NCERT SOLUTIONS

CLASS - 9th

Class : 9th
Subject : Maths
Chapter : 8
Chapter Name : QUADRILATERALS

Exercise 8.1

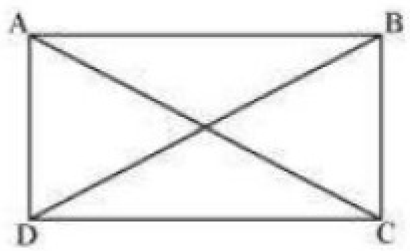
Q1 The angles of quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.

Answer. Let the common ratio between the angles be x. Therefore, the angles will be 3x, 5x, 9x, and 13x respectively.
As the sum Of all interior angles Of a quadrilateral is 360°,
Therefore, 3x + 5x + 9x + 13x = 360°
30x = 360°
x = 12°
Hence, the angles are
$3x = 3 \times 12 = 360$
$5x = 5 \times 12 = 60^{\circ}$
$9x = 9 \times 12 = 108^{\circ}$
$13x = 13 \times 12 = 156^{\circ}$

Page : 146 , Block Name : Exercise 8.1

Q2 If the diagonals of a parallelogram are equal, then show that it is a rectangle.

Answer.



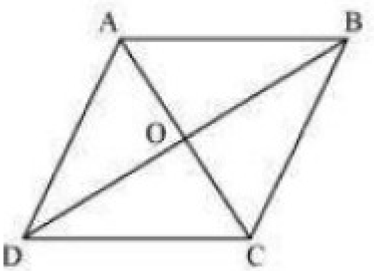
Let ABCD be a parallelogram. To show that ABCD is a rectangle, we have to prove that one of its interior angles is 90°.
In $\triangle$ABC and $\triangle$DCB,
AB = DC (Opposite sides Of a parallelogram are equal)
BC = BC (Common)
AC = DB (Given)
$\therefore \triangle ABC \cong \triangle DCB$ (By SSS Congruence rule)
$\Rightarrow \angle ABC = \angle DCB$

It is known that the sum of the measures of angles on the same side of transversal is 180°.
$\angle ABC + \angle DCB = 180^{\circ}(AB \| CD)$
$\Rightarrow \angle ABC + \angle ABC = 180^{\circ}$
$\Rightarrow 2\angle ABC = 180^{\circ}$
$\Rightarrow \angle ABC = 90^{\circ}$
Since ABCD is a parallelogram and one of its interior angles is 90° , ABCD is a rectangle.

Page : 146 , Block Name : Exercise 8.1

Q3 Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Answer.

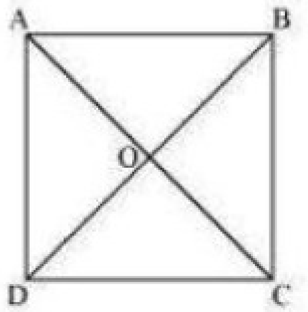


Let ABCD be a quadrilateral, whose diagonals AC and BD bisect each Other at right angle i.e., OA = OC, OB = OD, and $\angle AOB = \angle BOC = \angle COD = \angle AOD = 90^{\circ}$. To prove ABCD a rhombus, we have to prove ABCD is a parallelogram and all the sides of ABCD are equal.
In $\triangle$AOD and ΔCOD ,
OA = OC (Diagonals bisect each other)
$\angle AOD = \angle COD$ (Given)
OD = OD (Common)
$\therefore \triangle AOD = \Delta COD(By$ SAS congruence rule)
$\therefore AD = CD(1)$
Similarly, it can be proved that
AD = AB and CD = BC (2)
From equations (1) and (2) ,
AB = BC = CD = AD
Since opposite sides of quadrilateral ABCD are equal, it can be said that ABCD is a parallelogram.
Since all sides of a parallelogram ABCD are equal, it can be said that ABCD is a rhombus.

Page : 146 , Block Name : Exercise 8.1

Q4 Show that the diagonals of a square are equal and bisect each other at right angles.

Answer.



