NCERT SOLUTIONS

CLASS - 9th

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Class : 9th
Subject : Maths
Chapter : 12
Chapter Name : Heron's Formula

Exercise 12.1

Q1 A traffic signal board, indicating 'SCHOOL AHEAD', is an equilateral triangle with side 'a'. Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board?

Answer. Side of traffic signal board = a
Perimeter of traffic signal board
$2s = 3a \Rightarrow s = \frac{3}{2}a$
By Heron's formula,
Area of given triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$\text{Area of given triangle } = \sqrt{\frac{3}{2}a\left(\frac{3}{2}a-a\right)\left(\frac{3}{2}a-a\right)\left(\frac{3}{2}a-a\right)}$$

$$= \sqrt{\left(\frac{3}{2}a\right)\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)} \qquad \text{....(1)}$$

$$= \frac{\sqrt{3}}{4}a^2$$

Perimeter of traffic signal board = 180 cm
Side of traffic signal board $(a) = \left(\frac{180}{3}\right) \text{cm} = 60\text{cm}$

$$\text{Using equation (1), area of traffic signal board } = \frac{\sqrt{3}}{4}(60\text{cm})^2$$

$$= \left(\frac{3600}{4}\sqrt{3}\right)\text{cm}^2 = 900\sqrt{3}\text{cm}^2$$

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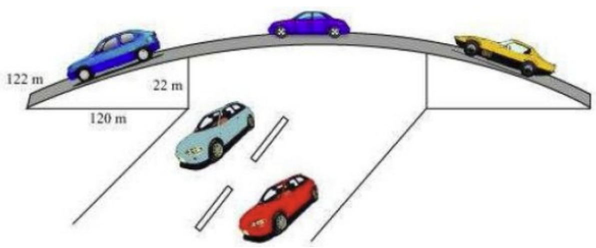
Q2 The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122 m, 22 m and 120 m (see Fig). The advertisements yield an earning of 5000 per m^2 per year. A company hired one of its walls for 3 months. How much rent did it pay?



Answer. The sides of the triangle (i.e., a, b, c) are of 122 m, 22 m, and 120 m respectively.
Perimeter of triangle = (122 + 22 + 120) m
2s = 264 m
s = 132 m
By Heron's formula,
Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$\text{Area of given triangle } = [\sqrt{132(132-122)(132-22)(132-120)}]\mathrm{m}^2$$
$$= [\sqrt{132(10)(110)(12)}]\mathrm{m}^2 = 1320\mathrm{m}^2$$

Rent of 1 m^2 area per year = Rs 5000
Rent of 1 m^2 area per month = Rs $\frac{5000}{12}$
Rent of 1320 m^2 area per 3 month = $\mathrm{Rs}\left(\frac{5000}{12} \times 3 \times 1320\right)$
= Rs (5000 x 330) = Rs 1650000
Therefore, the company had to pay Rs 1650000.

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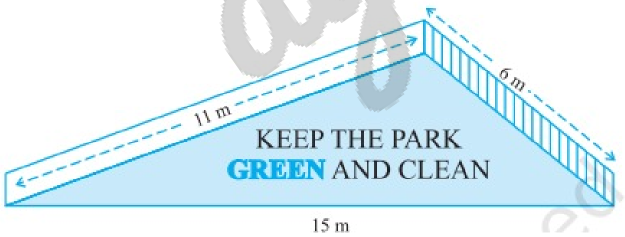
Q3 There is a slide in a park. One of its side walls has been painted in some colour with a message “KEEP THE PARK GREEN AND CLEAN” (see Fig). If the sides of the wall are 15 m, 11 m and 6 m, find the area painted in colour.



Answer. Sides of the triangular wall are 15 m, 11m, an 6m.
Semi perimeter of triangular wall (s) = (15 + 11 + 6) / 2 m = 16 m
Using Heron’s Formula,

$$\text{Area of the message} = \sqrt{\mathrm{s}(\mathrm{s}-\mathrm{a})(\mathrm{s}-\mathrm{b})(\mathrm{s}-\mathrm{c})}$$
$$= \sqrt{16(16-15)(16-11)(16-6)}\mathrm{m}^2$$
$$= \sqrt{16 \times 1 \times 5 \times 10}\mathrm{m}^2 = \sqrt{800}\mathrm{m}^2$$
$$= 20\sqrt{2}\mathrm{m}^2$$

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Q4 Find the area of a triangle two sides of which are 18cm and 10cm and the perimeter is 42cm.

