NCERT SOLUTIONS

CLASS - 9th

Class : 9th
Subject : Maths
Chapter : 1
Chapter Name : Number Systems

Exercise 1.1

Q1 Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and q ≠ 0?

Answer. Zero is a rational number as it can be represented as $\frac{0}{1}$ or $\frac{0}{2}$ or $\frac{0}{3}$ etc.

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Q2 Find six rational numbers between 3 and 4.

Answer. There are infinite rational numbers in between 3 and 4.
3 and 4 can be represented as $\frac{24}{8}$ and $\frac{32}{8}$
Therefore, rational numbers between 3 and 4 are
$\frac{25}{8}, \frac{26}{8}, \frac{27}{8}, \frac{28}{8}, \frac{29}{8}, \frac{30}{8}$

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Q3 Find five rational numbers between 3 5 and 4 5 .

Answer.
There are infinite rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$
$\frac{3}{5} = \frac{3\times6}{5\times6} = \frac{18}{30}$
$\frac{4}{5} = \frac{4\times6}{5\times6} = \frac{24}{30}$
Therefore, rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.
$\frac{19}{30}, \frac{20}{30}, \frac{21}{30}, \frac{22}{30}, \frac{23}{30}$

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Q4 State whether the following statements are true or false. Give reasons for your answers.
(i) Every natural number is a whole number.

(ii) Every integer is a whole number.
(iii) Every rational number is a whole number

Answer. (i) True; since the collection of whole numbers contains all natural numbers.
(ii) False; as integers may be negative but whole numbers are positive. For example: −3 is an integer but not a whole number.
(iii) False; as rational numbers may be fractional but whole numbers may not be. For Example :$\frac{1}{5}$ is rational number but not a whole number.

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Exercise 1.2

Q1 State whether the following statements are true or false. Justify your answers.
(i) Every irrational number is a real number.
(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.
(iii) Every real number is an irrational number.

Answer. (i) True; since the collection of real numbers is made up of rational and irrational numbers.
(ii) False; as negative numbers cannot be expressed as the square root of any other number.
(iii) False; as real numbers include both rational and irrational numbers. Therefore, every real number cannot be an irrational number.

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Q2 Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

Answer. If numbers such as $\sqrt{4}=2, \sqrt{9}=3$ are considered,
Then here, 2 and 3 are rational numbers. Thus, the square roots of all positive integers are not irrational.

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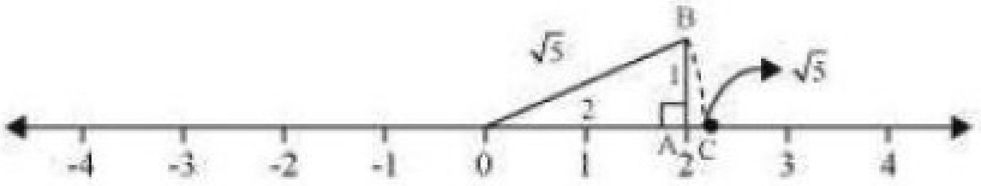
Q3 Show how $\sqrt{5}$ can be represented on the number line.

Answer.
We know that, $\sqrt{4}=2$
And, $\sqrt{5}=\sqrt{(2)^{2}+(1)^{2}}$



Mark a point 'A' representing 2 on number line. Now, construct AB Of unit length perpendicular to OA. Then, taking O as centre and 0B as radius, draw an arc intersecting number line at C.
C is representing $\sqrt{5}$.

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Exercise 1.3

Q1 Write the following in decimal form and say what kind of decimal expansion each has :
$(i)\frac{36}{100}(ii)\frac{1}{11}(iii)4\frac{1}{8}$
$\Big($ iv) $\frac{3}{13}(v)\frac{2}{11}(vi)\frac{329}{400}$

Answer.
$\frac{1}{11}=0.090909\ldots\ldots=0.\overline{09}$
Non-terminating repeating
(iii) $4\frac{1}{8}=\frac{33}{8}=4.125$
Terminating
(iv) $\frac{3}{13}=0.230769230769\ldots=0.230769$
Non-terminating repeating
(v) $\frac{2}{11}=0.18181818\ldots\ldots.=0.\overline{18}$
Non-terminating repeating
(vi) $\frac{329}{400}=0.8225$
Terminating

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Q2 You know that $\frac{1}{7}=0.\overline{142857}$. Can you predict what the decimal expansions of $\frac{2}{7},\frac{3}{7},\frac{4}{7},\frac{5}{7},\frac{6}{7}$ are, without actually doing the long division? If so, how?
[Hint : Study the remainders while finding the value of 1/7 carefully.]