Let ABCD be a square. Let the diagonals AC and BD intersect each other at a point O. To prove that the diagonals of a square are equal and bisect each other at right angles, we have to prove AC = BD, OA = OC, OB = OD, and $\angle AOB = 90^{\circ}$.

In $\triangle ABC$ and ΔDCB,

AB = DC (Sides Of a square are equal to each Other)

$\angle ABC = \angle DCB$ (All interior angles are of 90°)

BC = CB (Common side)

$\therefore \triangle \mathrm{ABC} = \Delta \mathrm{DCB}(\text{By SAS congruency })$

$\therefore \mathrm{AC} = \mathrm{DB}(\mathrm{ByCPCT})$

Hence, the diagonals of a square are equal in length.

In $\triangle \mathrm{AOB}$ and $\Delta \mathrm{COD}$,

$\angle AOB = \angle COD$ (Vertically Opposite Angle)

$\square ABO = \square CDO$ (Alternate interior angles)

AB = CD (Sides of a square are always equal)

$\square \triangle AOB \square \Delta COD(By \text{ AAS congruence rule})$

$\square AO = CO$ and $OB = OD(ByCPCT)$

Hence, the diagonals of a square bisect each other.

In $\triangle AOB$ and ΔCOB ,

As we had proved that diagonals bisect each other , therefore ,

AO = CO

AB = CB (Sides of a square are equal)

BO = BO (Common)

$\square \triangle AOB \square \Delta COB(By \text{ SSS congruence rule})$

$\square \square AOB = \square COB(ByCPCT)$

However, $\square AOB + \square COB = 180^{\circ}$ (Linear pair)

$2\square AOB = 180^{\circ}$

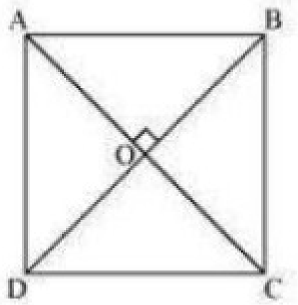
$\square AOB = 90^{\circ}$

Hence, the diagonals of a square bisect each other at right angles.

Page : 146 , Block Name : Exercise 8.1

Q5 Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Answer.



Let us consider a quadrilateral ABCD in which the diagonals AC and BD intersect each other at O. It is given that the diagonals of ABCD are equal and bisect each other at right angles. Therefore, AC = BD, OA = OC, 0B = OD, and $\angle AOB = \angle BOC = \angle COD = \angle AOD = 90^{\circ}$. To prove ABCD is a square, we have to prove that ABCD is a parallelogram, AB = BC = CD = AD, and one of its interior angles is 90° .

In $\triangle$AOB and ΔCOD,

AO = CO (Diagonals bisect each other)

OB = OD (Diagonals bisect each other)

$\angle AOB = \angle COD$(Vertically opposite angles)

$\square\triangle\mathrm{AOB} = \square\Delta\mathrm{COD}$(SAS congruence rule)

$\square AB = CD (ByCPCT) \ldots (1)$

And, $\square OAB = \square OCD (ByCPCT)$

However, these are alternate interior angles for line AB and CD and alternate interior angles are equal to each other only when the two lines are parallel.

$\square AB \| CD \ldots (2)$

From equations (1) and (2), we obtain

ABCD is a parallelogram.

In $\triangle$AOD and $\triangle$COD ,

AO = CO (diagonals bisect each other)

$\square AOD = \square COD$ (Given that each is 90°)

OD = OD (Common)

$\square\triangle AOD \square \Delta COD (SAS$ congruence rule)

$\square AD = DC \ldots (3)$

However, AD = BC and AB = CD (Opposite sides of parallelogram ABCD)

$\square AB = BC = CD = DA$

Therefore, all the sides of quadrilateral ABCD are equal to each other.