Answer. Let the third side of the triangle be x.
Perimeter of the given triangle 42 cm
18 cm + 10 cm + x = 42
x = 14 cm
$s=\frac{\text{Perimeter}}{2}=\frac{42\text{cm}}{2}=21\text{cm}$
By Heron's formula,
Area of a triangle == $\sqrt{s(s-a)(s-b)(s-c)}$

$$\begin{aligned}\text{Area of the given triangle } &= (\sqrt{21(21-18)(21-10)(21-14)})\text{cm}^2\\ &= (\sqrt{21(3)(11)(7)})\text{cm}^2\\ &= 21\sqrt{11}\text{cm}^2\end{aligned}$$

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Q5. Sides of a triangle are in the ratio of 12 : 17 : 25 and its perimeter is 540cm. Find its area.

Answer. Let the common ratio between the sides of the given triangle be x.
Therefore, the side Of the triangle will be 12x, 17x, and 25x.
Perimeter of this triangle = 540 cm
12x + 17x + 25x = 540 cm
54x = 540 cm
x = 10 cm
Sides of the triangle will be 120 cm, 170 cm, and 250 cm.
$s=\frac{\text{Perimeter of triangle}}{2}=\frac{540\text{cm}}{2}=270\text{cm}$
By Heron's formula,

$$\begin{aligned}\text{Area of triangle } &= \sqrt{s(s-a)(s-b)(s-c)}\\ &= [\sqrt{270(270-120)(270-170)(270-250)}]\text{cm}^2\\ &= [\sqrt{270\times 150\times 100\times 20}]\text{cm}^2\end{aligned}$$

Therefore, the area Of this triangle is 9000 cm^2 .

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Q6 An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle.

Answer. Let the third side Of this triangle be x.
Perimeter of triangle = 30 cm
12 cm + 12 cm + x = 30 cm

x = 6 cm

$s = \frac{\text{Perimeter of triangle}}{2} = \frac{30\text{cm}}{2} = 15\text{cm}$

By Heron's formula,

$$\begin{aligned} \text{Area of triangle } &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= [\sqrt{15(15-12)(15-12)(15-6)}]\text{cm}^2 \\ &= [\sqrt{15(3)(3)(9)}]\text{cm}^2 \\ &= 9\sqrt{15}\text{cm}^2 \end{aligned}$$

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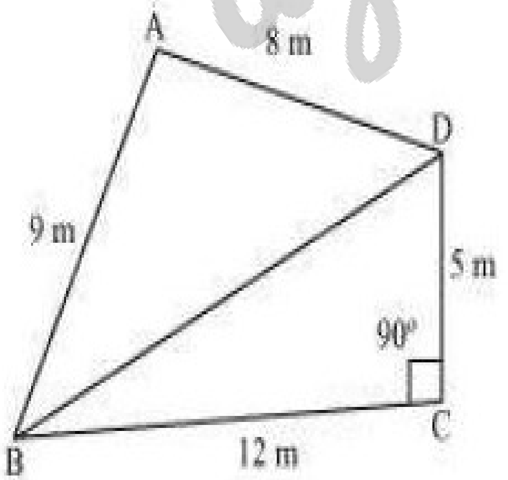
Exercise 12.2

Q1 A park, in the shape of a quadrilateral ABCD, has ∠ C = 90°, AB = 9 m, BC = 12 m, CD = 5 m and AD = 8 m. How much area does it occupy?

Answer. Let us join BD.

