PCD have a common base CD and these are lying between the same parallel lines AB and CD.
(ii)

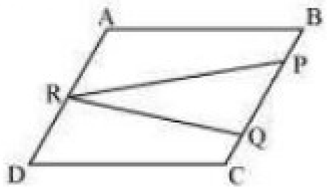


No. It can be observed that parallelogram PQRS and trapezium MNRS have a

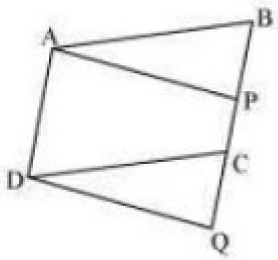
common base RS. However, their vertices, (i.e., opposite to the common base) P, Q of parallelogram and M, N of trapezium, are not lying on the same line.
(iii)



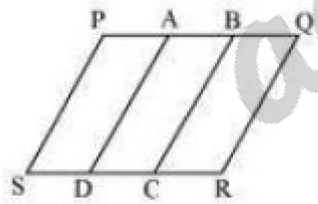
Yes. It can be observed that parallelogram PQRS and triangle TQR have a common base QR and they are lying between the same parallel lines PS and QR.
(iv)



No. It can be observed that parallelogram ABCD and triangle PQR are lying between same parallel lines AD and ac. However, these do not have any common base.
(v)



Yes. It can be observed that parallelogram ABCD and parallelogram APQD have a common base AD and these are lying between the same parallel lines AD and 3Q.
(vi)



No. It can be observed that parallelogram PBCS and PQRS are lying on the same base PS. However, these do not lie between the same parallel lines.

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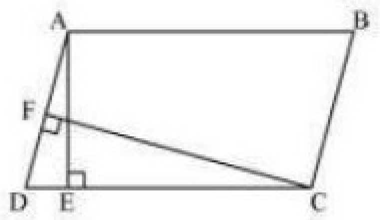
Exercise 9.2

Q1 In Figure, ABCD is a parallelogram, $AE \perp DC$ and CF ⊥ AD. If AB = 16 cm, AE = 8 cm and CF = 10 cm, find AD.



Answer. In parallelogram ABCD, CD = AB = 16 cm
[Opposite sides of a parallelogram are equal]
We know that
Area of a parallelogram = Base x Corresponding altitude
Area of parallelogram ABCD = CD x AE = AD x CF
Area of parallelogram = Base x Corresponding altitude
Area of parallelogram ABCD = CD x AE = AD x CF
$16\mathrm{cm} \times 8\mathrm{cm} = \mathrm{AD} \times 10\mathrm{cm}$
$\mathrm{AD} = \frac{16\times 8}{10}\mathrm{cm} = 12.8\mathrm{cm}$
Thus, the length of AD is 12.8 cm.

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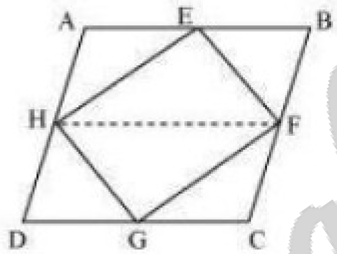
Q2 If E,F,G and H are respectively the mid-points of the sides of a parallelogram ABCD, show that $\mathrm{ar}(\mathrm{EFGH}) = \frac{I}{2}\mathrm{ar}(\mathrm{ABCD})$.

Answer.



Let us join HF.
In parallelogram ABCD,
AD = BC and $\mathrm{AD} \| \mathrm{BC}$ (Opposite sides of a parallelogram are equal and parallel)
AB = CD (Opposite sides of a parallelogram are equal)
$\Rightarrow \frac{1}{2}\mathrm{AD} = \frac{1}{2}\mathrm{BC}$ and
$\mathrm{AH} \| \mathrm{BF}$
Therefore, ABFH is a parallelogram.
Since $\Delta\mathrm{HEF}$ and parallelogram ABFH are on the same base HF and between the same parallel lines AB and HF,
$\therefore$ Area $(\Delta\mathrm{HEF}) = \frac{1}{2}$ Area (ABFH) $\ldots$ (1) Similarly, it can be proved that
Area $(\Delta HGF) = \frac{1}{2}$ Area (HDCF) $\ldots$ (2)
On adding equations (1) and (2), we obtain
Area $(\Delta\mathrm{HEF}) +$ Area $(\Delta\mathrm{HGF}) = \frac{1}{2}$ Area (ABFH) $+ \frac{1}{2}$ Area (HDCF)
$= \frac{1}{2}[$ Area (ABFH) $+$ Area (HDCF)$]$
$\Rightarrow$ Area (EFGH) $= \frac{1}{2}$ Area (ABCD)

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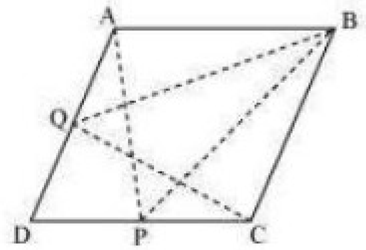
Q3 P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD.

Show that ar (APB) = ar (BQC).

Answer.



It can be observed that $\triangle BQC$ and paralleogram ABCD lie on the same base BC and these are between the same parallel lines AD and BC.
$\therefore$ Area $(\Delta BQC)=\frac{1}{2}$ Area (ABCD) $\ldots(1)$
Similarly, $\triangle\mathrm{APB}$ and parallelogram ABCD lie on the same base AB and between the same parallel lines AB and DC.
$\therefore$ Area $(\Delta\mathrm{APB})=\frac{1}{2}$ Area (ABCD) $\ldots(2)$
From equation (1) and (2), we obtain
Area $(\Delta BQC)=$ Area (ΔAPB)

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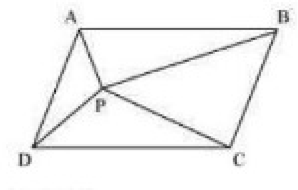
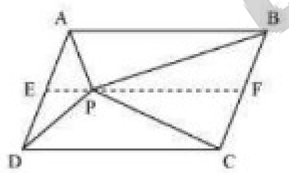
Q4 In Figure, P is a point in the interior of a parallelogram ABCD. Show that
(i) $\mathrm{ar}(\mathrm{APB})+\mathrm{ar}(\mathrm{PCD})=\frac{1}{2}\mathrm{ar}(\mathrm{ABCD})$
(ii) $\mathrm{ar}(\mathrm{APD})+\mathrm{ar}(\mathrm{PBC})=\mathrm{ar}(\mathrm{APB})+\mathrm{ar}(\mathrm{PCD})$
[Hint : Through P, draw a line parallel to AB.]



Answer.



(i) Let us draw a line segment EF, passing through point P and parallel to line segment AB.
In parallelogram ABCD,
$\mathrm{AB}\|\mathrm{EF}(\text{By construction })\ldots(1)$
ABCD is a parallelogram.
$\therefore$ $\mathrm{AD}\ \|\ \mathrm{BC}$ (Opposite sides of a parallelogram)
$\Rightarrow \mathrm{AE}\|\mathrm{BF}\ldots(2)$
From equations (1) and (2), we obtain
$\mathrm{AB}\|\mathrm{EF}$ and $\mathrm{AE}\|\mathrm{BF}$
Therefore, quadrilateral ABFE is a parallelogram
It can be observed that $\triangle\mathrm{APB}$ and parallelogram ABFE are lying on the same base AB and between the same parallel lines AB and EF.
$\therefore$ Area $(\Delta\mathrm{APB})=\frac{1}{2}$ Area (ABFE) $\ldots(3)$
Similarly, for $\Delta\mathrm{PCD}$ and parallelogram EFCD,

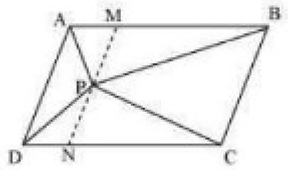
Area ($\triangle$PCD) = $\frac{1}{2}$ $\text{Area }(EFCD)\ldots(4)$

Adding equation (3) and (4), we obtain

$(\Delta \mathrm{APB})+\text{ Area }(\Delta \mathrm{PCD})=\frac{1}{2}[\text{ Area }(\mathrm{ABFE})+\text{ Area }(\mathrm{EFCD})]$

$(\Delta \mathrm{APB})+\text{ Area }(\Delta \mathrm{PCD})=\frac{1}{2}\text{ Arca }(\mathrm{ABCD})\ldots(5)$

(ii)



Let us draw a line segment MN, passing through point P and parallel to line segment AD.
In parallelogram ABCD,
MN || AD (By construction)...(6)
ABCD is a parallelogram.