In $\triangle ADC$ and ΔBCD, ,

AD = BC (Already proved)

AC = BD (Given)

DC =CD (Common)

$\square\triangle\mathrm{ADC}\square\Delta\mathrm{BCD}$(SSS Congruence rule)

$\square\square\mathrm{ADC} = \square\mathrm{BCD}(\mathrm{ByCPCT})$

However, $\square ADC + \square BCD = 180^{\circ}$(Co-interior angles)

$\square\square ADC + \square ADC = 180^{\circ}$

$\square 2 \square ADC = 180^{\circ}$

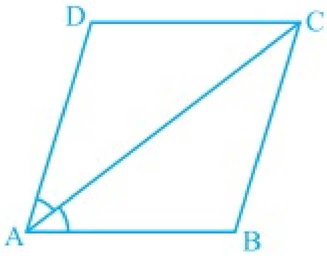
$\square\square ADC = 90^{\circ}$

One of the interior angles of quadrilateral ABCD is a right angle.

Thus, we have obtained that ABCD is a parallelogram, AB = BC = CD = AD and one of its interior angles is 90° .Therefore, ABCD is a square.

Page : 146 , Block Name : Exercise 8.1

Q6 Diagonal AC of a parallelogram ABCD bisects ∠ A



Show that
(i) it bisects ∠ C also,
(ii) ABCD is a rhombus.

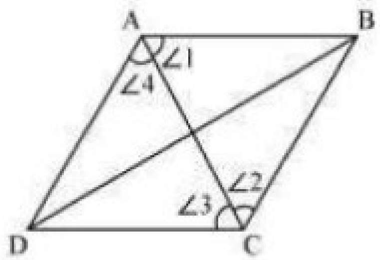
Answer. (i) ABCD is a parallelogram.
$\square\square DAC = \square BCA$ (Alternate interior angles) .....(1)
And, $\square BAC = \square DCA$ (Alternate interior angles) .....(2)
However, it is given that AC bisects $\square$A .
$\square\square DAC = \square BAC \ldots (3)$
From equations (1), (2), and (3), we obtain
$\square\text{DAC} = \square BCA = \square BAC = \square DCA \ldots (4)$
$\square\square DCA = \square BCA$
Hence, AC bisects $\square$C .

(ii) From equation (4) , we obtain
$\square DAC = \square DCA$
$\square DA = DC($ Side opposite to equal angles are equal$)$
However, DA = BC and AB = CD (Opposite sides of a parallelogram)
$\square AB = BC = CD = DA$
Hence, ABCD is a rhombus.

Page : 146 , Block Name : Exercise 8.1

Q7 ABCD is a rhombus. Show that diagonal AC bisects ∠ A as well as ∠ C and diagonal BD bisects ∠ B as well as ∠ D.

Answer.



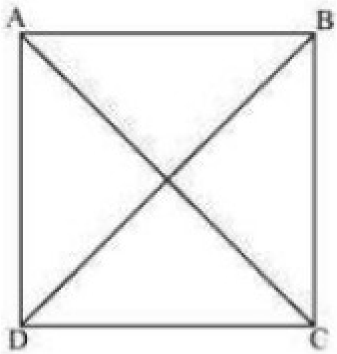
Let us join AC.

In $\triangle ABC$,
BC = AB (Sides of a rhombus are equal to each other)
∠1=∠2 (Angles opposite to equal sides of a triangle are equal)
However, ∠1=∠3 (Alternate interior angles for parallel lines AB and CD)
∠2 = ∠3
Therefore, AC bisects ∠C.
Also, □2 = □4 (Alternate interior angles for $\|$ lines BC and DA
□□1 = □4
Therefore, AC bisects □A .
Similarly, it can be proved that BD bisects \angl$B$ and □D as well.

Page : 146 , Block Name : Exercise 8.1

Q8 ABCD is a rectangle in which diagonal AC bisects ∠ A as well as ∠ C. Show that:
(i) ABCD is a square
(ii) diagonal BD bisects ∠ B as well as ∠ D.

Answer.