In A3CD, applying Pythagoras theorem,

$$\text{BD}^2 = \text{BC}^2 + \text{CD}^2$$
$$= (12)^2 + (5)^2$$
$$= 144 + 25$$
$$\text{BD}^2 = 169$$
$$\text{BD} = 13\text{m}$$



Area of Δ BCD

$$= \frac{1}{2} \times \text{BC} \times \text{CD} = \left(\frac{1}{2} \times 12 \times 5\right) \text{m}^2 = 30\text{m}^2$$

For ΔABD_r

$s = \frac{\text{Perimeter}}{2} = \frac{(9+8+13)\text{m}}{2} = 15\text{m}$

By Heron's formula,

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

Area of ΔABD $= [\sqrt{15(15-9)(15-8)(15-13)}]\text{m}^2$

$= (\sqrt{15 \times 6 \times 7 \times 2})\text{m}^2$

$= 6\sqrt{35}\text{m}^2$

$= (6 \times 5.916)\text{m}^2$

$= 35.496\text{m}^2$

Area of the park $=$ Area of ΔABD $+$ Area of ΔBCD

$= 35.496 + 30\text{m}^2 = 65.496\text{m}^2 = 65.5\text{m}^2$(approximately)

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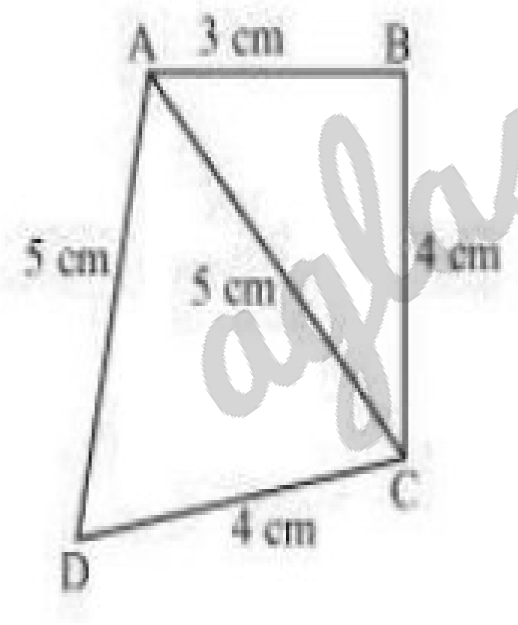
Q2 Find the area of a quadrilateral ABCD in which AB = 3 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 5 cm.

Answer.



For $\triangle ABC$,

$AC^2 = AB^2 + BC^2$

$(5)^2 = (3)^2 + (4)^2$

Therefore, $\triangle ABC$ is a right-angled triangle, right-angled at point B.

Area of Δ ABC, $= \frac{1}{2} \times \text{AB} \times \text{BC} = \frac{1}{2} \times 3 \times 4 = 6\text{cm}^2$

For ΔADC,

Perimeter $= 2s = \mathrm{AC} + \mathrm{CD} + \mathrm{DA} = (5+4+5)\mathrm{cm} = 14\mathrm{cm}$

$s = 7\mathrm{cm}$

By Heron's formula,

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$\begin{aligned}\text{Area of } \Delta\mathrm{ADC} &= [\sqrt{7(7-5)(7-5)(7-4)}]\mathrm{cm}^2 \\ &= (\sqrt{7\times 2\times 2\times 3})\mathrm{cm}^2 \\ &= 2\sqrt{21}\mathrm{cm}^2 \\ &= (2\times 4.583)\mathrm{cm}^2 \\ &= 9.166\mathrm{cm}^2\end{aligned}$$

Area of ABCD $=$ Area of ΔABC $+$ Area of ΔACD

$= (6+9.166)\mathrm{cm}^2 = 15.166\mathrm{cm}^2 = 15.2\mathrm{cm}^2$(approximately)

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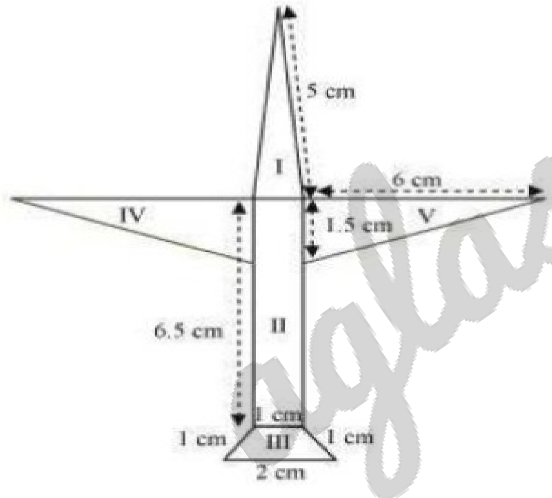
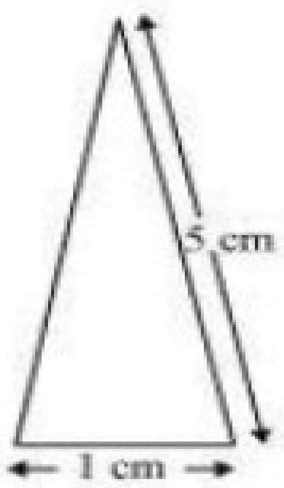
Q3 Radha made a picture of an aeroplane with coloured paper as shown in Fig 12.15. Find the total area of the paper used.



