## NCERT

## SOLUTIONS

## CLASS - 9th


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Class: 9th
Subject: Maths
Chapter: 7
Chapter Name : Triangles

## Exercise 7.1

Q1 In quadrilateral $\mathrm{ACBD}, \mathrm{AC}=\mathrm{AD}$ and AB bisects $\angle \mathrm{A}$ (see Fig). Show that $\Delta \mathrm{ABC} \cong \triangle \mathrm{ABD}$. What can you say about BC and BD ?


Answer. In $\triangle \mathrm{ABC}$ and $\triangle \mathrm{ABD}$,
$\mathrm{AC}=\mathrm{AD}$ (Given)
$\angle \mathrm{CAB}=\angle \mathrm{DAB}(\mathrm{AB}$ bisects $\angle \mathrm{A})$
$\mathrm{AB}=\mathrm{AB}$ (Common)
Therefore, $\Delta \mathrm{ABC} \cong \Delta \mathrm{ABD}$ ( By SAS congruence rule)
$\mathrm{BC}=\mathrm{BD}(\mathrm{By} \mathrm{CPCT})$
Therefore, BC and BD are Of equal lengths.
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Q 2 ABCD is a quadrilateral in which $\mathrm{AD}=\mathrm{BC}$ and $\angle \mathrm{DAB}=\angle \mathrm{CBA}$ (see Fig). Prove that
(i) $\Delta \mathrm{ABD} \cong \triangle \mathrm{BAC}$
(ii) $\mathrm{BD}=\mathrm{AC}$
(iii) $\angle \mathrm{ABD}=\angle \mathrm{BAC}$.


Answer. In $\angle \mathrm{ABD}$ and $\angle \mathrm{BAC}$,
$\mathrm{AD}=\mathrm{BC}$ (Given)
$\angle \mathrm{DAB}=\angle \mathrm{CBA}$ (Given)
$\mathrm{AB}=\mathrm{BA}$ (Common)
$\therefore \triangle \mathrm{ABD}=\triangle \mathrm{BAC}($ By SAS congruence rule $)$
$\therefore \mathrm{BD}=\mathrm{AC}(\mathrm{ByCPCT})$
And, $\angle \mathrm{ABD}=\angle \mathrm{BAC}(\mathrm{By} \mathrm{CPCT})$
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Q3 AD and BC are equal perpendiculars to a line segment AB (see Fig.). Show that CD bisects AB.


Answer. In $\triangle \mathrm{BOC}$ and $\triangle \mathrm{AOD}$,
$\angle \mathrm{BOC}=\angle \mathrm{AOD}$ (Vertically opposite angles )
$\angle \mathrm{CBO}=\angle \mathrm{DAO}\left(\operatorname{Each} 90^{\circ}\right)$
$\mathrm{BC}=\mathrm{AD}$ ( Given )
$\therefore \triangle \mathrm{BOC}=\triangle \mathrm{AOD}$ (AAS congruence rule)
$\therefore \mathrm{BO}=\mathrm{AO}(\mathrm{ByCPCT})$
$\Rightarrow \mathrm{CD}$ bisects AB
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Q4 1 and m are two parallel lines intersected by another pair of parallel lines p and q (see Fig). Show that $\Delta \mathrm{ABC} \cong \Delta \mathrm{CDA}$.


Answer. In $\Delta \mathrm{ABC}$ and $\Delta \mathrm{CDA}$,

$$
\angle B A C=\angle D C A(\text { Alternate interior angles, as } p \| q)
$$

$$
A C=C A(\text { Common })
$$

$\angle B C A=\angle D A C$ ( Alternate interior angles, as $l \| m$ )
$\triangle A B C=\triangle C D A(B y$ ASA congruence rule $)$
Page : 119 , Block Name : Exercise 7.1
Q5 Line $l$ is the bisector of an angle $\angle \mathrm{A}$ and B is any point on l . BP and BQ are perpendiculars from B to the arms of $\angle \mathrm{A}$ (see Figure). Show that:
(i) $\Delta \mathrm{APB} \cong \Delta \mathrm{AQB}$
(ii) $\mathrm{BP}=\mathrm{BQ}$ or B is equidistant from the arms of $\angle \mathrm{A}$.