(i) It is given that ABCD is a rectangle.
$\angle A = \angle C$
$\Rightarrow \frac{1}{2}\angle A = \frac{1}{2}\angle C$ (AC bisects ∠A and ∠C)
$\Rightarrow \angle DAC = \angle DCA$
CD = DA (Sides opposite to equal angles are also equal)
However, DA = BC and AB = CD (Opposite sides of a rectangle are equal)
□$AB = BC = CD = DA$
ABCD is a rectangle and all of its sides are equal.
Hence, ABCD is a square.

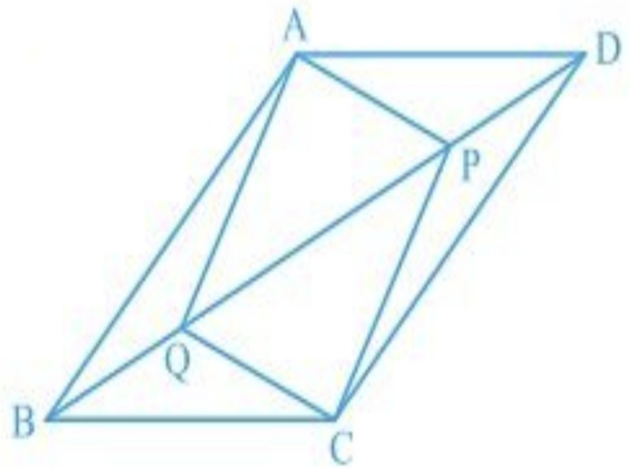
(ii) Let us join BD.
In ΔBCD ,
BC = CD (Sides of a square are equal to each other.)
$\angle CDB = \angle CBD$ (Angles opposite to equal sides are equal)
However, ∠CDB = ∠ABD (Alternate interior angles for AB$\|$CD)
$\triangle CBD = \triangle ABD$
BD bisects $\angle B$
Also, △CBD = △ADB (Alternate interior angles for BC II AD)

$\angle CDB = \angle ABD$

BD bisects $\angle D$

Page : 146 , Block Name : Exercise 8.1

Q9 In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ



Show that:
(i) Δ APD ≅ Δ CQB
(ii) AP = CQ
(iii) Δ AQB ≅Δ CPD
(iv) AQ = CP
(v) APCQ is a parallelogram.

Answer. (i) In $\triangle APD$ and ΔCQB, ,
$\square ADP = \square CBQ$ (Alternate interior angles for BC II AD)
AD = CB (Opposite sides of a parallelogram ABCD)
DP = BQ (Given)
$\square \triangle APD \square \Delta CQB$ (Using SAS Congruence rule)

(ii) As we have observed that $\triangle \mathrm{APD} \square \Delta \mathrm{CQB}$,
$\square AP = CQ (CPCT)$

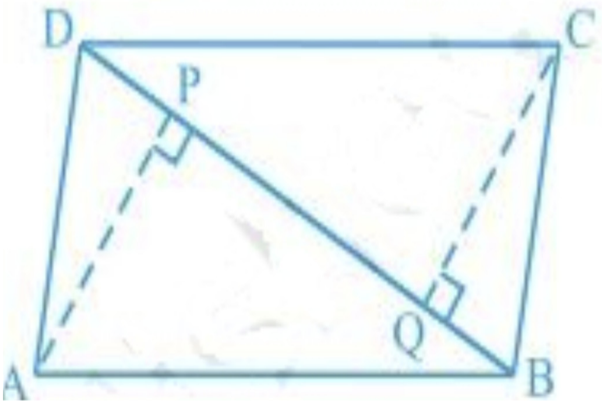
(iii) In $\triangle \mathrm{AQB}$ and $\Delta \mathrm{CPD}$
$\square ABQ = \square CDP($ Alternate interior angles for $AB \| CD)$
AB = CD (Opposite sides of a parallelogram ABCD)
BQ = DP (Given)
$\square \triangle AQB \square \Delta CPD (U$sing SAS congruence rule)

(iv) As we had observed that $\triangle \mathrm{AQB} \square \Delta \mathrm{CPD}$,
$\square AQ = CP (CPCT)$

(v) From the result obtained in (ii) and (iv) ,
AQ = CP and AP = CQ
Since opposite sides in quadrilateral APCQ are equal to each other, APCQ is a parallelogram.