Answer.



This triangle is an isosceles triangle.

Perimeter $= 2s = (5+5+1)\text{cm} = 11\text{cm}$

$s = \frac{11\text{cm}}{2} = 5.5\text{cm}$

Area of the triangle $-\sqrt{s(s-a)(s-b)(s-c)}$

$= [\sqrt{5.5(5.5-5)(5.5-5)(5.5-1)}]\text{cm}^2$

$= [\sqrt{(5.5)(0.5)(0.5)(4.5)}]\text{cm}^2$

$= 0.75\sqrt{11}\text{cm}^2$

$= (0.75 \times 3.317)\text{cm}^2$

$= 2.488\text{cm}^2$ (approximately)

For quadrilateral II

This quadrilateral is a rectangle.

Area $= I \times b = (6.5 \times 1)\text{cm}^2 = 6.5\text{cm}^2$

For quadrilateral Ill

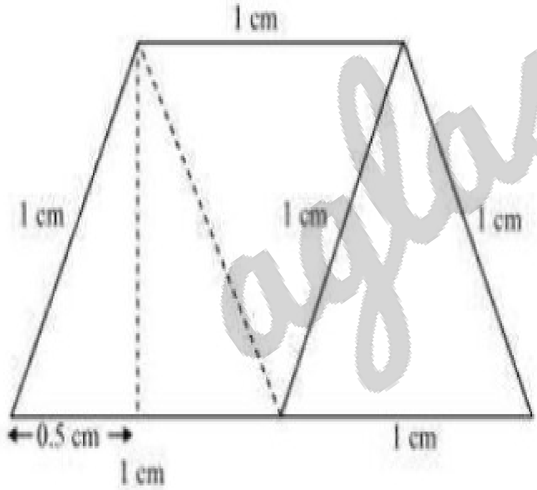
This quadrilateral is a trapezium.

Perpendicular height of parallelogram $= \left(\sqrt{1^2 - (0.5)^2}\right)$ cm

$= \sqrt{0.75}\text{cm} = 0.866\text{cm}$

Area = Area of parallelogram + Area of equilateral triangle

$= (0.866)1 + \frac{\sqrt{3}}{4}(1)^2 = 0.866 + 0.433 = 1.299\text{cm}^2$



Area of triangle (IV) = Area of triangle in (V)

$= \left(\frac{1}{2} \times 1.5 \times 6\right) \text{cm}^2 = 4.5\text{cm}^2$

Total area of the paper used = 2.488 + 6.5 + 1.299 + 4.5 x 2

$= 19.287\text{cm}^2$

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Q4 A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 26 cm, 28 cm and 30 cm, and the parallelogram stands on the base 28 cm, find the height of the parallelogram.

Answer. For triangle

Perimeter of triangle = (26 + 28 + 30) cm = 84 cm

2s 84 cm

s = 42 cm

By Heron's formula,

Area of triangle = = $\sqrt{s(s-a)(s-b)(s-c)}$

Area of triangle $= [\sqrt{42(42-26)(42-28)(42-30)}]\mathrm{cm}^2$

$= [\sqrt{42(16)(14)(12)}]\mathrm{cm}^2 = 336\mathrm{cm}^2$= 336 cm^2

Let the height of the parallelogram be h.

Area of parallelogram = Area of triangle

h x 28 cm = 336 cm^2

h = 12 cm

Therefore, the height of the parallelogram is 12 cm.