Answer. Yes. It can be done as follows.

$\frac{2}{7} = 2 \times \frac{1}{7} = 2 \times 0.\overline{142857} = 0.\overline{285714}$

$\frac{3}{7} = 3 \times \frac{1}{7} = 3 \times 0.\overline{142857} = 0.\overline{428571}$

$\frac{4}{7} = 4 \times \frac{1}{7} = 4 \times 0.142857 = 0.571428$

$\frac{5}{7} = 5 \times \frac{1}{7} = 5 \times 0.\overline{142857} = 0.714285$

$\frac{6}{7} = 6 \times \frac{1}{7} = 6 \times 0.\overline{142857} = 0.\overline{857142}$

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Q3 Express the following in the form p/q , where p and q are integers and q ≠ 0.
(i)$0.\overline{6}$ (ii)$0.4\overline{7}$ (iii)$0.\overline{001}$

Answer.

(i) $0.\overline{6} = 0.666\ldots$

Let $x = 0.666\ldots$

$10x = 6.666\ldots$

$10x = 6 + x$

$9x = 6$

$x = \frac{2}{3}$

(ii) $0.\overline{47} = 0.4777\ldots\ldots$

$= \frac{4}{10} + \frac{0.777}{10}$

Let $x = 0.777\ldots$

$10x = 7.777\ldots$

$10x = 7 + x$

$x = \frac{7}{9}$

$$\frac{4}{10} + \frac{0.777\ldots}{10} = \frac{4}{10} + \frac{7}{90}$$

$$= \frac{36+7}{90} = \frac{43}{90}$$

$(iii)\overline{0.001} = 0.001001\ldots$

Let $x = 0.001001\ldots$

$1000x = 1.001001\ldots$

$1000x = 1 + x$

$999x = 1$

$x = \frac{1}{999}$

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Q4 Express 0.99999 .... in the form p/q . Are you surprised by your answer? With your teacher and classmates discuss why the answer makes sense.

Answer. Let x = 0.9999....
10x = 9.9999....
10x = 9 + x
9x = 9
x = 1

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Q5 What can the maximum number of digits be in the repeating block of digits in the decimal expansion of 1/17 ? Perform the division to check your answer.

Answer. It can be observed that,
$\frac{1}{17} = 0.\overline{0588235294117647}$
There are 16 digits in the repeating block Of the decimal expansion of $\frac{1}{17}$.

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Q6 Look at several examples of rational numbers in the form p/q (q ≠ 0), where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

Answer. Terminating decimal expansion will occur when denominator q of rational number p/q is either of 2, 4, 5, 8, 10, and so on...
$\frac{9}{4} = 2.25$
$\frac{11}{8} = 1.375$
$\frac{27}{5} = 5.4$
It can be observed that terminating decimal may be obtained in the situation where prime factorisation of the denominator Of the given fractions has the power Of 2 only
or 5 only or both.

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Q7 Write three numbers whose decimal expansions are non-terminating non-recurring.

Answer. 3 numbers whose decimal expansions are non-terminating non-recurring are as follows.
0. 505005000500005000005...
0.7207200720007200007200000...
0.080080008000080000080000008...