Page : 147 , Block Name : Exercise 8.1

Q10 ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD



.

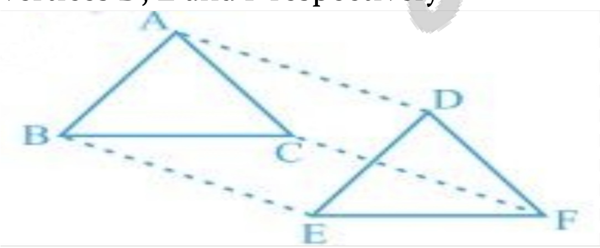
Show that
(i) Δ APB ≅ Δ CQD
(ii) AP = CQ

Answer. (i) Δ APB and Δ CQD
$\square APB = \square CQD\,(Each 90^{\circ})$
AB = CD (Opposite sides of a parallelogram ABCD)
$\square ABP = \square CDQ$ (Alternate interior angles for AB II CD)
$\square\triangle \mathrm{APB} \square \Delta \mathrm{CQD}$(By AAS congruency)

(ii) By using the above result
$\triangle APB \square \Delta CQD$, we obtain
AP = CQ (By CPCT)

Page : 147 , Block Name : Exercise 8.1

Q11 In Δ ABC and Δ DEF, AB = DE, AB || DE, BC = EF and BC || EF. Vertices A, B and C are joined to vertices D, E and F respectively



.

Show that
(i) quadrilateral ABED is a parallelogram
(ii) quadrilateral BEFC is a parallelogram
(iii) AD || CF and AD = CF
(iv) quadrilateral ACFD is a parallelogram
(v) AC = DF
(vi) Δ ABC ≅ Δ DEF.

Answer. It is given that AB = DE and AB II DE.
If two opposite sides of a quadrilateral are equal and parallel to each other , then it will be a

parallelogram.
Therefore, quadrilateral ABED is a parallelogram.

(ii) Again, BC = EF and BC II EF
Therefore, quadrilateral BCEF is a parallelogram.

(iii) As we had observed that ABED and BEFC are parallelograms, therefore
AD = BE and AD II BE
(Opposite sides of a parallelogram are equal and parallel)
And, BE = CF and BE II CF
(Opposite sides of a parallelogram are equal and parallel)
$\Box AD = CF$ and $AD \| CF$

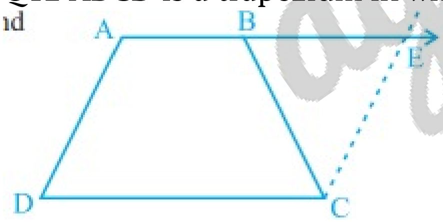
(iv) As we had observed that one pair of opposite sides (AD and CF) of a quadrilateral ACFD are equal and parallel to each other, therefore , it is a parallelogram.

(v) As ACFD is a parallelogram, therefore, the pair of opposite sides will be equal and parallel to each other.
$\Box AC \| DF$ and AC = DF

(vi) $\triangle$ABC and ΔDEF ,
AB = DE (Given)
BC = EF (Given)
AC = DF (ACFD is parallelogram)
$\Box \triangle ABC \Box \Delta DEF$ (By SSS Congruence Rule)

Page : 147 , Block Name : Exercise 8.1

Q12 ABCD is a trapezium in which AB || CD and AD = BC



Show that
(i) ∠ A = ∠ B
(ii) ∠ C = ∠ D
(iii) Δ ABC ≅ Δ BAD
(iv) diagonal AC = diagonal BD
[Hint : Extend AB and draw a line through C parallel to DA intersecting AB produced at E.]