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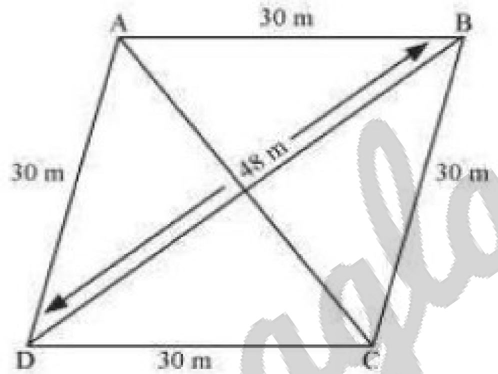
Q5 A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 m, how much area of grass field will each cow be getting?

Answer.



Let ABCD be a rhombus-shaped field.

For ΔBCD,

$s = \frac{(48+30+30)\mathrm{cm}}{2} = 54\mathrm{m}$

By Heron's formula,

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

Therefore, area of $\Delta BCD = [\sqrt{54(54-48)(54-30)(54-30)}]\mathrm{m}^2$

$= \sqrt{54(6)(24)(24)} = 3 \times 6 \times 24 = 432\mathrm{m}^2$

Area Of field = 2 x Area Of ABCD

$= (2 \times 432)\mathrm{m}^2 = 864\mathrm{m}^2$

Area for grazing for 1 cow $= \frac{864}{18} = 48\mathrm{m}^2$

Each cow will get $48\mathrm{m}^2$ area of grass field.

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Q6 An umbrella is made by stitching 10 triangular pieces of cloth of two different colours (see Fig), each piece measuring 20 cm, 50 cm and 50 cm. How much cloth of each colour is required for the umbrella?

Answer. For each triangular piece,

$$s=\frac{(20+50+50)\mathrm{cm}}{2}=60\mathrm{cm}$$

Semi-perimeter,

By Heron's formula,

$$\text{Area of triangle } = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{Area of each triangular piece } = [\sqrt{60(60-50)(40-50)(60-20)}]\mathrm{cm}^2$$

$$= [\sqrt{60(10)(10)(40)}]\mathrm{cm}^2 = 200\sqrt{6}\mathrm{cm}^2$$

Since there are 5 triangular pieces made of two different coloured cloths,

$$\text{Area of each cloth required } = (5\times 200\sqrt{6})\mathrm{cm}^2$$

$$= 1000\sqrt{6}\mathrm{cm}^2$$

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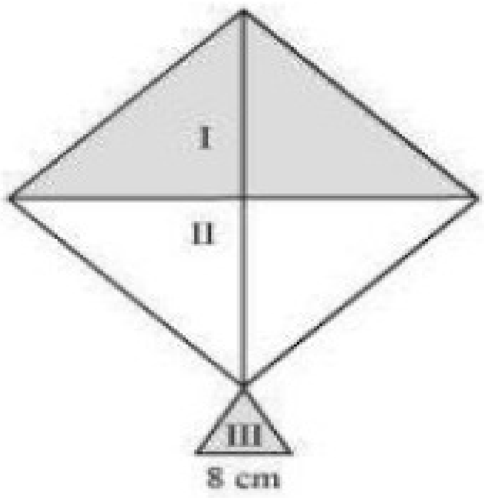
Q7 A kite in the shape of a square with a diagonal 32 cm and an isosceles triangle of base 8 cm and sides 6 cm each is to be made of three different shades as shown in Fig. How much paper of each shade has been used in it?



Answer. We know that

$$\text{Area of square } = \frac{1}{2}(\text{ diagonal })^2$$

$$\text{Area of the given kite } = \frac{1}{2}(32\text{cm})^2 = 512\text{cm}^2$$

$$\text{Area of 1st shade } = \text{ Area of } 2^{\text{nd}} \text{ shade}$$

$$= \frac{512\text{cm}^2}{2} = 256\text{cm}^2$$

Therefore, the area of paper required in each shape is 256 cm^2.

For III triangle

$$\text{Semi-perimeter, } s = \frac{(6+6+8)\text{cm}}{2} = 10\text{cm}$$

By Heron's formula,

$$\text{Area of triangle } = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{Area of IIIrd triangle } = \sqrt{10(10-6)(10-6)(10-8)}$$