In $\triangle A P B$ and $\triangle A Q B$.
Answer.

$$
\angle A P B=\angle A Q B\left(E a c h 90^{\circ}\right)
$$

$\angle P A B=\angle Q A B(1$ is the angle bisector of $\angle A)$
$A B=A B$ (Common)
$\therefore \triangle \mathrm{APB} \cong \triangle \mathrm{AQB}$ (By AAS congruence rule)
$\therefore \mathrm{BP}=\mathrm{BQ}(\mathrm{ByCPCT})$
Or, it can be said that $B$ is equidistant from the arms of $\angle A$.
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Q 6 In Figure, $\mathrm{AC}=\mathrm{AE}, \mathrm{AB}=\mathrm{AD}$ and $\angle \mathrm{BAD}=\angle \mathrm{EAC}$. Show that $\mathrm{BC}=\mathrm{DE}$.


Answer. It is given that $\angle \mathrm{BAD}=\angle \mathrm{EAC}$
$\angle \mathrm{BAD}+\angle \mathrm{DAC}=\angle \mathrm{EAC}+\angle \mathrm{DAC}$
$\angle B A C=\angle D A E$
In $\triangle B A C$ and $\triangle D A E$,
$A B=A D(G$ iven $)$
$\angle B A C=\angle D A E$ (Proved above)
$\mathrm{AC}=\mathrm{AE}$ (Given)
Therefore, $\triangle \mathrm{BAC} \cong \triangle \mathrm{DAE}$ (By SAS congruence rule)
Therefore, BC = DE (By CPCT)
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Q7 $A B$ is a line segment and $P$ is its mid-point. $D$ and $E$ are points on the same side of $A B$ such that $L$ $\mathrm{BAD}=\angle \mathrm{ABE}$ and $\angle \mathrm{EPA}=\angle \mathrm{DPB}$ (see Figure). Show that
(i) $\triangle \mathrm{DAP} \cong \triangle \mathrm{EBP}$
(ii) $\mathrm{AD}=\mathrm{BE}$


Answer. It is given that $\angle \mathrm{EPA}=\angle \mathrm{DPB}$
$\angle \mathrm{EPA}+\angle \mathrm{DPE}=\angle \mathrm{DPB}+\angle \mathrm{DPE}$
Therefore, $\angle \mathrm{DPA}=\angle \mathrm{EPB}$
In $\angle \mathrm{DAP}$ and $\angle \mathrm{EBP}$, $\angle \mathrm{DAP}=\angle \mathrm{EPB}$ (Given)
$\mathrm{AP}=\mathrm{BP}(\mathrm{P}$ is mid-point of AB$)$
$\angle D P A=\angle E P B$ (From above)
Therefore, $\triangle \mathrm{DAP} \cong \Delta$ EBP (ASA congruence rule)
Therefore, AD - BE (By CPCT)
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Q8 In right triangle $A B C$, right angled at $C$, $M$ is the mid-point of hypotenuse $A B$. $C$ is joined to $M$ and produced to a point D such that $\mathrm{DM}=\mathrm{CM}$. Point D is joined to point B (see Figure). Show that:
(i) $\Delta \mathrm{AMC} \cong \Delta \mathrm{BMD}$
(ii) $\angle \mathrm{DBC}$ is a right angle.
(iii) $\Delta \mathrm{DBC} \cong \triangle \mathrm{ACB}$
(iv) $\mathrm{CM}=1 / 2 \mathrm{AB}$


Answer. (i) In $\Delta \mathrm{AMC}$ and $\Delta \mathrm{BMD}$,
$\mathrm{AM}=\mathrm{BM}$ ( M is the mid-point Of AB )
$\angle A M C=\angle B M D$ (Vertically opposite angles)
CM = DM (Given)
Therefore, $\Delta \mathrm{AMC} \cong \Delta \mathrm{BMD}$ (By SAS congruence rule)
Therefore AC $=\mathrm{BD}(\mathrm{By} \mathrm{CPCT})$
And, $\angle \mathrm{ACM}=\angle \mathrm{BDM}(\mathrm{By} \mathrm{CPCT})$
(ii) $\angle \mathrm{ACM}=\angle \mathrm{BDM}$

However, $\angle \mathrm{ACM}$ and $\angle \mathrm{BDM}$ are alternate interior angles.
Since alternate angles are equal,
It can be said that DB II AC
$\angle \mathrm{DBC}+\angle \mathrm{ACB}=180^{\circ}$ (Co-interior angles)
$\angle \mathrm{DBC}+90^{\circ}=180^{\circ}$
$\angle D B C=90^{\circ}$
(iii) In $\triangle \mathrm{DBC}$ and $\triangle \mathrm{ACB}$,