Answer. Let us extend AB. Then, draw a line through C, which is parallel to AD, intersecting AE at point E. It is clear that AECD is a parallelogram.
(i) AD = CE (Opposite sides of a parallelogram AECD)
However, AD = BC (Given)
Therefore, BC = CE
$\Box CEB = \Box CBE$ (Angle opposite to equal sides are also equal)

Consider parallel lines AD and CE. AE is the transversal line for them.
$\square A+\square CEB=180^{\circ}$ (Angles on the same side of transversal)
$\square A+\square CBE=180^{\circ}(\text{ Using the relation } \square CEB=\square CBE) \ldots (1)$
However, $\square B+\square CBE=180^{\circ}(\text{ Linear pair angles }) \ldots .(2)$
From equations (1) and (2) , we obtain
$\square A=\square B$

(ii) AB II CD
$\square A+\square D=180^{\circ}$ (Angles on the same side are transversal)
Also, $\square C+\square B=180^{\circ}$ (Angles on the same side are transversal)
$\square\square A+\square D=\square C+\square B$
However, $\square A=\square B$ [Using the result obtained in (i)]
$\square\square C=\square D$

(iii) In $\triangle ABC$ and ΔBAD, ,
AB = BA (Common side)
BC = AD (Given)
$\square B=\square A(\text{ Proved before })$
$\square\triangle ABC\square\Delta BAD$ (SAS congruence rule)

(iv) We had observed that,
$\triangle \mathrm{ABC} \square \Delta \mathrm{BAD}$
$\square \mathrm{AC}=\mathrm{BD}(\mathrm{By CPCT})$

Page : 147 , Block Name : Exercise 8.1

Exercise 8.2

Q1 ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA

AC is a diagonal. Show that :
(i) SR || AC and SR = $\frac{1}{2}$ AC
(ii) PQ = SR
(iii) PQRS is a parallelogram.

Answer. (i) In $\triangle$ADC, S and R are the mid-points of sides AD and CD respectively.
In a triangle, the line segment joining the mid-points of any two sides of the triangle is parallel to the 3rd side and is half of it.
$\square$ SR $\| AC$ and $SR=\frac{1}{2} AC \ldots (1)$

(ii) In $\triangle \mathrm{ABC}$. P and Q are the mid-points of sides AB and BC respectively. Therefore, by using mid-point theoram,

$\mathrm{PQ} \| \mathrm{AC}$ and $\mathrm{PQ}=\frac{1}{2} \mathrm{AC} \ldots(2)$

Using equations (1) and (2) , we obtain

PQ II SR and PQ = SR ..... (3)

$\square \mathrm{PQ}=\mathrm{SR}$

(iii) From equation (3), we obtained

PQ II SR and PQ = SR

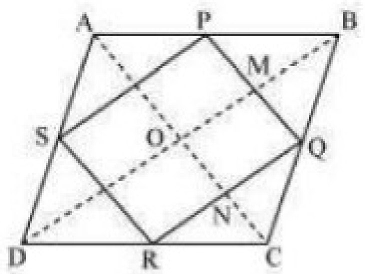
Clearly, one pair of opposite sides of quadrilateral PQRS is parallel and equal.

Hence, PQRS is a parallelogram.

Page : 150 , Block Name : Exercise 8.2

Q2 ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

Answer.



In $\triangle \mathrm{ABC}$, P and Q are the mid-points of sides AB and BC respectively.

$\square \mathrm{PQ} \| \mathrm{AC}$ and $\mathrm{PQ}=\frac{1}{2} \mathrm{AC}($ Using mid-point theorem $) \ldots$ (1)

In $\triangle \mathrm{ADC}$,

R and S are the mid-points of sides CD and AD respectively.

$\square \mathrm{RS} \| \mathrm{AC}$ and $\mathrm{RS}=\frac{1}{2} \mathrm{AC}($ Using mid-point theorem $) \ldots$ (2)

From equations (1) and (2) , we obtain

PQ II RS and PQ = RS

Since in quadrilateral PQRS, one pair of opposite sides is equal and parallel to each other, it is a parallelogram.

Let the diagonals of rhombus ABCD intersect each other at point O.

In quadrilateral OMPQ,

MQ II ON ($(\because \mathrm{PQ} \| \mathrm{AC})$

QN II OM ($(\because \mathrm{QR} \| \mathrm{BD})$

Therefore, OMQN is a parallelogram.