$$= (\sqrt{10 \times 4 \times 4 \times 2})\text{cm}^2$$

$$= (4 \times 2\sqrt{5})\text{cm}^2$$

$$= \Big(4 \times 2\sqrt{5}\text{cm}^2$$

$$= (8 \times 2.24)\text{cm}^2$$

$$= 17.92\text{cm}^2$$

Area of paper required for III^{rd} shade = 17.92 cm^2

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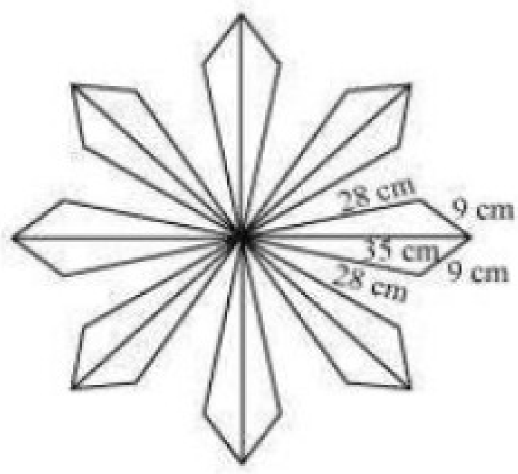
Q8 A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being 9 cm, 28 cm and 35 cm (see Fig.). Find the cost of polishing the tiles at the rate of 50p per cm^2.



Answer. It can be observed that

Semi-perimeter of each triangle-shaped tile, $s = \frac{(35+28+9)\text{cm}}{2} = 36\text{cm}$

By Heron's formula,

Area of triangle= $= \sqrt{s(s-a)(s-b)(s-c)}$

$\text{Area of each tile } = [\sqrt{36(36-35)(36-28)(36-9)}]\text{cm}^2$

$= [\sqrt{36 \times 1 \times 8 \times 27}]\text{cm}^2$

$= 36\sqrt{6}\text{cm}^2$

$= (36 \times 2.45)\text{cm}^2$

$= 88.2\text{cm}^2$

$\text{Area of 16 tiles } = (16 \times 88.2)\text{cm}^2 = 1411.2\text{cm}^2$

Cost of polishing per cm^2 area = 50 p

Cost of polishing 1411.2 cm^2 area = Rs (1411.2 x 0.50) = Rs 705.60

Therefore. It will cost Rs 705.60 while polishing all the tiles.

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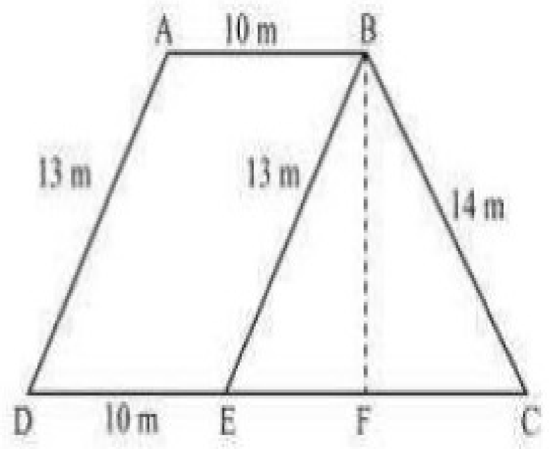
Q9 A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. The non-parallel sides are 14 m and 13 m. Find the area of the field.

Answer.



Draw a line BE parallel to AD and draw a perpendicular 3F on CO.
It can be observed that ABED is a parallelogram.
BE = AD = 13 m
ED = AB = 10 m
EC = 25 - ED = 15m
For Δ BEC,
Semi-perimeter, $s=\frac{(13+14+15)\mathrm{m}}{2} = 21\mathrm{m}$
By Heron’s formula,

$$\text{Area of triangle } = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{Area of } \Delta BEC = [\sqrt{21(21-13)(21-14)(21-15)}]m^2$$

$$= [\sqrt{21(8)(7)(6)}]_{m^2 = 84\mathrm{m}^2}$$

$$\text{Area of } \Delta BEC = \frac{1}{2} \times CE \times BF$$

$$84\mathrm{cm}^2 = \frac{1}{2} \times 15\mathrm{cm} \times \mathrm{BF}$$

$$\mathrm{BF} = \left(\frac{168}{15}\right)\mathrm{cm} = 11.2\mathrm{cm}$$

$$\text{Area of ABED} = \mathrm{BF} \times \mathrm{DE} = 11.2 \times 10 = 112\mathrm{m}^2$$

$$\text{Area of the field } = 84 + 112 = 196\mathrm{m}^2$$

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