$\therefore\ \mathrm{AB} \| \mathrm{DC}$ (Opposite sides of a parallelogram)

$\Rightarrow \mathrm{AM} \| \mathrm{DN} \ldots(7)$

From equations (6) (7), we obtain

$\mathrm{MN} \| \mathrm{AD}$ and $\mathrm{AM} \| \mathrm{DN}$

Therefore, quadrilateral AMND is a parallelogram.
It can be observed that $\triangle APD$ and parallelogram AMND are lying on the same base AD and between the same parallel lines AD and MN.

$\therefore\ \text{Area }(\Delta APD)=\frac{1}{2}\text{ Area }(\mathrm{AMND})\ \ldots(8)$

Similarly, for $\triangle$PCB and parallelogram MNCB,

$\text{Area }(\Delta \mathrm{PCB})=\frac{1}{2}\text{ Area }(\mathrm{MNCB})\ \ldots(9)$

Adding equations (8) and (9), we obtain

$(\Delta \mathrm{APD})+\text{ Area }(\Delta \mathrm{PCB})=\frac{1}{2}[\text{ Area }(\mathrm{AMND})+\text{ Area }(\mathrm{MNCB})]$

$(\Delta \mathrm{APD})+\text{ Area }(\Delta \mathrm{PCB})=\frac{1}{2}\mathrm{Arca}(\mathrm{ABCD})\quad\ldots(10)$

On comparing equations (5) and (10), we obtain

$(\Delta \mathrm{APD})+\text{ Area }(\Delta \mathrm{PBC})=\text{ Area }(\Delta \mathrm{APB})+\text{ Area }(\Delta \mathrm{PCD})$

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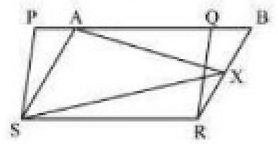
Q5 In Figure, PQRS and ABRS are parallelograms and X is any point on side BR. Show that

(i) $\mathrm{ar}(\mathrm{PQRS})=\mathrm{ar}(\mathrm{ABRS})$

(ii) $\mathrm{ar}(\mathrm{AXS})=\frac{1}{2}\mathrm{\ ar\ }(\mathrm{PQRS})$



Answer. (i) It can be observed that paralleogram PQRS and ABRS lie on the same base SR and also, these lie in between the same parallel lines SR and PB.

$\therefore\ \text{Area }(\mathrm{PQRS})=\text{ Area }(\mathrm{ABRS})\ \ldots(1)$

(ii) Consider $\triangle AXS$ and paralleogram ABRS.
As these lie on the same base and are between the same parallel lines AS and BR,

$\therefore\ \text{Area }(\Delta \mathrm{A}\times \mathrm{S})=\frac{1}{2}\text{ Area }(\mathrm{ABRS})\ \ldots(2)$

From equations (1) and (2), we obtain

Area $(\Delta \mathrm{AXS})$ = $=\frac{1}{2}\ Area(PQRS)$

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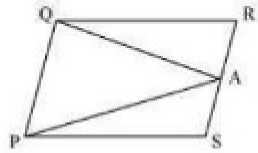
Q6 A farmer was having a field in the form of a parallelogram PQRS. She took any point A on RS and joined it to points P and Q. In how many parts the fields is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it?

Answer.



From the figure, it can be that point A divides the field into three parts.
These parts are triangular in shape - $\triangle \mathrm{PSA}, \Delta \mathrm{PAQ}$, and $\triangle \mathrm{QRA}$
Are of $\Delta \mathrm{PSA}+$ Area of $\Delta \mathrm{PAQ}+$ Area of $\Delta \mathrm{QRA}$ = Area of $\|$gm pQRS $\ldots$ (1)
$\therefore$ Area $(\Delta \mathrm{PAQ})=\frac{1}{2}$ Area $(\mathrm{PQRS}) \ldots(2)$
From equation (1) and (2), we obtain
Area $(\Delta \mathrm{PSA})+$ Area $(\Delta \mathrm{QRA})=\frac{1}{2}\ Area(PQRS)\ldots(3)$
Clearly, it can be observed that the farmer must sow wheat in triangular part PAQ and pulses in other two triangular parts PSA and QRA or wheat in triangular parts PSA and QRA and pulses in triangular parts PAQ.

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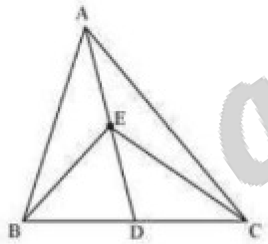
Exercise 9.3

Q1 In Figure, E is any point on median AD of a Δ ABC. Show that ar (ABE) = ar (ACE).



Answer. AD is the median of $\triangle$ABC. Therefore, it will divide $\triangle$ABC in to two triangles of equal areas.
$\therefore$ Area $(\Delta \mathrm{ABD})=$ Area $(\Delta \mathrm{ACD}) \ldots(1)$
ED is the median of $\triangle EBC$
$\therefore$ Area $(\Delta EBD)=$ Area $(\Delta ECD) \ldots(2)$
On subtracting equation (2) from equations (1), we obtain
Area $(\Delta ABD)-$ Area $(EBD)=$ Area $(\Delta ACD)-$ Area (ΔECD)
Area $(\Delta ABE)=$ Area (ΔACE)

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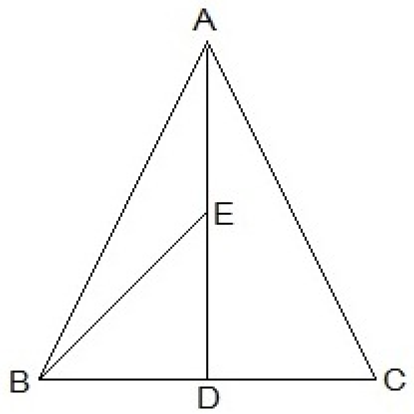
Q2 In a triangle ABC, E is the mid-point of median AD. Show that ar (BED) = 1/4 ar(ABC).

Answer.



$\text{ar}(BED) = (1/2) \times BD \times DE$

As E is the mid-point of AD

Thus, $AE = DE$

As AD is the median on side BC of triangle ABC,

Thus, $BD = DC$

Therefore,

DE = (½)AD ….(i)
BD = (½) BC …(ii)
From (i) and (ii)

$\text{ar(BED)} = (1/2) \times (1/2)\text{BC} \times (1/2)\text{AD}$

$\Rightarrow \text{ar(BED)} = (1/2) \times (1/2)\,\text{ar(ABC)}$

$\Rightarrow \text{ar(BED)} = 1/4\,\text{ar(ABC)}$

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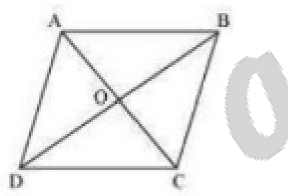
Q3 Show that the diagonals of a parallelogram divide it into four triangles of equal area.

Answer.



We know that diagonals of parallelogram bisect each other.
Therefore, O is the mid-point of AC and BD.
BO is the median in $\triangle$ABC. Therefore, it will divide it into two triangles of equal areas.
$\therefore$ Area $(\Delta AOB) =$ Area $(\Delta BOC) \ldots (1)$
In $\triangle BCD, CO$ is the nedian.
$\therefore$ Area $(\Delta BOC) =$ Area $(\Delta COD) \ldots (2)$
Similarly, Area $(\Delta \text{COD}) =$ Area $(\Delta \text{AOD}) \ldots (3)$
From equations (1), (2), and (3), we obtain
Area $(\Delta AOB) =$ Area $(\Delta BOC) =$ Area $(\Delta COD) =$ Area (ΔAOD)
Therefore, it is evident that the diagonals of a paralleogram divide it into four triangles of equal area.

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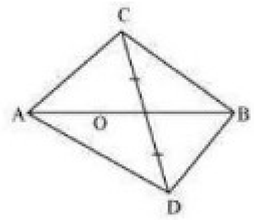
Q4 In Figure, ABC and ABD are two triangles on the same base AB. If line- segment CD is bisected by AB at O, show that ar(ABC) = ar (ABD).