DB $=\mathrm{AC}$ (Already proved)
$\angle \mathrm{DBC}=\angle \mathrm{ACB}\left(\right.$ Each $\left.90^{\circ}\right)$
$\mathrm{BC}=\mathrm{CB}$ (Common)
Therefore, $\triangle \mathrm{DBC} \cong \triangle \mathrm{ACB}$ (SAS congruence rule)
(iv) $\triangle \mathrm{DBC} \cong \triangle \mathrm{ACB}$
$\mathrm{AB}=\mathrm{DC}(\mathrm{By} \mathrm{CPCT})$
$\mathrm{AB}=2 \mathrm{CM}$
$\therefore C M=\frac{1}{2} A B$
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## Exercise 7.2

Q1. In an isosceles triangle ABC , with $\mathrm{AB}=\mathrm{AC}$, the bisectors of $\angle \mathrm{B}$ and $\angle \mathrm{C}$ intersect each other at O. Join A to O. Show that :
(i) $\mathrm{OB}=\mathrm{OC}$
(ii) AO bisects $\angle \mathrm{A}$

Answer.

(i) It is given that in triangle $\mathrm{ABC}, \mathrm{AB}=\mathrm{AC}$
$\angle A C B=\angle A B C$ (Angles opposite to equal sides Of a triangle are equal)
$\frac{1}{2} \angle A C B=\frac{1}{2} \angle A B C$
$\angle \mathrm{OCB}=\angle \mathrm{OBC}$
Therefore, $\mathrm{OB}=\mathrm{OC}$ (Sides opposite to equal angles of a triangle are also equal)
(ii) In $\triangle \mathrm{OAB}$ and $\triangle \mathrm{OAC}$,
$\mathrm{AO}=\mathrm{AO}($ Common $)$
$\mathrm{AB}=\mathrm{AC}$ (Given)
OB = OC (Proved above)
Therefore, $\triangle \mathrm{OAB} \cong \triangle \mathrm{OAC}$ (By SSS congruence rule)
$\angle \mathrm{BAO}=\angle \mathrm{CAO}$ (СРСТ)
Therefore, AO bisects $\angle A$.
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Q2 In $\triangle \mathrm{ABC}, \mathrm{AD}$ is the perpendicular bisector of BC (see Fig). Show that $\Delta \mathrm{ABC}$ is an isosceles triangle in which $\mathrm{AB}=\mathrm{AC}$.


Answer. In $\triangle \mathrm{ADC}$ and $\triangle \mathrm{ADB}$,
$\mathrm{AD}=\mathrm{AD}$ (Common)
$\angle \mathrm{ADC}=\angle \mathrm{ADB}\left(\right.$ Each $\left.90^{\circ}\right)$
$\mathrm{CD}=\mathrm{BD}$ ( AD is the perpendicular bisector of BC )
Therefore, $\triangle \mathrm{ADC} \cong \triangle \mathrm{ADB}$ (By SAS congruence rule)
$\mathrm{AB}=\mathrm{AC}(\mathrm{By} \mathrm{CPCT})$
Therefore, ABC is an isosceles triangle in which $\mathrm{AB}=\mathrm{AC}$
Page : 123 , Block Name : Exercise 7.2
Q3 ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively (see Fig). Show that these altitudes are equal.


Answer. In $\triangle \mathrm{AEB}$ and $\triangle \mathrm{AFC}$,
$\angle A E B$ and $\angle A F C$ (Each $90^{\circ}$ )
$\angle \mathrm{A}=\angle \mathrm{A}$ (Common angle)
$\mathrm{AB}=\mathrm{AC}$ (Given)
Therefore, $\triangle \mathrm{AEB} \cong \triangle \mathrm{AFC}$ (By AAS congruence rule)
$\mathrm{BE}=\mathrm{CF}(\mathrm{By} \mathrm{CPCT})$
Page : 124 , Block Name : Exercise 7.2
Q4 ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (see Fig). Show that (i) $\triangle \mathrm{ABE} \cong \triangle \mathrm{ACF}$
(ii) $\mathrm{AB}=\mathrm{AC}$, i.e., ABC is an isosceles triangle.