$\square \square MQN=\square NOM$

$\square \square PQR=\square NOM$

However, $\square \mathrm{NOM}=90^{\circ}$ (Diagonals of a rhombus are perpendicular to each other).

$\square \square PQR=90^{\circ}$

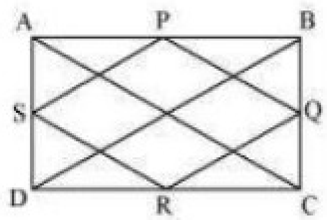
Clearly, PQRS is a parallelogram having one of its interior angles as 90°.

Hence, PQRS is a rectangle.

Page : 150 , Block Name : Exercise 8.2

Q3 ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

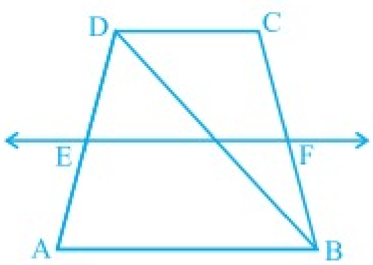
Answer.



Let us join AC and BD.
In $\triangle ABC$,
P and Q are the mid-points of AB and BC respectively.
$\square PQ \| AC$ and $PQ = \frac{1}{2} AC($ Mid-point theorem$) \ldots$ (1)
Similarly in $\triangle ADC$,
$SR \| AC$ and $SR = \frac{1}{2} AC($ Mid-point theorem $) \ldots$ (2)
Clearly, PQ II SR and PQ = SR
Since in quadrilateral PQRS, one pair of opposite sides is equal and parallel to each other, it is a parallelogram.
$\square$ PS || QR and PS = QR (Opposite sides of parallelogram)... (3)
In $\triangle BCD$, Q and R are the mid-points of side BC and CD respectively.
$\square QR \| BD$ and $QR = \frac{1}{2} BD$ (Mid-point theoram) ...... (4)
However,the diagonals of a rectangle are equal.
$\square AC = BD \ldots (5)$
By using equation (1), (2), (3), (4), and (5), we obtain
PQ = QR = SR = PS
Therefore, PQRS is a rhombus.

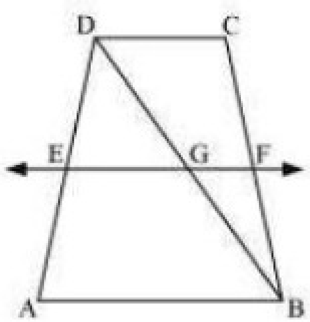
Page : 150 , Block Name : Exercise 8.2

Q4 ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F



Show that F is the mid-point of BC.

Answer. Let EF intersect DB at G.



By converse of mid-point theoram, we know that a line drawn through the mid-point of any side of a triangle and parallel to another side, bisects the third side.

In $\triangle \mathrm{ABD}$,

EF II AB and E is the mid-point of AD.

Therefore, G will be the mid-point of DB.

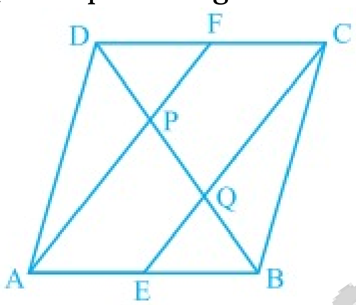
As EF II AB and AB II CD ,

$\square \mathrm{EF} \| \mathrm{CD}$ (Two lines parallel to the same line are parallel to each other.)

In $\triangle BCD$, GF II CD and G is the mid-point of line BD. Therefore, by using converse of mid-point theoram, F is the mid-point of BC.

Page : 150 , Block Name : Exercise 8.2

Q5 In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively



Show that the line segments AF and EC trisect the diagonal BD.

Answer. ABCD is a parallelogram.

$\square AB \| CD$

And hence, AE II FC

Again, AB = CD (Opposite sides of a parallelogram ABCD)

$\frac{1}{2} AB = \frac{1}{2} CD$

AE = FC (E and F are mid-points of side AB and CD)

In quadrilateral AECF, one pair of opposite sides (AE and CF) is parallel and equal to each other.