Answer. Consider $\triangle \mathrm{ACD}$
Line-segment CD is bisected by AB at O. Therefore, AO is the median of $\triangle \mathrm{ACD}$
$\therefore$ Area $(\Delta \mathrm{ACO}) =$ Area $(\Delta \mathrm{ADO}) \ldots (1)$
Considering ΔBCD, BO is the median.
$\therefore$ Area $(\Delta BCO) =$ Area $(\Delta BDO) \ldots (2)$
Adding equations (1) and (2), we obtain
Area $(\Delta ACO) +$ Area $(\Delta BCO) =$ Area $(\triangle ADO) +$ Area (ΔBDO)
$\Rightarrow$ Area $(\Delta ABC) =$ Area (ΔABD)

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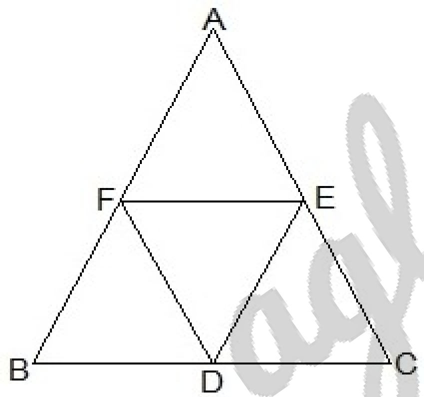
Q5 D, E and F are respectively the mid-points of the sides BC, CA and AB of a Δ ABC. Show that
(i) $BDEF is a parallelogram.$
(ii) $\mathrm{ar}(\mathrm{DEF}) = \frac{1}{4}\mathrm{ar}(\mathrm{ABC})$
(iii) $\mathrm{ar}(\mathrm{BDEF}) = \frac{1}{2}\mathrm{ar}(\mathrm{ABC})$

Answer.



(i) In Δ ABC
$\mathrm{EF} \| \mathrm{BC}$ and $\mathrm{EF} = 1/2\mathrm{BC}$ (by mid point theorem)
also,
$\mathrm{BD} = 1/2\mathrm{BC}$(D is the mid point)
So , BD = EF
also,
BF and DE will also parallel and equal to each other.
Thus, the pair opposite sides are equal in length and parallel
to each other.
$\therefore$ BDEF is a parallelogram.

(ii) Proceeding from the result of (i)
BDEF, DCEF, AFDE are parallelograms.
Diagonal of a parallelogram divides it into two triangles of equal area.
$\therefore \mathrm{ar}(\Delta \mathrm{BFD}) = \mathrm{ar}(\Delta \mathrm{DEF})$(For parallelogram BDEF) …. (i)

Also, $\mathrm{ar}(\Delta \mathrm{AFE}) = \mathrm{ar}(\Delta \mathrm{DEF})$(For parallelogram DCEF) …. (ii)
$\mathrm{ar}(\Delta \mathrm{CDE}) = \mathrm{ar}(\Delta \mathrm{DEF})$(For parallelogram AFDE) …. (iii)
From (i), (ii) and (iii)

$\mathrm{ar}(\Delta BFD) = \mathrm{ar}(\Delta AFE) = \mathrm{ar}(\Delta CDE) = \mathrm{ar}(\Delta DEF)$

$\Rightarrow \mathrm{ar}(\Delta BFD) + \mathrm{ar}(\Delta AFE) + \mathrm{ar}(\Delta CDE) + \mathrm{ar}(\Delta DEF) =$
$\mathrm{arar}(\Delta ABC)$

$\Rightarrow 4\,\mathrm{ar}(\Delta DEF) = \mathrm{ar}(\Delta ABC)$

$\Rightarrow \mathrm{ar}(DEF) = 1/4\,\mathrm{ar}(ABC)$

(iii) Area (parallelogram BDEF) = ar (Δ DEF) + ar(Δ BDE)
= ar (parallelogram BDEF) = ar (Δ DEF) + ar(Δ DEF)
= ar (parallelogram BDEF) = 2 x ar (Δ DEF)
= $\mathrm{ar}(\text{parallelogram BDEF}) = 2 \times 1/4\ \mathrm{ar}(\ (\Delta ABC) \Rightarrow$ $\mathrm{ar}(\text{parallelogram BDEF}) = 1/2\,\mathrm{ar}(\Delta ABC)$

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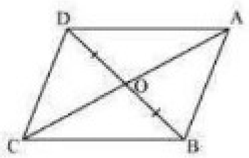
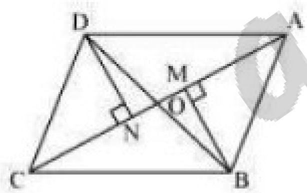
Q6 In Figure, diagonals AC and BD of quadrilateral ABCD intersect at O such that OB = OD.
If AB = CD, then show that:
(i) ar (DOC) = ar (AOB)
(ii) ar (DCB) = ar (ACB)
(iii) DA || CB or ABCD is a parallelogram.
[Hint: From D and B, draw perpendiculars to AC.]



Answer.



Let us draw DN ⊥ AC and BN ⊥ AC.
(i) In $\triangle DON$ and $\triangle BOM$
$\angle \mathrm{DNO} = \angle \mathrm{BMO}$ (By construction)
$\angle DON = \angle BOM$ (Vertically opposite angles)
$OD = OB (Given)$
By AAS congruence rule,
$\triangle \mathrm{DON} \cong \triangle \mathrm{BOM}$
$\mathrm{DN} = \mathrm{BM} \ldots(1)$
We know that congruent triangles have equal areas.
$\text{Area } (\Delta \mathrm{DON}) = \text{ Area } (\Delta \mathrm{BOM}) \ldots(2)$
In $\triangle \mathrm{DNC}$ and $\triangle \mathrm{BMA}$,
$\angle DNC = \angle BMA (By$ construction)
$\mathrm{CD} = \mathrm{AB}(\text{given})$
$\mathrm{DN} = \mathrm{BM}[\text{Using Equation } (1)]$
$\therefore \triangle \mathrm{DNC} \cong \triangle \mathrm{BMA}(\text{RHS congruence rule})$

$\therefore$ Area $(\Delta\mathrm{DNC}) =$ Area $(\Delta\mathrm{BMA})$ ..(3)
On adding Equations (2) and (3), we obtain
Area $(\Delta\mathrm{DON}) +$ Area $(\Delta\mathrm{DNC}) =$ Area $(\Delta\mathrm{BOM}) +$ Area $(\Delta\mathrm{BMA})$
Therefore, Area $(\Delta\mathrm{DOC}) =$ Area $(\triangle\mathrm{AOB})$
(ii) We obtained,
Area $(\Delta DOC) =$ Area (ΔAOB)
$\therefore$ Area $(\Delta DOC) +$ Area $(\Delta OCB) = (\Delta AOB) +$ Area (ΔOCB)
(Adding Area $(\triangle OCB)$to both sides)
$\therefore$ Area $(\Delta DCB) = Ar$ ca(ΔACB)
(iii) We obtained,
Area $(\Delta\mathrm{DCB}) =$ Area $(\triangle\mathrm{ACB})$
If two triangles have the same base and equal areas, then these will lie between the same parallels.
$\therefore \mathrm{DA} \| \mathrm{CB}$ …(4)
In quadrilateral ABCD, one pair of opposite sides is equal (AB = CD) and the other part of opposite sides is parallel $(\mathrm{DA} \| \mathrm{CB})$

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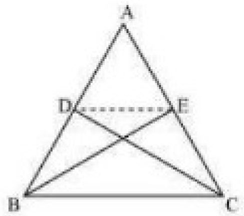
Q7 D and E are points on sides AB and AC respectively of Δ ABC such that ar (DBC) = ar (EBC). Prove that DE || BC.

Answer.



Since $\triangle BCE$ and $\triangle BCD$ are lying on a common base BC and also have equal areas, $\triangle BCE$ and ΔBCD will lie between the same parallel lines.
$\therefore \mathrm{DE} \| \mathrm{BC}$

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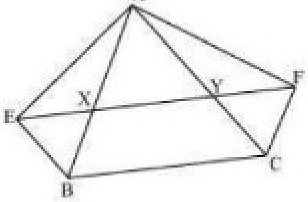
Q8 XY is a line parallel to side BC of a triangle ABC. If BE || AC and CF || AB meet XY at E and F respectively, show that
ar (ABE) = ar (ACF)

Answer.



It is given that
$XY \| BC = EY \| BC$
$BE \| AC = BE \| CY$
Therefore,EBYC is a parallelogram.
It is given that
$XY \| BC = XF \| BC$
$FC \| AB = FC \| XB$

Therefore, BCFX is a parallelogram.
Parallelograms EBYC and BCFX are on the same base BC and between the same parallels BC and EF.
$\therefore$ Area $(EBCY) = \frac{1}{2}$ Area $(BCFX) \quad \ldots(1)$
Consider parallelogram EBYC and $\triangle$AEB
These lie on the same base BE and are between the same parallels BE and AC.
$\therefore$ Area $(\Delta \mathrm{ABE}) = \frac{1}{2}$ ..(2)
Also, parallelogram ΔCFX and $\triangle$ACF are on the same base CF and between the same parallels CF and AB.
$\therefore$ Area $(\Delta \mathrm{ACF}) = \frac{1}{2}$ Area (BCFX) $\quad \ldots(3)$
From equations (!), (2), and (3), we obtain
Area $(\triangle ABE) =$ Area (ΔACF)

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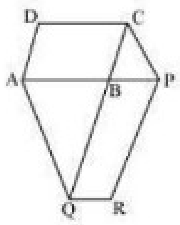
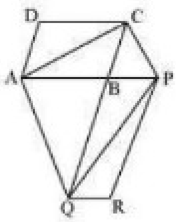
Q9 The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed (see Figure). Show that ar (ABCD) = ar (PBQR).
[Hint : Join AC and PQ. Now compare ar (ACQ) and ar (APQ).]