Answer. (i) $\Delta \mathrm{ABE}$ and $\Delta \mathrm{ACF}$,
$\angle A B E$ and $\angle A C F\left(E a c h 90^{\circ}\right)$
$\angle A=\angle A$ ( Common angle )
$B E=C F(G$ iven $)$
Therefore, $\triangle \mathrm{ABE} \cong \triangle \mathrm{ACF}$ (By AAS congruence rule)
(ii) It has already been proved that
$\triangle \mathrm{ABE} \cong \triangle \mathrm{ACF}$
Therefore, $\mathrm{AB}=\mathrm{AC}(\mathrm{By} \mathrm{CPCT})$
Page : 124 , Block Name : Exercise 7.2
Q5 ABC and DBC are two isosceles triangles on the same base BC (see Fig. 7.33). Show that $\angle \mathrm{ABD}=$ $\angle \mathrm{ACD}$


Answer.


Let us join AD.
In $\triangle \mathrm{ABD}$ and $\triangle \mathrm{ACD}$,
$\mathrm{AB}=\mathrm{AC}$ (Given)
$\mathrm{BD}=\mathrm{CD}$ (Given)
$\mathrm{AD}=\mathrm{AD}$ (Common side)
Therefore, $\triangle \mathrm{ABD} \cong \triangle \mathrm{ACD}$ (By SSS congruence rule)
$\angle \mathrm{ABD}=\angle \mathrm{ACD}$ (By CPCT)
Page : 124 , Block Name : Exercise 7.2
Q6 $\triangle \mathrm{ABC}$ is an isosceles triangle in which $\mathrm{AB}=\mathrm{AC}$. Side BA is produced to D such that $\mathrm{AD}=\mathrm{AB}$ (see Fig). Show that $\angle B C D$ is a right angle.


Answer. In $\triangle \mathrm{ABC}$,

## $\mathrm{AB}=\mathrm{AC}$ ( Given)

$\therefore \angle \mathrm{ACB}=\angle \mathrm{ABC}$ (Angles opposite to equal sides of a triangle are also equal)
In $\triangle \mathrm{ACD}$,
$\mathrm{AC}=\mathrm{AD}$ (Given)
$\therefore \angle \mathrm{ADC}=\angle \mathrm{ACD}$ (Angles opposite to equal sides of a triangle are also equal)
In $\triangle B C D$,
$\angle \mathrm{ABC}+\angle \mathrm{BCD}+\angle \mathrm{ADC}=1800$ (Angle sum property of a triangle)
$\angle \mathrm{ACB}+\angle \mathrm{ACB}+\angle \mathrm{ACD}+\angle \mathrm{ACD}=180^{\circ}$
$2(\angle \mathrm{ACB}+\angle \mathrm{ACD})=180^{\circ}$
$2(\angle \mathrm{BCD})=180^{\circ}$
Therefore, $\angle B C D=90^{\circ}$
Page : 124 , Block Name : Exercise 7.2
Q7 ABC is a right angled triangle in which $\angle \mathrm{A}=90^{\circ}$ and $\mathrm{AB}=\mathrm{AC}$. Find $\angle \mathrm{B}$ and $\angle \mathrm{C}$.
Answer.


It is given that
$\therefore \angle C=\angle B$ ( Angles opposite to equal sides are also equal)
In $\triangle A B C$.
$\mathrm{AB}=\mathrm{AC} \angle A+\angle B+\angle C=180^{\circ}$ (Angle sum property of a triangle)
$90^{\circ}+\angle B+\angle C=180^{\circ}$
$90^{\circ}+\angle B+\angle B=180^{\circ}$
$2 \angle B=90^{\circ}$
$\angle B=45^{\circ}$
$\therefore \angle B=\angle C=45^{\circ}$
Page : 124 , Block Name : Exercise 7.2
Q8 Show that the angles of an equilateral triangle are $60^{\circ}$ each.
Answer.