Therefore, AECF is a parallelogram.

$\square$ AF $\|$EC (Opposite sides of a parallelogram)

In ΔDQC, F is the mid-point of side DC and FP II CQ (as AF II EC). Therefore, by using converse of mid-point theoram , it can be said that P is the mid-point of DQ.

DP=PQ

Similarly, in $\triangle$APB, E is the mid-point of side AB and EQ II AP (as AF II EC).

Therefore, by using converse of mid-point theoram, it can be said that Q is the mid-point of PB.

PQ=QB

From equations (1) and (2),

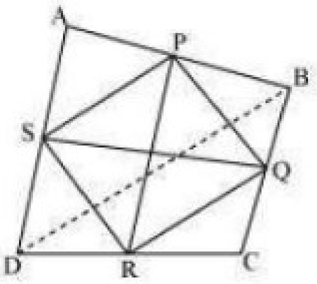
DP = PQ = BQ

Hence, the line segments AF and EC trisect the diagonal BD.

Page : 151 , Block Name : Exercise 8.2

Q6 Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Answer.



Let ABCD is a quadrilateral in which P, Q, R, and S are the mid-points of sides AB, BC, CD, and DA respectively. JOin PQ, QR, RS, SP, and BD.

In $\triangle ABD$, S and P are the mid-points Of AD and AB respectively. Therefore, by using mid-point theorem, it can be said that

$\mathrm{SP} \| \mathrm{BD}$ and $\mathrm{SP} = \frac{1}{2}\mathrm{BD} \ldots (1)$

Similarly in $\triangle BCD$,

$\mathrm{QR} \| \mathrm{BD}$ and $\mathrm{QR} = \frac{1}{2}\mathrm{BD} \ldots (1)$

From equations (1) and (2), we obtain

$\mathrm{SP} \parallel \mathrm{QR}$ and $SP = QR$

In quadrilateral SPQR, one pair of opposite sides is equal and parallel to each other. Therefore, SPQR is a parallelogram.

We know that diagonals Of a parallelogram bisect each other.

Hence, PR and QS bisect each other.

Page : 151 , Block Name : Exercise 8.2

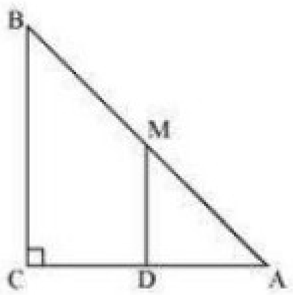
Q7 ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that

(i) D is the mid-point of AC

(ii) MD $\perp$AC

(iii) CM = MA = $\frac{1}{2}$ AB

Answer.



(i) In $\triangle$ABC,

It is given that M is the mid-point of AB and MD II BC.

Therefore, D is the mid-point of AC. (Converse of mid-point theorem)

(ii) As DM II CB and AC is a transversal line for them, therefore,

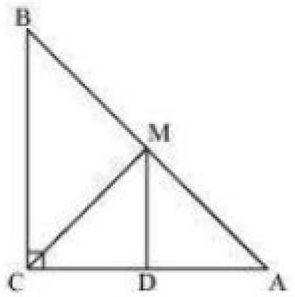
$\angle MDC + \angle DCB = 180^\circ (Co-\text{ interior angles })$

$\square MDC + 90^\circ = 180^\circ$

$\square MDC = 90^\circ$

$\square MD \square AC$

(iii) Join MC.



In $\triangle$AMD and ΔCMD,

AD = CD (D is the mid-point Of side AC)

$\angle ADM = \angle CDM \ (Each 90^\circ)$

DM = DM (Common)

$\square \triangle AMD = \square \Delta CMD$ (By SAS Congruence Rule)

Therefore, AM = CM (By CPCT)

However, AM = $\frac{1}{2}{}_{AB}$ (M is the mid-point of AB)

Therefore, it can be said that

$\text{CM} = \text{AM} = \frac{1}{2}\text{AB}$

Page : 151 , Block Name : Exercise 8.2