Answer.



Let us join AC and PQ.
$\triangle$ACQ and $\triangle$AQP are the same base AQ and between the same parallels AQ and CP.
Area $(\Delta ACQ) =$ Area (ΔAPQ)
$(\Delta \mathrm{ACQ}) -$ Area $(\Delta \mathrm{ABQ}) =$ Area $(\Delta \mathrm{APQ}) -$ Area $(\Delta \mathrm{ABQ})$
$(\Delta \mathrm{ABC}) =$ Area $(\Delta \mathrm{QBP})$ ...(1)
Since AC and PQ are diagonals of parallelograms ABCD and PBQR respectively.
Area $(\Delta ABC) = \frac{1}{2}$ Area $(ABCD) \ldots(2)$
Area $(\Delta QBP) = \frac{1}{2}$ Area (PBQR) $\quad \ldots(3)$
From equations(!),(2) and (3), we obtain
$\frac{1}{2}$ Area $(ABCD) = \frac{1}{2}\ Area(PBQR)$
Area(ABCD) = Area(PBQR)

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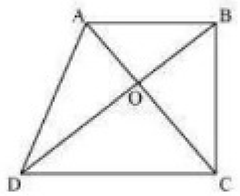
Q10 Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O. Prove that ar (AOD) = ar (BOC).

Answer.



It can be observed that $\triangle DAC$ and ΔDBC lie on the same base DC and between the same parallels AB and CD.
Area $(\Delta DAC) = Arca(\Delta DBC)$
Area $(\Delta DAC) - Area(\Delta DOC) = \text{Arca}(\Delta\text{DBC}) - \text{Arca}(\Delta\text{DOC})$
$(\Delta AOD) = \text{ Arca }(\Delta BOC)$

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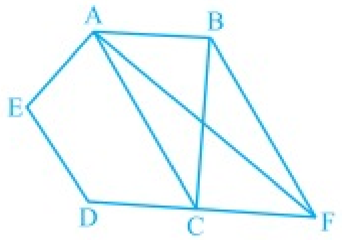
Q11 In Figure, ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F. Show that
(i) ar (ACB) = ar (ACF)
(ii) ar (AEDF) = ar (ABCDE)



Answer. (i) $\triangle\text{ACB}$ and $\triangle\text{ACF}$ lie on the same base AC and are between
The same parallels AC and BF.
(ii) It can be observed that
Area $(\Delta ACB) = \text{ Area }(\Delta\text{ACF})$
Area $(\triangle ACB) + \text{ Area }(ACDE) = \text{ Area }(\Delta\text{ACF}) + \text{ Area }(\text{ACDE})$
Area (ABCDE) = Area (AEDF)

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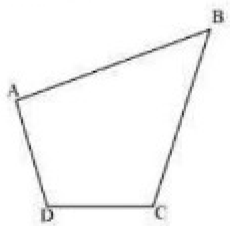
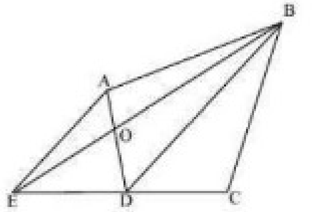
Q12 A villager Itwaari has a plot of land of the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portion of his plot from one of the corners to construct a Health Centre. Itwaari agrees to the above proposal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.

Answer.



Let quadrilateral ABCD be the original shape of the field.
The proposal may be implemented as follows.
Join diagonal BD and draw a line parallel to BD through point A. Let it meet the extended side CD of ABCD at point E. Join BE and AD. Let them intersect each other at O. Then, portion $\triangle\text{AOB}$ can be cut from the original field so that the new shape of the field will b ΔBCE.(se figure).

We have to prove that the area of $\triangle$AOB (portion that was cut so as to construct Health Centre) is equal to the area of $\triangle$DEO (portion added to the field so as to make the area of the new field so formed equal to the area of the original field).



It can be observed that $\triangle DEB$ and $\triangle DAB$ lie on the same base BD and are between the same parallels BD and AE.

$\text{Area } (\Delta DEB) = \text{ Area } (\Delta DAB)$

$\text{Area } (\Delta DEB) - \text{ Area } (\Delta DOB) = (\Delta DAB) - \text{ Area } (\Delta DOB)$

$\text{Area } (\Delta DEO) = Area(\Delta AOB)$

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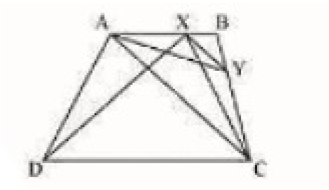
Q13 ABCD is a trapezium with AB || DC. A line parallel to AC intersects AB at X and BC at Y. Prove that ar (ADX) = ar (ACY).
[Hint: Join CX.]

Answer.



It can be observed that $\triangle ADX$ and $\triangle ACX$ lie on the same base AX and are between the same parallels AB and DC.

$\text{Area } (\Delta ADX) = \text{ Arca } (\Delta ACX) \ldots(1)$

$\triangle$ACY and $\triangle$ACX lie on the same base AC and are between the same parallels AC and XY.

$\text{Area } (\Delta ACY) = \text{ Area } (ACX) \ldots(2)$

From Equations (1) and (2), we obtain

$\text{Area } (\Delta ADX) = Area(\Delta ACY)$

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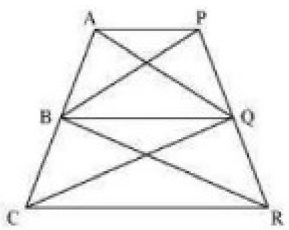
Q14 In Figure, AP || BQ || CR. Prove that ar (AQC) = ar (PBR).



Answer. Since $\triangle ABQ$ and $\triangle PBQ$ lie on the same base BQ and are between the same parallels AP and BQ.

$\therefore \text{ Area } (\Delta \text{ABQ}) = \text{ Arca } (\Delta \text{PBQ}) \ldots(1)$

Again, $\triangle BCQ$ and $\triangle BRQ$ lie on the same BQ and are between the same parallels BQ and CR.

$\therefore$ Area (ΔBCQ) = Area (\DeltaBRQ) $\quad \ldots$ ….(2)

On adding Equations(1) and (2), we obtain

$\text{Area } (\Delta ABQ) + Arca(\Delta BCQ) = (\Delta PBQ) + \text{ Area } (\Delta BRQ)$

$\therefore$ Area $(\Delta AQC) =$ Area (ΔPBR)

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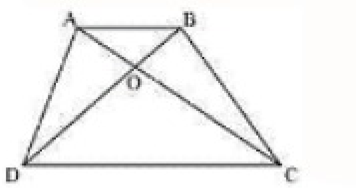
Q15 Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that ar (AOD) = ar (BOC). Prove that ABCD is a trapezium.



Answer.
It is given that
Area $(\triangle AOD) =$ Area (ΔBOC)
Area $(\Delta AOD) +$ Area $(\Delta AOB) = (\Delta BOC) +$ Area (ΔAOB)
Area $(\Delta ADB) =$ Area $(\triangle ACB)$
We know that triangles on the same base having area equal to each other lie between the same parallels.
Therefore, these triangles, $\triangle ADB$ and $\triangle ACB$, are lying between the same parallels.
i.e., AB || CD
Therefore, ABCD is a trapezium.

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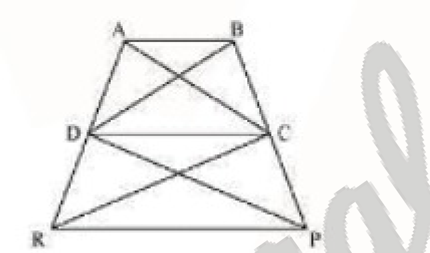
Q16 In Figure, ar (DRC) = ar (DPC) and ar (BDP) = ar (ARC). Show that both the quadrilaterals ABCD and DCPR are trapeziums.



It is given that
Area $(\Delta DRC) =$ Area (ΔDPC)
As $\triangle DRC$ and ΔDPC lie on the same base DC and have equal area, therefore, they must lie between the same parallel lines.
$\therefore$ DC$\|$ RP
Therefore, DCPR is a trapezium
It is also given that
Area $(\Delta BDP) =$ Area $(\triangle ARC)$
Area $(\Delta BDP) -$ Area $(\Delta DPC) = (\Delta ARC) -$ Area (ΔDRC)
$\therefore$ Area $(\Delta BDC) = Area(\Delta ADC)$
Since $\triangle BDC$ and $\triangle ADC$ are on the same base CD and have equal areas, they must lie between the same parallel lines.
$\therefore$ AB$\|$CD
Therefore, ABCD is a trapezium.