Let us consider that ABC is an equilateral triangle.
Therefore, $\mathrm{AB}=\mathrm{BC}+\mathrm{AC}$
$\mathrm{AB}=\mathrm{AC}$
Therefore, $\angle \mathrm{C}=\angle \mathrm{B}$ (Angles opposite to equal sides of a triangle are equal)
Also,
$\mathrm{AB}=\mathrm{AC}$
$\angle \mathrm{B}=\angle \mathrm{A}$ (Angles opposite to equal sides of a triangle are equal)
Therefore, we obtain
$\angle \mathrm{A}=\angle \mathrm{B}=\angle \mathrm{C}$
In $\triangle A B C$,
$\angle A+\angle B+\angle C=180^{\circ}$
$\angle A+\angle A+\angle A=180^{\circ}$
$\angle A=180^{\circ}$
$\angle A=60^{\circ}$
$\therefore \angle A=\angle B=\angle C=60^{\circ}$
Hence, in a equilateral triangle, all interior angles are of measure $60^{\circ}$.
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## Exercise 7.3

$\mathrm{Q} 1 \Delta \mathrm{ABC}$ and $\triangle \mathrm{DBC}$ are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see Figure). If AD is extended to intersect BC at P , show that
(i) $\Delta \mathrm{ABD} \cong \triangle \mathrm{ACD}$
(ii) $\triangle \mathrm{ABP} \cong \triangle \mathrm{ACP}$
(iii) AP bisects $\angle \mathrm{A}$ as well as $\angle \mathrm{D}$.
(iv) AP is the perpendicular bisector of BC


Answer. (i) In $\triangle A B D$ and $\triangle A C D$,

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\(A B=A C(G\) wen \()\)
\(B D=C D(\) Given \()\)
\(A D=A D\) (Common)
\(\triangle A B D \cong \triangle A C D(B y\) SSS congruence rule \()\)
\(\angle B A D=\angle C A D(B y C P C T)\)
\(\angle B A P=\angle C A P\)
        .(1)
(ii) In \(\triangle \mathrm{ABP}\) and \(\triangle \mathrm{ACP}\),
\(A B=A C\) ( Given )
\(\angle B A P=\angle C A P[\) From equation (1)]
\(A P=A P(\) Common \()\)
\(\therefore \triangle \mathrm{ABP} \cong \triangle \mathrm{ACP}\) ( By SAS congruence rule)
\(\therefore B P=C P(B y C P C T) \ldots\) (2)
(iii) From equation (1),
\(\angle B A P=\angle C A P\)
Hence, AP bisects \(\angle A\),
In \(\Delta B D P\) and \(\Delta C D P\)
\(\mathrm{BD}=\mathrm{CD}(\) Given \()\)
\(\mathrm{DP}=\mathrm{DP}(\) Common \()\)
\(\mathrm{BP}=\mathrm{CP}[\) From equation (2)]
\(\therefore \triangle B D P \cong \triangle C D P\) (By SSS congruence rule)
\(\triangle \angle B D P=\angle C D P(B y C P C T) \ldots\) (3)
Hence, AP bisects \(\angle \mathrm{D}\)
(iv) \(\triangle B D P \cong \triangle C D P\)
\(\therefore \angle B P D=\angle C P D(B y C P C T) \ldots\) (4)
\(\angle B P D+\angle C P D=180^{\circ}\) ( Linear pair angles)
\(\angle B P D+\angle B P D=180^{\circ}\)
\(2 \angle 8 P D=180^{\circ}[\) From Equation (4)]
\(\angle B P D=90^{\circ}\)

FRom equations (2) and (5), it can be said that AP is the perpendicular bisector of BC.
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Q2 AD is an altitude of an isosceles triangle \(A B C\) in which \(A B=A C\). Show that
(i) AD bisects BC
(ii) AD bisects \(\angle \mathrm{A}\).

Answer.

(i) In \(\triangle B A D\) and \(\triangle C A D\),
\(\angle \mathrm{ADB}=\angle \mathrm{ADC}\left(\mathrm{Each} 90^{\circ}\right.\) as AD is an altitude \()\)
\(A B=A C(G\) iven \()\)
\(A D=A D(\) Common \()\)
\(\therefore \triangle B A D \cong \triangle C A D\) (By RHS Congruence rule)
\(\triangle B D=C D(\) Ву СРСТ \()\)

Hence, AD bisects BC.
(ii) Also, by CPCT,
\(\angle B A D=\angle C A D\)
Hence, AD bisects \(\angle \mathrm{A}\)
Page : 128 , Block Name : Exercise 7.3
Q3 Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of \(\triangle \mathrm{PQR}\) (see Figure). Show that:
(i) \(\triangle \mathrm{ABM} \cong \triangle \mathrm{PQN}\)
(ii) \(\triangle \mathrm{ABC} \cong \triangle \mathrm{PQR}\)