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Q8 Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Answer.
$\frac{5}{7} = 0.714285$
$\frac{9}{11} = 0.\overline{81}$
3 irrational numbers are as follows.
$0.73073007300073000073..$
$0.75075007500075000075..$
$0.79079007900079000079\ldots$

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Q9 Classify the following numbers as rational or irrational :
(i) $\sqrt{23}$ (ii) $\sqrt{225}$ (iii) 0.3796
(iv) 7.478478 (v) $1.101001000100001\ldots$

Answer. (i) $\sqrt{23} = 4.79583152331\ldots$....
As the decimal expansion of this number is non-terminating non-recurring, therefore, it is an irrational number.
$\sqrt{225} = 15 = \frac{15}{1}$
It is a rational number as it can be represented in $\frac{p}{q}$ form.
(iii) 0.3796
As the decimal expansion of this number is terminating, therefore, it is a rational number.
(iv) $7.478478\ldots = 7.\overline{478}$
As the decimal expansion Of this number is non-terminating recurring, therefore, it is a rational number.
(v) As the decimal expansion of this number is non-terminating non-repeating, therefore, it is an irrational number.

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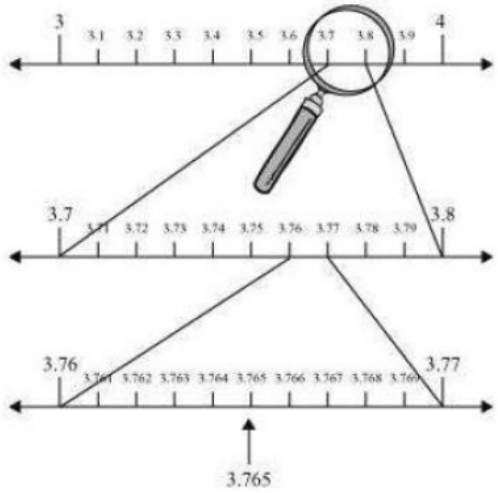
Exercise 1.4

Q1 Visualise 3.765 on the number line, using successive magnification.

Answer. 3.765 can be visualised as in the following steps.



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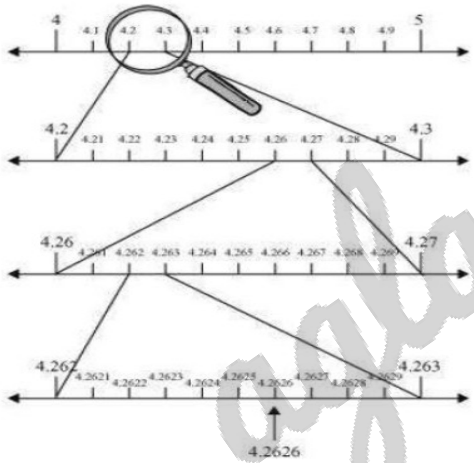
Q2 Visualise $4.\overline{26}$ on the number line, up to 4 decimal places.

Answer. $4.\overline{26} = 4.2626\ldots$
4.2626 can be visualised as in the following steps.



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Exercise 1.5

Q1 Classify the following numbers as rational or irrational:

(i) $2-\sqrt{5}$ (ii) $(3+\sqrt{23})-\sqrt{23}$ (iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$

$(iv)\frac{1}{\sqrt{2}}(v)2\pi$

Answer. $(\mathrm{i})^{2-\sqrt{5}}=2-2.2360679\ldots$
$=-0.2360679\ldots$
As the decimal expansion Of this expression is non-terminating non-recurring, therefore, it is an irrational number.
$\Big($ ii$)\ ^{(3+\sqrt{23})-\sqrt{23}}=3=\frac{3}{1}$
As it can be represented in p/q form, therefore, it is a rational number.
(iii) $\frac{2\sqrt{7}}{7\sqrt{7}}=\frac{2}{7}$
As it can be represented in p/q form, therefore, it is a rational number.
(iv) $\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}=0.7071067811\ldots$
As the decimal expansion Of this expression is non-terminating non-recurring, therefore, it is an irrational number.
(v) 2 n=2(3.1415 \dots)\) = 6.2830....
As the decimal expansion Of this expression is non-terminating non-recurring, therefore, it is an irrational number.