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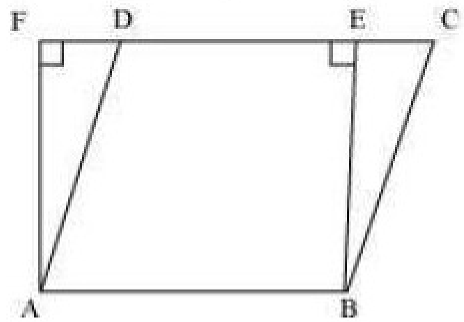
Exercise 9.4

Q1 Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.

Answer. As the parallelogram and the rectangle have the same base and equal area, therefore, these will also lie between the same parallels.
Consider the parallelogram ABCD and rectangle ABEF as follows.



Here, it can be observed that parallelogram ABCD and rectangle ABEF are between the same parallels AB and CF.
We know that opposite sides of parallelogram or a rectangle are of equal lengths.
Therefore,
AB = EF (For rectangle)
AB = CD (For parallelogram)
$\therefore CD = EF$
$\therefore AB + CD = AB + EF$...(1)
Of all the lines segments that can be drawn to a given line from a point not lying on it, the perpendicular line segment is the shortest.
$\therefore \mathrm{AF} < \mathrm{AD}$
And similarly, BE < BC
$\therefore \mathrm{AF} + \mathrm{BE} < \mathrm{AD} + \mathrm{BC} \quad \ldots(2)$
FRom equations (1) and (2), we obtain
AB + EF + AF + BE < AD + BC + AB + CD
Perimeter of rectangle ABEF < Perimeter of parallelogram ABCD.

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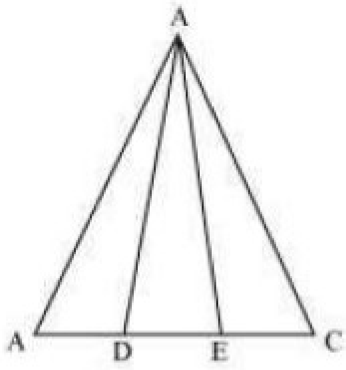
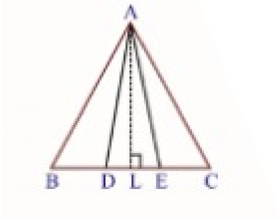
Q2 In Figure, D and E are two points on BC such that BD = DE = EC. Show that ar (ABD) = ar (ADE) = ar (AEC).



Can you now answer the question that you have left in the 'Introduction' of this chapter, whether the field of Budhia has been actually divided into three parts of equal area?
[Remark: Note that by taking BD = DE = EC, the triangle ABC is divided into three triangles ABD, ADE and AEC of equal areas. In the same way, by dividing BC into n equal parts and joining the points of division so obtained to the opposite vertex of BC, you can divide ΔABC into n triangles of equal areas.]

Answer. Let us draw a line segment $AL \perp BC$



We know that,
Area of a triangle = $\frac{1}{2} \times \text{ Base } \times \text{ Altitude}$
Area $(\triangle ADE) = \frac{1}{2} \times DE \times AL$
Area $(\Delta ABD) = \frac{1}{2} \times BD \times AL$
Area $(\triangle AEC) = \frac{1}{2} \times EC \times AL$
It is given that DE = BD =EC
$\frac{1}{2} \times DE \times AL = \frac{1}{2} \times BD \times AL = \frac{1}{2} \times EC \times AL$
Area $(\triangle ADE) = \text{ Area } (\Delta ABD) = \text{ Area } (\triangle AEC)$
It can be observed that Bhudhia has divided her field into 3 equal parts.

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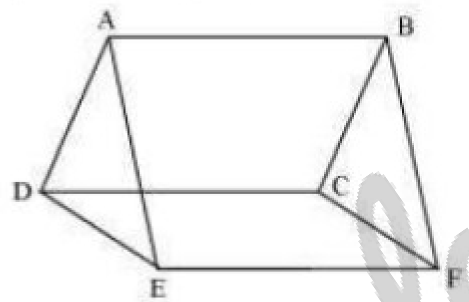
Q3 In Figure, ABCD, DCFE and ABFE are parallelograms. Show that ar (ADE) = ar (BCF).



Answer. It is given that ABCD is a parallelogram. We know that opposite sides of a parallelogram are equal.
$\therefore$ AD = BC …(1)
Similarly, for parallelograms DCEF and ABFE, it can be proved that
DE = CF…(2)
And, EA = FB….(3)
In $\triangle$ADE and ΔBCF,
AD = BC [Using equation (1)]
DE = CF [Using equation (2)]
EA = FB [Using Equation (3)]
$\therefore \triangle \text{ADE} \cong \triangle \text{BCF}$(SSS congruence rule)
$\therefore (\Delta ADE) = \text{ Arca } (\Delta BCF)$

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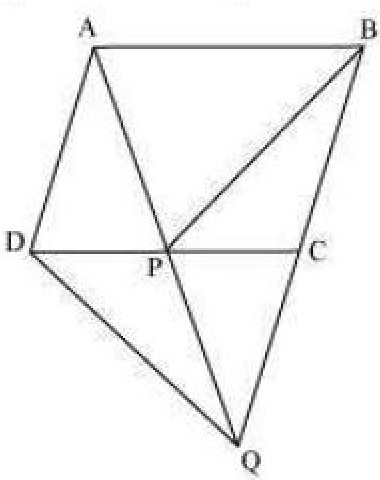
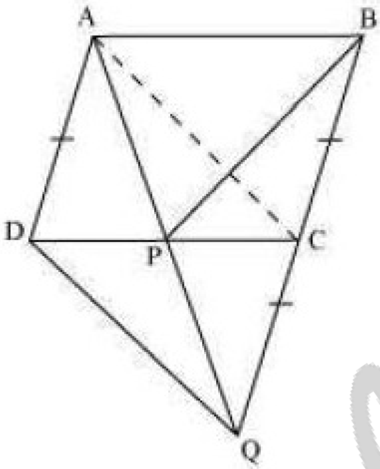
Q4 In Figure, ABCD is a parallelogram and BC is produced to a point Q such that AD = CQ. If AQ intersect DC at P, show that ar (BPC) = ar (DPQ).
[Hint : Join AC.]



Answer. It is given that ABCD is a parallelogram.
AD || BC and AB || DC(Opposite sides of a parallelogram are parallel to each other)
Join point A to point C.



Consider $\triangle \mathrm{APC}$ and $\triangle \mathrm{BPC}$
$\triangle \mathrm{APC}$ and $\Delta \mathrm{BPC}$ are lying on the same base PC and between the same parallels PC and AB.
Therefore,
$\text{Area } (\Delta \mathrm{APC}) = \text{ Area } (\Delta \mathrm{BPC}) \ldots (1)$
In quadrilateral ACDQ, it is given that
AD = CQ
Since ABCD is a parallelogram,
AD || BC (Opposite sides of a parallelogram are parallel)
CQ is a line segment which is obtained when line segment BC is produced.
$\therefore$ AD || CQ
We have,
AC = DQ and AC || DQ
Hence, ACQD is a parallelogram.
Consider BDCQ and BACQ
These are on the same base CQ and between the same parallels CQ and AD.
Therefore,
Area $(\Delta DCQ) = A\,\text{rca}(\Delta ACQ)$
$\therefore$ Area $(\Delta DCQ) - \text{ Arca } (\Delta PQC) = \text{ Area } (\Delta ACQ) - \text{ Area } (\Delta PQC)$
$\therefore \text{ Area } (\Delta \mathrm{DPQ}) = \text{ Arca } (\Delta \mathrm{APC}) - (2)$
From equations (1) and (2), we obtain
Area (ΔBPC) = Area (ΔDPQ)

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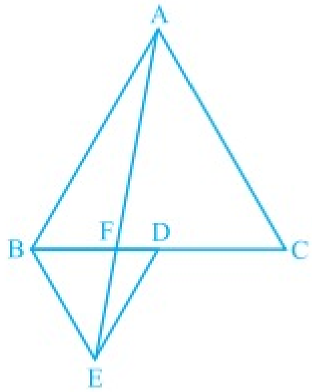
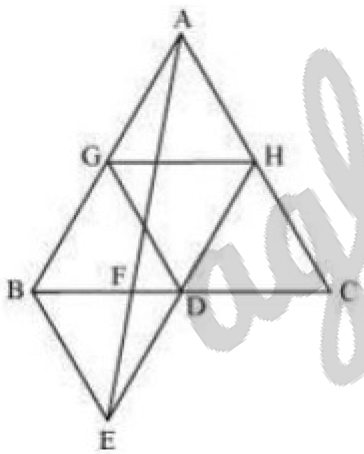
Q5 In Figure, ABC and BDE are two equilateral triangles such that D is the mid-point of BC. If AE intersects BC at F, show that