Answer. (i) In \(\triangle A B C, A M\) is the median to BC.
\(\therefore \mathrm{BM}=\frac{1}{2} \mathrm{BC}\)
In \(\triangle \mathrm{PQR}, \mathrm{PN}\) is the median to QR .
\(\therefore Q N=\frac{1}{2} Q R\)
However, \(\mathrm{BC}=\mathrm{QR}\)
\(\therefore \frac{1}{2} B C=\frac{1}{2} Q R\)
\(\therefore \mathrm{BM}=\mathrm{QN} \quad \ldots(1)\)
In \(\triangle A B M\) and \(\triangle P Q N\),
\(\mathrm{AB}=\mathrm{PQ}\) (Given)
\(\mathrm{BM}=\mathrm{QN}[\) From Equation (1)]
\(\mathrm{AM}=\mathrm{PN}(\) Given \()\)
\(\mathrm{ABM} \cong \triangle \mathrm{PQN}\) (By SSS congruence rule
\(\angle \mathrm{ABM}=\angle \mathrm{PQN}(\mathrm{ByCPCT})\)
\(\angle \mathrm{ABC}=\angle \mathrm{PQR}\)
(iii) In \(\triangle A B C\) and \(\triangle P Q R\),
\(\mathrm{AB}=\mathrm{PQ}\) ( Given )
\(\angle \mathrm{ABC}=\angle \mathrm{PQR}[\) From Equation (2)]
\(\mathrm{BC}=\mathrm{QR}\) ( Given )
\(\therefore \triangle A B C \cong \triangle P Q R\) (By SAS congruence rule)
Page : 128 , Block Name : Exercise 7.3
Q4 BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle \(A B C\) is isosceles.

Answer.


In \(\triangle B E C\) and \(\triangle C F B\),
\(\angle B E C=\angle C F B\left(E a c h 90^{\circ}\right)\)
\(\mathrm{BC}=\mathrm{CB}(\) Common \()\)
\(\mathrm{BE}=\mathrm{CF}\) (Given)
\(\therefore \triangle B E C \cong \triangle C F B\) (By RHS congruency)
\(\therefore \angle B C E=\angle C B F(B y C P C T)\)
\(\therefore A B=A C\) (Sides opposite to equal angles of a triangle are equal)
Hence, \(\triangle \mathrm{ABC}\) is isosceles.
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Q 5 ABC is an isosceles triangle with \(\mathrm{AB}=\mathrm{AC}\). Draw \(\mathrm{AP} \perp \mathrm{BC}\) to show that \(\angle \mathrm{B}=\angle \mathrm{C}\).

\section*{Answer.}


In \(\triangle A P B\) and \(\triangle A P C\),
\(\angle A P B=\angle A P C\left(E a c h 90^{\circ}\right)\)
\(A B=A C\) (Given)
\(A P=A P(\) Common \()\)
\(\therefore \triangle A P B \cong \triangle A P C\) (Using RHS congruence rule)
\(\therefore \angle B=\angle C\) (By using CPCT)
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\section*{Exercise 7.4}

Q1 Show that in a right angled triangle, the hypotenuse is the longest side.
Answer.


Let us consider a right-angled triangle ABC, right-angled at B .
In \(\triangle A B C\),
\(\angle A+\angle B+\angle C=180^{\circ}\) (Angle sum property of a triangle)
\(\angle A+90^{\circ}+\angle C=180^{\circ}\)
\(\angle A+\angle C=90^{\circ}\)
Hence, the other two angles have to be acute(i.e., less than \(90^{\circ}\) ).
\(\angle B\) is the largest angle in \(\triangle A B C\).
\(\angle B>\angle A\) and \(\angle B>\angle C\)
\(A C>B C\) and \(A C>A B\)
[In any triangle, the side opposite to the larger (greater) angle is longer.]
Therefore, AC is the largest side in \(\triangle A B C\)
However, AC is the hypotenuse of \(\triangle \mathrm{ABC}\). Therefore, hypotenuse is the longest side in a right-angled triangle.

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Q2 In Figure, sides AB and AC of \(\triangle \mathrm{ABC}\) are extended to points P and Q respectively. Also, \(\angle \mathrm{PBC}<\) \(\angle\) QCB. Show that AC \(>A B\).