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Q2 Simplify each of the following expressions:
$(\mathrm{i})^{(3+\sqrt{3})(2+\sqrt{2})}(\mathrm{ii})(3+\sqrt{3})(3-\sqrt{3})$
$(\mathrm{iii})(\sqrt{5}+\sqrt{2})^2_{(\mathrm{iv})}(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

Answer.
$(\mathrm{\ i})\quad (3+\sqrt{3})(2+\sqrt{2})=3(2+\sqrt{2})+\sqrt{3}(2+\sqrt{2})$
$=6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$
$(\mathrm{ii})\ ^{(3+\sqrt{3})(3-\sqrt{3})}=(3)^2-(\sqrt{3})^2$
$=9-3=6$
$(\mathrm{ii})\ (\sqrt{5}+\sqrt{2})^2=(\sqrt{5})^2+(\sqrt{2})^2+2(\sqrt{5})(\sqrt{2})$
$=5+2+2\sqrt{10}=7+2\sqrt{10}$

$(\mathrm{ii})(\sqrt{5}+\sqrt{2})^2=(\sqrt{5})^2+(\sqrt{2})^2+2(\sqrt{5})(\sqrt{2})$
$=5+2+2\sqrt{10}=7+2\sqrt{10}$

$(\mathrm{v})(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})=(\sqrt{5})^2-(\sqrt{2})^2$
$=5-2=3$

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Q3 Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is, π = c/d · This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

Answer. There is no contradiction. When we measure a length with scale or any other instrument, we only obtain an approximate rational value. We never obtain an exact value. For this reason, we may not realise that either c or d is irrational. Therefore, the fraction c/d is irrational. Hence, n is irrational.

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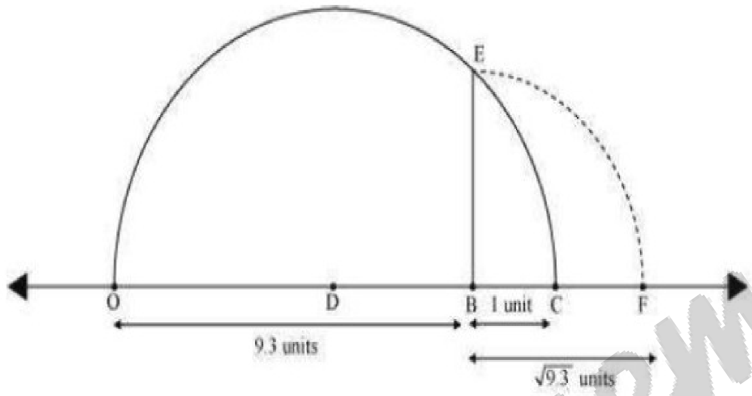
Q4 Represent $\sqrt{9.3}$ on the number line.

Answer. Mark a line segment 0B 9.3 on number line. Further, take BC of 1 unit. Find the mid-point D of OC and draw a semi-circle on OC while taking D as its centre. Draw a perpendicular to line OC passing through point B. Let it intersect the semi-circle at E. Taking B as centre and BE as radius, draw an arc intersecting number line at F. BF is $\sqrt{9.3}$.



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Q5 Rationalise the denominators of the following:

(i) $\frac{1}{\sqrt{7}}$ (ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$ (iv) $\frac{1}{\sqrt{7}-2}$

Answer. (i) $\frac{1}{\sqrt{7}} = \frac{1\times\sqrt{7}}{1\times\sqrt{7}} = \frac{\sqrt{7}}{7}$

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{1}{(\sqrt{7}-\sqrt{6})(\sqrt{7}+\sqrt{6})}$

$= \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7})^2-(\sqrt{6})^2}$

$= \frac{\sqrt{7}+\sqrt{6}}{7-6} = \frac{\sqrt{7}+\sqrt{6}}{1} = \sqrt{7}+\sqrt{6}$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}=\frac{(\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})}$
$=\frac{\sqrt{5}-\sqrt{2}}{(\sqrt{5})^2-(\sqrt{2})^2}=\frac{\sqrt{5}-\sqrt{2}}{5-2}$
$=\frac{\sqrt{5}-\sqrt{2}}{3}$
(iv) $\frac{1}{\sqrt{7}-2}=\frac{1}{(\sqrt{7}-2)(\sqrt{7}+2)}$
$=\frac{\sqrt{7}+2}{(\sqrt{7})^2-(2)^2}$
$=\frac{\sqrt{7}+2}{7-4}=\frac{\sqrt{7}+2}{3}$

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Exercise 1.6

Q1 Find : (i) $64^{\frac{1}{2}}$ (ii) $32^{\frac{1}{5}}$ (iii) $125^{\frac{1}{3}}$.