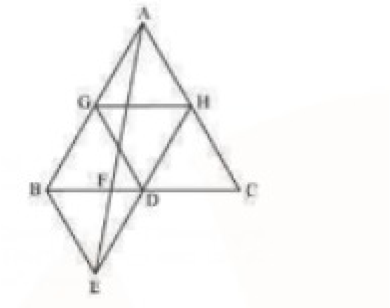
(i) $\mathrm{ar(BDE)} = \frac{1}{4}\mathrm{ar(ABC)}$
(ii) $\mathrm{ar(BDE)} = \frac{1}{2}\mathrm{ar(BAE)}$
(iii) $\mathrm{ar(ABC)} = 2\mathrm{ar(BEC)}$
(iv) $\mathrm{ar(BFE)} = \mathrm{ar(AFD)}$
(v) $\mathrm{ar(BFE)} = 2\mathrm{ar(FED)}$
(vi) $\mathrm{ar(FED)} = \frac{1}{8}\mathrm{ar(AFC)}$
[Hint : Join EC and AD. Show that BE || AC and DE || AB, etc.]

Answer. (i) Let G and H be the mid-points of side AB and AC respectively.
Line segment GH is joining the mid-points. Therefore, it will be parallel to third side BC and also its length will be half of the length of 3C (mid-point theorem).



$\therefore GH = \frac{1}{2}BC$ and $GH \| BD$
$\therefore \mathrm{GH = BD = DC}$ and $\mathrm{GH \| BD (D}$ is the mid-point of $\mathrm{BC)}$
Similarly,
GD = HC = HA
HD = AG BG
Therefore, clearly $\triangle$ABC is divided into 4 equal equilateral triangles viz
$\triangle BGD, \triangle AGH, \Delta DHC$ and ΔGHD
In other words, $\Delta BGD = \frac{1}{4}\Delta ABC$
In other words, $\Delta BGD = \frac{1}{4}\Delta ABC$
Now consider $\triangle BDG$ and ΔBDE
BD = BD(Common base)
As both triangles are equilateral triangle, we can say
BG = BE
DG = DE

Therefore, $\Delta BDG \cong \triangle BDE[By$ SSS congruency $]$
Thus, area $(\Delta BDG)=$ area(ΔBDE)
ar$(\Delta BDE)=\frac{1}{4}$ ar $[\Delta ABC)$
Hence proved



(ii) Area $(\Delta BDE)=$ Area $(\triangle AED)$ (Common base DE and DE ∥ AB)
Area $(\Delta BDE)-$ Arca $(\Delta FED)=Area(\Delta AED)-Area(\Delta FED)$
Area $(\Delta BEF)=Area(\Delta AFD)$ ..(1)
Now, Area $(\Delta ABD)=$ Arca $(\Delta ABF)+$ Area (ΔAFD)
Area $(\Delta ABD)=$ Area $(\Delta ABF)+$ Area (ΔBEF) [From equation (1)]
Area $(\Delta ABD)=$ Area $(\Delta ABE)\quad \ldots(2)$
AD is the median in $\triangle ABC$
$(\Delta \mathrm{ABD})=\frac{1}{2}$ ar $(\Delta \mathrm{ABC})$
$=\frac{4}{2}$ar(ΔBDE) (As proved earlier)
ar (ΔABD) = 2 ar (ΔBDE) (3)
From (2) and (3), we obtain
2 ar ((\Delta BDE)=\text { ar }(\Delta A B E) \)
ar $(BDE)=\frac{1}{2}$ ar (BAE)
(iii) ar $(\Delta ABE)=$ ar (ΔBEC) (Common base BE and BE||AC)
ar ((\Delta A B F)+\text { ar }(\Delta B E F)= \) ar $(\triangle BEC)$
Using equations (1), we obtain
$\operatorname{ar}(\Delta \mathrm{ABF})+\operatorname{ar}(\Delta \mathrm{AFD})=\operatorname{ar}(\Delta \mathrm{BEC})$
$\operatorname{ar}(\Delta \mathrm{ABD})=\operatorname{ar}(\Delta \mathrm{BEC})$
$\frac{1}{2}\operatorname{ar}(\Delta \mathrm{ABC})=\operatorname{ar}(\Delta \mathrm{BEC})$
$\operatorname{ar}(\Delta ABC)=2$ ar (ΔBEC)
(iv) It is seen that $\triangle BDE$ and ar $\triangle AED$ lie on the base (DE) and between the parallels DE and AB.
$\therefore \operatorname{ar}(\Delta \mathrm{BDE})=\operatorname{ar}(\Delta \mathrm{AED})$
$\therefore \operatorname{ar}(\Delta BDE)-\operatorname{ar}(\Delta FED)=\operatorname{ar}(\Delta AED)-\operatorname{ar}(\Delta FED)$
$\therefore \operatorname{ar}(\Delta BFE)=\operatorname{ar}(\Delta AFD)$
(v) Let h be the height of vertex E, corresponding to the side BD in $\triangle$BDE .
Let H be the height of vertex A, corresponding to the side BC in $\triangle$ABC.
In (i), it was shown that $\operatorname{ar}(\mathrm{BDE})=\frac{1}{4}\operatorname{ar}(\mathrm{ABC})$
$\therefore \frac{1}{2}\times \mathrm{BD}\times h=\frac{1}{4}\left(\frac{1}{2}\times \mathrm{BC}\times H\right)$
$\Rightarrow \mathrm{BD}\times h=\frac{1}{4}(2\mathrm{BD}\times H)$
$\Rightarrow h=\frac{1}{2}H$
In (iv), it was shown that $\operatorname{ar}(\Delta BFE)=\operatorname{ar}(\Delta AFD)$
$\therefore$ ar $(\triangle BFE)=$ ar $(\Delta \mathrm{AFD})$
= 2 ar (ΔFED)
Hence,
(vi) Area $(\mathrm{AFC})=\operatorname{area}(\mathrm{AFD})+\operatorname{area}(\mathrm{ADC})$
$=\operatorname{ar}(\mathrm{BFE})+\frac{1}{2}\operatorname{ar}(\mathrm{ABC})\quad [\ln(\mathrm{iv}), \operatorname{ar}(\mathrm{BFE})=\operatorname{ar}(\mathrm{AFD}); \mathrm{AD}$ is median of $\Delta \mathrm{ABC}]$

$= \text{ar(BFE)} + \frac{1}{2} \times 4\,\text{ar(BDE)} \quad \left[\text{In(i)}, \text{ar(BDE)} = \frac{1}{4}\text{ar(ABC)}\right]$
$= \text{ar(BFE)} + 2\,\text{ar(BDE)}$ ...(5)
Now, by (v), ar $\text{(BFE)} = 2\,\text{ar(FED)}$ ....(6)
$\text{arc(BDE)} = \text{ar(BFE)} + \text{ar(FED)} = 2\,\text{ar(FED)} + \text{ar(FED)} = 3\,\text{ar(FED)}$ ....(7)
Therefore, from equations (5), (6), and (7), we ger:
$\text{ar(AFC)} = 2\,\text{ar(FED)} + 2 \times 3\,\text{ar(FED)} = 8\text{ar(FED)}$
$\therefore \text{ar(AFC)} = \text{sar(FED)}$
Hence, $\text{ar(FED)} = \frac{1}{8}\text{ar(AFC)}$

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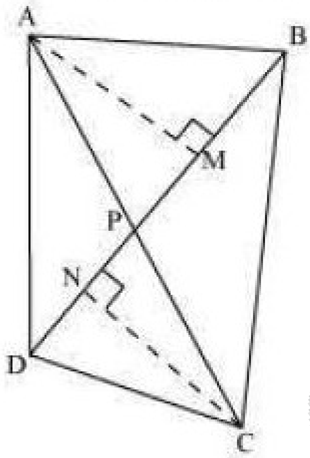
Q6 Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that ar (APB) × ar (CPD) = ar (APD) × ar (BPC).
[Hint :From A and C, draw perpendiculars to BD.]

Answer. A quadrilateral ABCD, in which diagonals AC and BD intersect each other at point E.