Answer. In the given figure,
\[
\begin{align*}
& \angle A B C+\angle P B C=180^{\circ}(\text { Linear pair }) \\
& \angle A B C=180^{\circ}-\angle P B C \quad \ldots(1) \tag{1}
\end{align*}
\]

Also,
\(\angle A C B+\angle Q C B=180^{\circ}\)
\(\angle A C B=180^{\circ}-\angle Q C B\)
As \(\angle P B C<\angle Q C B\)
\(180^{\circ}-\angle P B C>180^{\circ}-\angle Q C B\)
\(\angle A B C>\angle A C B\) [ From Equations (1) and (2)]
\(A C>A B\) ( Side opposite to the larger angle is larger.)
Hence proved AC > AB
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Q3 In Figure, \(\angle \mathrm{B}<\angle \mathrm{A}\) and \(\angle \mathrm{C}<\angle \mathrm{D}\). Show that \(\mathrm{AD}<\mathrm{BC}\).


\footnotetext{
Answer. In \(\triangle \mathrm{AOB}\)
}
\(\angle B<\angle A\)
\(A O<B O\) ( Side opposite to smaller angle is smaller)
In \(\triangle \mathrm{COD}\),
\(\angle C<\angle D\)
\(\mathrm{OD}<\mathrm{OC}\) ( Side opposite to smaller angle is smaller)
On adding Equations (1) and (2), we obtain
\(\mathrm{AO}+\mathrm{OD}<\mathrm{BO}+\mathrm{OC}\)
\(\mathrm{AD}<\mathrm{BC}\), proved
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Q4 AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD (see Figure). Show that \(\angle \mathrm{A}>\angle \mathrm{C}\) and \(\angle \mathrm{B}>\angle \mathrm{D}\).


Answer.


Let us join AC.
In \(\triangle \mathrm{ABC}\)
\(\mathrm{AB}<\mathrm{BC}\) ( AB is the smallest side of quadrilateral ABCD )
\(\angle 2<\angle 1\) ( Angle opposite to the smaller side is smaller)...(1)
In \(\triangle A D C\)
\(A D<C D\) (CD is the largest side of quadrilateral ABCD)
\(\angle 4<\angle 3\) (Angle opposite to the smaller side i smaller)...(2)
On adding equations(1) and (2),we obtain
\(\angle 2+\angle 4<\angle 1+\angle 3\)
\(\angle C<\angle A\)
\(\angle A>\angle C\)
Let us join BD.


In \(\triangle A B D\)
\(A B<A D(A B\) is the smallest side of quadriateral ABCD\()\)
\(\angle 8<\angle 5\) (Angle opposite to the smaller side is smaller)...(3)
In \(\triangle B D C\)
\(\mathrm{BC}<\mathrm{CD}(\mathrm{CD}\) is the largest side of quadriateral ABCD\()\)
\(\angle 7<\angle 6\) (Angle opposite to the smaller side is smaller)...(4)
On adding equations (3) and (4), we obtain
\(\angle 8+\angle 7<\angle 5+\angle 6\)
\(\angle D<\angle B\)
\(\angle B>\angle D\) ( Hence, proved )
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Q5 In Figure, \(\mathrm{PR}>\mathrm{PQ}\) and PS bisects \(\angle \mathrm{QPR}\). Prove that \(\angle \mathrm{PSR}>\angle \mathrm{PSQ}\).


Answer. As \(P R>P Q\)
\(\angle P Q R>\angle P R Q\) (Angle opposite to larger side is larger)
PS is the bisector of \(\angle Q P R\).
\(\angle Q P S=\angle R P S \quad \ldots(2)\)
\(\angle P S R\) is the exterior angle of \(\triangle P Q S\)
\(\angle P S R=\angle P Q R+\angle Q P S \quad-(3)\)
\(\angle P S Q\) is the exterior angle of \(\triangle P R S\).
\(\angle P S Q=\angle P R Q+\angle R P S\)
Adding Equations (1) and (2), we obtain
\(\angle P Q R+\angle Q P S>\angle P R Q+\angle R P S\)
\(\angle \mathrm{PSR}>\angle \mathrm{PSQ}[\) Using the values of Equations (3) and (4)]
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Q6 Show that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

Answer.