Answer. (i)
$64^{\frac{1}{2}}=\left(2^6\right)^{\frac{1}{2}}$
$=2^{6.\frac{1}{2}}$ $\quad\left[\left(a^m\right)^n=a^m\right]$
$=2^3=8$
(ii)
$32^{\frac{1}{5}}=\left(2^5\right)^{\frac{1}{5}}$ $\quad\left[\left(a^m\right)^n=a^m\right]\backslash)$
$=(2)^{5\frac{1}{5}}$
$=2^1=2$
(iii)
$(125)^{\frac{1}{3}}=\left(5^3\right)^{\frac{1}{3}}$ $\quad\left[\left(a^m\right)^n=a^m\right]$
$=5^{3\frac{1}{3}}$
$=5^1=5$

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Q2 Find:
(i) $^{\frac{3}{2}}\Big($ ii) $32^{\frac{2}{5}}$ (iii) $16^{\frac{3}{4}}$
(iv) $125^{\frac{-1}{3}}$

Answer.

(i)

$$9^{\frac{3}{2}} = \left(3^{2}\right)^{\frac{3}{2}} \qquad \left[(a^{m})^{n} = a^{m}\right]$$
$$= 3^{2\frac{3}{2}}$$
$$= 3^{3} = 27$$

(ii)

$$(32)^{\frac{2}{5}} = \left(2^{5}\right)^{\frac{2}{5}} \qquad \left[(a^{m})^{n} = a^{m}\right]$$
$$= 2^{5\times\frac{2}{5}}$$
$$= 2^{2} = 4$$

(iii)

$$(16)^{\frac{3}{4}} = \left(2^{4}\right)^{\frac{3}{4}}$$
$$= 2^{4+\frac{3}{4}} \qquad \left[(a^{m})^{n} = a^{m}\right]$$
$$= 2^{3} = 8$$

(iv)

$$(125)^{\frac{-1}{3}} = \frac{1}{(125)^{\frac{1}{3}}} \qquad \left[a^{-n} = \frac{1}{a^{n}}\right]$$
$$= \frac{1}{\left(5^{3}\right)^{\frac{1}{3}}}$$
$$= \frac{1}{5^{3\frac{1}{3}}} \qquad \left[(a^{m})^{n} = a^{m}\right]$$
$$= \frac{1}{5}$$

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Q3 Simplify:

(i) $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}}$ (ii) $(\frac{1}{3^{3}})^{7}$ (iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$

(iv) $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}}$.

Answer.

(i)

$$2^{\frac{2}{3}}.2^{\frac{1}{5}} = 2^{\frac{2}{3}+\frac{1}{5}} \qquad \left[a^{m}.a^{n} = a^{m+n}\right]$$
$$= 2^{\frac{10+3}{15}} = 2^{\frac{13}{15}}$$

(ii)

$$\left(\frac{1}{3^{3}}\right)^{7} = \frac{1}{3^{3\times7}} \qquad \left[\left(a^{m}\right)^{n} = a^{mn}\right]$$
$$= \frac{1}{3^{21}}$$
$$= 3^{-21} \qquad \left[\frac{1}{a^{m}} = a^{-m}\right]$$

(iii)

$$\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}=11^{\frac{1}{2}-\frac{1}{4}} \qquad \left[\frac{a^m}{a^n}=a^{m-n}\right]$$

$$=11^{\frac{2-1}{4}}=11^{\frac{1}{4}}$$

(iv)

$$7^{\frac{1}{2}}.8^{\frac{1}{2}}=(7\times8)^{\frac{1}{2}} \qquad [a^m.b^m=(ab)^m]$$

$$=(56)^{\frac{1}{2}}$$

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