To Prove : ar $(\triangle AED) \times \text{arc}(\Delta BEC)$
$= \text{arc}(\triangle ABE) \times \text{arc}(\Delta CDE)$
Construction From A, draw $AM \perp BD$ and $CN \perp BD$
Proof : : $\text{ar}(\Delta\text{ABE}) = \frac{1}{2} \times \text{BE} \times \text{AM}$ .....(i)
$ar(\Delta AED) = \frac{1}{2} \times DE \times AM$ ....(ii)
Dividing eq.(ii) by (i), we get,
$\frac{\text{ar}(\Delta\text{AED})}{\text{ar(GABE)}} = \frac{\frac{1}{2}\times\text{DE}\times\text{AM}}{\frac{1}{2}\times\text{BE}\times\text{AM}}$
$\Rightarrow \frac{\text{ar}(\Delta\text{AED})}{\text{ar}(\Delta\text{ABE})} = \frac{\text{DE}}{\text{BE}}$ ....(iii)
Similarly $\frac{\text{ar}(\Delta\text{CDE})}{\text{ar}(\Delta\text{BEC})} - \frac{\text{DE}}{\text{BE}}$ ....(iv)
From eq.(iii) and (iv), we get
$\frac{\text{ar}(\Delta\text{AED})}{\text{ar}(\triangle\text{ABE})} == \frac{\text{ar}(\Delta\text{CDE})}{\text{ar}(\Delta\text{BEC})}$
$\Rightarrow \text{arc}(\Delta AED) \times \text{ar}(\Delta BEC) = \text{arc}(AABE) \times \text{arc}(\Delta CDE)$
Hence proved.

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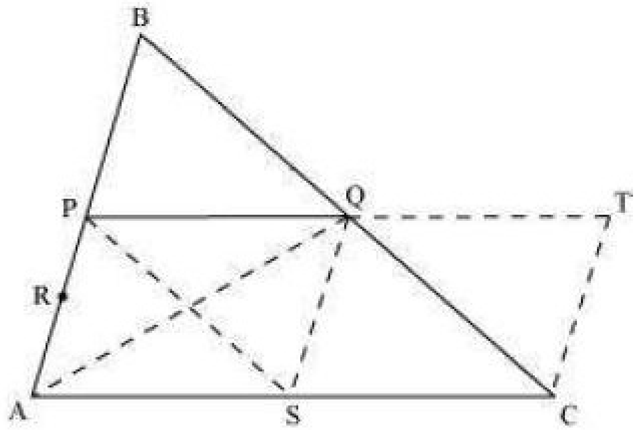
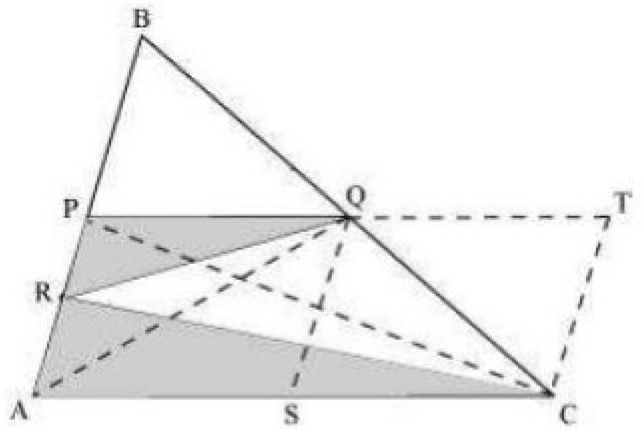
Q7 P and Q are respectively the mid-points of sides AB and BC of a triangle ABC and R is the mid-point of AP, show that
(i) $\text{ar(PRQ)} = \frac{1}{2}\text{ar(ARC)}$

(ii) $\mathrm{ar}(\mathrm{RQC}) = \frac{3}{8}\mathrm{ar}(\mathrm{ABC})$
(iii) $\mathrm{ar}(\mathrm{PBQ}) = \mathrm{ar}(\mathrm{ARC})$

Answer. Take a point S on AC such that S is the mid-point of AC.
Extend PQ to T such that PQ = QT.
Join TC, QS, PS, and AQ.



In $\triangle\mathrm{ABC}$, P and Q are the mis-points of AB and BC respectively. Hence, by using mid-point theorem, we obatin
$\mathrm{PQ} \| \mathrm{AC}$ and $\mathrm{PQ} = \frac{1}{2}\mathrm{AC}$
$\therefore$ PQ || AC and PQ = AS (As S is the mid-point of AC)
$\therefore$ PQSA is a parallelogram. We know that diagonals of a parallelogram bisect it into equal areas of triangles.
$\therefore$ ar $(\Delta\mathrm{PAS}) = \mathrm{ar}(\Delta\mathrm{SQP}) = \mathrm{ar}(\Delta\mathrm{PAQ}) = \mathrm{ar}(\Delta\mathrm{SQA})$
Similarly, it can also be proved that quadrilaterals PSCQ, QSCT, AND PSQB are also parallelograms and therefore,
$\mathrm{ar}(\Delta\mathrm{PSQ}) = \mathrm{ar}(\Delta\mathrm{CQS})(\text{ For parallelogram PSCQ})$
$\mathrm{ar}(\Delta\mathrm{QSC}) = \mathrm{ar}(\Delta\mathrm{CTQ})(\text{ For parallelogram QSCT})$
$\mathrm{ar}(\Delta\mathrm{PSQ}) = \mathrm{ar}(\Delta\mathrm{QBP})(\text{ For parallelogram PSQB})$
Thus,
$\mathrm{ar}\ (\Delta\mathrm{PAS}) = \mathrm{ar}(\Delta\mathrm{SQP}) = \mathrm{ar}(\Delta\mathrm{PAQ}) = \mathrm{ar}(\Delta\mathrm{SQA}) = \mathrm{ar}(\Delta\mathrm{QSC}) = \mathrm{ar}(\Delta\mathrm{CTQ}) = \mathrm{ar}\ (\Delta\mathrm{QBP})\ldots(1)$
$(\Delta\mathrm{ABC}) = \mathrm{ar}(\Delta\mathrm{PBQ}) + \mathrm{ar}(\Delta\mathrm{PAS}) + \mathrm{ar}(\Delta\mathrm{PQS}) + \mathrm{ar}(\Delta\mathrm{QSC})$
$\mathrm{ar}(\Delta\mathrm{ABC}) = \mathrm{ar}(\Delta\mathrm{PBQ}) + \mathrm{ar}(\Delta\mathrm{PBQ}) + \mathrm{ar}(\Delta\mathrm{PBQ}) + \mathrm{ar}(\Delta\mathrm{PBQ})$
$= \mathrm{ar}(\Delta PBQ) + \mathrm{ar}(\Delta PBQ) + \mathrm{ar}(\Delta PBQ) + \mathrm{ar}(\Delta PBQ)$
$= 4\ \mathrm{ar}\ (\Delta\mathrm{PBQ})$
$\therefore$ ar $\mathrm{ar}(\Delta\mathrm{PBQ}) = \frac{1}{4}\mathrm{ar}(\Delta\mathrm{ABC})\ldots(2)$
(i) Join point P to C.



In ΔPAQ, QR is the median.

$\therefore \operatorname{ar}(\Delta PRQ) = \frac{1}{2}\operatorname{ar}(\Delta PAQ) = \frac{1}{2} \times \frac{1}{4}\operatorname{ar}(\Delta ABC) = \frac{1}{8}\operatorname{ar}(\Delta ABC) \ldots \ldots (3)$

In $\triangle ABC$, P and Q are the mid-points of AB and BC respectively. Hence, by using mid-point theorem, we obtain

PQ = 1/2 AC

$AC = 2PQ \Rightarrow AC = PT$

Also, $PQ \| AC \Rightarrow PT \| AC$

Hence, PACT is a parallelogram.