Let us take a line l and from point P (i.e., not on line l ), draw two line segments PN and PM. Let PN be perpendicular to line L and PM is drawn at some other angle.
In \(\triangle\) PNM
\(\angle N=90^{\circ}\)
\(\angle P+\angle N+\angle M=180^{\circ}\) ( Angle sum property of a triangle)
\(\angle P+\angle M=90^{\circ}\)
Clearly, \(\angle M\) is an acute angle
\(\angle M<A N\)
\(P N<P M\) ( side opposite to the smaller angle is smaller)
Similarly, by drawing different line segments from \(P\) to \(l\), it can be proved that \(P N\) is smaller in comparison to them.
Therefore, it can be observed that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

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\section*{Exercise 7.5}

Q1 ABC is a triangle. Locate a point in the interior of \(\triangle \mathrm{ABC}\) which is equidistant from all the vertices of \(\triangle \mathrm{ABC}\).

Answer. Circumference of a triangle is always equidistant from all the vertices of that triangle.
Circumference is the point where perpendicular bisectors of all the sides of the triangle meet together.


In \(\triangle \mathrm{ABC}\), we can find the circumference by drawing the perpendicular bisectors of sides \(\mathrm{AB}, \mathrm{BC}\), AND CA of this triangle. O is the point where these bisectors are meeting together. Therefore, O is the point which is equidistant from all the vertices of \(\triangle \mathrm{ABC}\).

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Q2 In a triangle locate a point in its interior which is equidistant from all the sides of the triangle.
Answer. The point which is equidistant from all the sides of a triangle is called the incentre of the triangle. Incentre of a triangle is the intersection point of the angle bisectors of the interior angles of that triangle.


Here, in \(\triangle \mathrm{ABC}\), we can find the incentre of this triangle by drawing the angle bisectors of the interior angles of this triangle. I is the point where these angle bisectors are intersecting each other. Therefore, I is the point equidistant from all the sides of \(\triangle \mathrm{ABC}\)

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Q3 In a huge park, people are concentrated at three points (see Figure):
A : where there are different slides and swings for children,
B : near which a man-made lake is situated,
C : which is near to a large parking and exit. Where should an ice cream parlour be set up so that maximum number of persons can approach it?
(Hint : The parlour should be equidistant from A, B and C)


B*

Answer. Maximum number of persons can approach the ice-cream parlour if it is equidistant from \(\mathrm{A}, \mathrm{B}\) and C from a triangle. In a triangle, the circumcentre is the only point that is equidistant from its
vertices. So, the ice-cream parlour should be set up at the circumcentre O of \(\triangle \mathrm{ABC}\)


In this situation, maximum number of persons can approach it. We can find circumcentre O of this triangle by drawing perpendicular bisectors of the ised of this triangle.

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Q4 . Complete the hexagonal and star shaped Rangolies [see Figure (i) and (ii)] by filling them with as many equilateral triangles of side 1 cm as you can. Count the number of triangles in each case. Which has more triangles?

(I)

(II)

Answer. It can be observed that hexagonal-shaped rangoli has 6 equilateral triangles in it.


Area of \(\triangle \mathrm{OAB}=\frac{\sqrt{3}}{4}(\text { side })^{2}=\frac{\sqrt{3}}{4}(5)^{2}\)
\(=\frac{\sqrt{3}}{4}(25)=\frac{25 \sqrt{3}}{4} \mathrm{~cm}^{2}\)
Area of hexagonal-shaped rangoli \(=6 \times \frac{25 \sqrt{3}}{4}=\frac{75 \sqrt{3}}{2} \mathrm{~cm}^{2}\)
Area of equilateral triangle having its sides as \(1 \mathrm{~cm}=\frac{\sqrt{3}}{4}(1)^{2}=\frac{\sqrt{3}}{4} \mathrm{~cm}^{2}\)
Number of equilateral triangles of 1 cm side that can be filled In this hexagonal-shaped rangoli
\(=\frac{\frac{75 \sqrt{3}}{2}}{\frac{\sqrt{3}}{4}}=150\)
Star-shaped rangoli has 12 equilateral triangles of side 5 cm in it.


Area of star-shaped rangoli \(=12 \times \frac{\sqrt{3}}{4} \times(5)^{2}=75 \sqrt{3}\)
Number of equilateral triangles of 1 cm side that can be filled in this star-shaped rangoli \(=\frac{75 \sqrt{3}}{\frac{\sqrt{3}}{4}}=300\) Therefore, star-shaped rangoli has more equilateral triangles in it.

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