$\operatorname{ar}(PACT) = \operatorname{ar}(PACQ) + \operatorname{ar}(\Delta QTC)$

$= \operatorname{ar}(PACQ) + \operatorname{ar}(\Delta PBQ[\text{Using equation (1)}]$

$\therefore \operatorname{ar}(PACT) = \operatorname{ar}(\Delta ABC) \ldots (4)$

$\operatorname{ar}(\Delta ARC) = \frac{1}{2}\operatorname{ar}(\Delta PAC) \quad (\text{CR is the median of } \Delta PAC)$

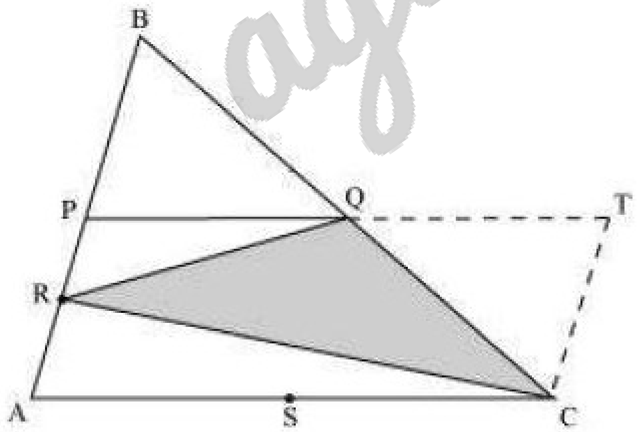
$= \frac{1}{2} \times \frac{1}{2}\operatorname{ar}(PACT)(\text{PC is the diagonal of parallelogram PACT})$

$= \frac{1}{4}\operatorname{ar}(\Delta PACT) = \frac{1}{4}\operatorname{ar}(\Delta ABC)$

$\Rightarrow \frac{1}{2}\operatorname{ar}(\Delta ARC) = \frac{1}{8}\operatorname{ar}(\Delta ABC)$

$\Rightarrow \frac{1}{2}\operatorname{ar}(\Delta ARC) = \operatorname{ar}(\Delta PRQ)[\text{ Using equation (3)}] \ldots (5)$

(ii)



$\operatorname{ar}(PACT) = \operatorname{ar}(\Delta PRQ) + \operatorname{ar}(\Delta ARC) + \operatorname{ar}(\Delta QTC) + \operatorname{ar}(\Delta RQC)$

$(1), (2), (3), (4), \text{ and } (5)$, we obtain

$\operatorname{ar}(\Delta ABC) = \frac{1}{8}\operatorname{ar}(\Delta ABC) + \frac{1}{4}\operatorname{ar}(\Delta ABC) + \frac{1}{4}\operatorname{ar}(\Delta ABC) + \operatorname{ar}(\Delta RQC)$

$\operatorname{ar}(\Delta ABC) = \frac{5}{8}\operatorname{ar}(\Delta ABC) + \operatorname{ar}(\Delta RQC)$

$\operatorname{ar}(\Delta RQC) = \left(1 - \frac{5}{8}\right)\operatorname{ar}(\Delta ABC)$

$\operatorname{ar}(\Delta RQC) = \frac{3}{8}\operatorname{ar}(\Delta ABC)$

(iii) In parallelogram PACT,

$\mathrm{ar}(\Delta \mathrm{ARC})=\frac{1}{2} \mathrm{ar}(\Delta \mathrm{PAC}) \quad$ (CR is the median of $\Delta \mathrm{PAC}$)
$=\frac{1}{2} \times \frac{1}{2} \mathrm{ar}(\mathrm{PACT})$(PC is the diagonal of parallelogram PACT)
$=\frac{1}{4} \mathrm{ar}(\Delta \mathrm{PACT})$
$=\frac{1}{4} \mathrm{ar}(\Delta \mathrm{ABC})$
$=\mathrm{ar}(\Delta \mathrm{PBQ})$

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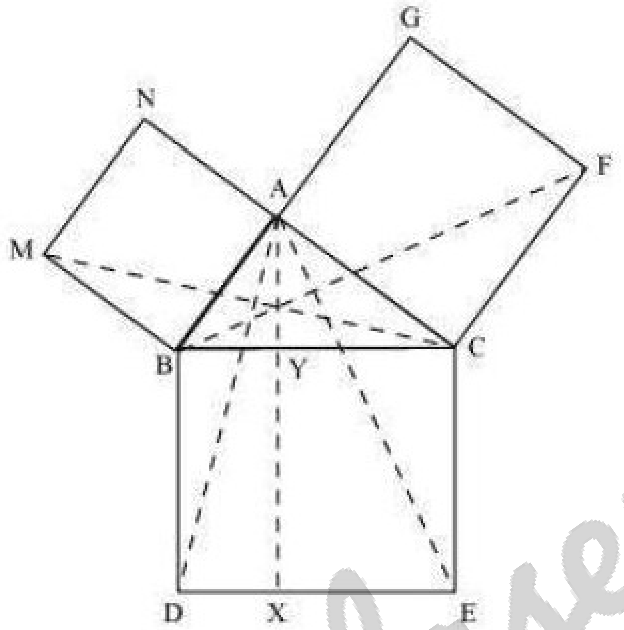
Q8 In Figure, ABC is a right triangle right angled at A. BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment AX $\perp$ DE meets BC at Y. Show that:



(i) $\Delta \mathrm{MBC} \cong \triangle \mathrm{ABD}$
(ii) $\mathrm{ar}(\mathrm{BYXD})=2\, \mathrm{ar}(\mathrm{MBC})$
(iii) ar $(\mathrm{BYXD})=\mathrm{ar}(\mathrm{ABMN})$
(iv) $\Delta \mathrm{FCB} \cong \triangle \mathrm{ACE}$
(v) $\mathrm{ar}(\mathrm{CYXE})=2 \mathrm{ar}(\mathrm{FCB})$
(vi) $\mathrm{ar}(\mathrm{CYXE})=\mathrm{ar}(\mathrm{ACFG})$
(vii) $\mathrm{ar}(\mathrm{BCFD})=\mathrm{ar}(\mathrm{ABMN})+\mathrm{ar}(\mathrm{ACFG})$
Note : Result (vii) is the famous Theorem of Pythagoras. You shall learn a simpler proof of this theorem in Class X.

Answer. (i) We know that each angle of a square is 90° .
Hence, $\angle ABM=\angle DBC=90^{\circ}$
$\therefore \angle ABM+\angle ABC=\angle DBC+\angle ABC$
$\therefore \angle MBC=\angle ABD$
In $\triangle MBC$ and $\triangle ABD$
$\angle MBC=\angle ABD$(Proved above)
$\mathrm{MB}=\mathrm{AB}$(Sides of square ABMN)
$BC=BD$(Sides of square BCED)
$\therefore \triangle \mathrm{MBC} \cong \triangle \mathrm{ABD}$ (SAS congruence rule)
(ii) We have
$\triangle MBC \cong \triangle ABD$
$\therefore(\Delta MBC)=$ ar $(\Delta ABD) \ldots(1)$
It is given that $AX \perp DE$ and $BD \perp DE$ (Adjacent sides of square BDEC)
$\therefore \mathrm{BD} \| \mathrm{AX}$ (Two lines perpendicular to same line are parallel to each other)

$\triangle ABD$ and parallelogram BYXD are on the same base BD and between the same parallels BD and AX.
Area $(\Delta YXD) = 2$ Area (ΔMBC) [Using equation (1)]...(2)
(iii) $\triangle$MBC and parallelogram ABMN are lying on the same base MB and between same parallels MB and NC.
2 ar $(\Delta MBC) = ar(ABMN)$
ar $(\Delta YXD) =$ ar $(ABMN)$ [Using equation (2)]...(3)
(iv) We know that each angle of a square is 90°
$\therefore \angle FCA = \angle BCE = 90^{\circ}$
$\therefore \angle FCA + \angle ACB = \angle BCE + \angle ACB$
$\therefore \angle FCB = \angle ACE$
In ΔFCB and $\triangle ACE$
$\angle FCB = \angle ACE$
$\mathrm{FC} = \mathrm{AC}(\text{ Sides of square ACFG})$
$\mathrm{CB} = \mathrm{CE}(\text{ Sides of square BCED})$
$\Delta\mathrm{FCB} \cong \triangle\mathrm{ACE}$ (SAS congruence rule)
(v) It is given that $AX \perp DE$ and $CE \perp DE$ (Adjacent sides of square BDEC)
Hence, CE || AX (Two lines perpendicular to th same line are parallel to each other)
Consider BACE and parallelogram CYXE
BACE and parallelogram CYXE are on the same base CE and between the same parallels CE and AX.
$\therefore$ ar $(\Delta YXE) = 2$ ar $(\Delta ACE) \ldots (4)$
We had proved that
$\triangle\Delta\mathrm{FCB} \cong \triangle\mathrm{ACE}$
$\mathrm{ar}(\Delta FCB) \cong \mathrm{ar}(\Delta ACE) - (5)$
On comparing equations (4) and (5), we obtain
$(\mathrm{CYXE}) = 2$ ar $(\Delta\mathrm{FCB})$ ..(6)
(vi) Consider BFCB and parallelogram ACFG
BFCB and parallelogram ACFG are lying on the same base CF and between the same parallels CF and BG.
$\therefore$ ar $(\mathrm{ACFG}) = 2\mathrm{ar}(\Delta\mathrm{FCB})$
$\therefore$ ar $(\mathrm{ACFG}) = \mathrm{ar}(\mathrm{CYXE})$ [Using equation(6)]...(7)
(vii) From the figure, it is evident that
ar $(\Delta CED) = \mathrm{ar}(\Delta YXD) + \mathrm{ar}(CYXE)$
$\therefore$ ar $(\Delta CED) = \mathrm{ar}(ABMN) + \mathrm{ar}(ACFG)$ [Using equations (3) and (7)].

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