GA - General Aptitude

Q1 - Q5 carry one mark each.

Q.No. 1 Rajiv Gandhi Khel Ratna Award was conferred_____Mary Kom, a six-time world champion in boxing, recently in a ceremony_____the Rashtrapati Bhawan (the President's official residence) in New Delhi.

(A) with, at
(B) on, in
(C) on, at
(D) to, at

Q.No. 2 Despite a string of poor performances, the chances of K. L. Rahul's selection in the team are_____.

(A) slim
(B) bright
(C) obvious
(D) uncertain

Q.No. 3 Select the word that fits the analogy:

Cover : Uncover :: Associate : _____

(A) Unassociate
(B) Inassociate
(C) Misassociate
(D) Dissociate

Q.No. 4 Hit by floods, the kharif (summer sown) crops in various parts of the country have been affected. Officials believe that the loss in production of the kharif crops can be recovered in the output of the rabi (winter sown) crops so that the country can achieve its food-grain production target of 291 million tons in the crop year 2019-20 (July-June). They are hopeful that good rains in July-August will help the soil retain moisture for a longer period, helping winter sown crops such as wheat and pulses during the November-February period.

Which of the following statements can be inferred from the given passage?

(A) Officials declared that the food-grain production target will be met due to good rains.
(B) Officials want the food-grain production target to be met by the November-February period.
(C) Officials feel that the food-grain production target cannot be met due to floods.
(D) Officials hope that the food-grain production target will be met due to a good rabi produce.

Q.No. 5 The difference between the sum of the first $2n$ natural numbers and the sum of the first n odd natural numbers is _____.

(A) $n^2 - n$
(B) $n^2 + n$
(C) $2n^2 - n$
(D) $2n^2 + n$

Q6 - Q10 carry two mark each.

Q.No. 6

Repo rate is the rate at which Reserve Bank of India (RBI) lends commercial banks, and reverse repo rate is the rate at which RBI borrows money from commercial banks.

Which of the following statements can be inferred from the above passage?

(A) Decrease in repo rate will increase cost of borrowing and decrease lending by commercial banks.

(B) Increase in repo rate will decrease cost of borrowing and increase lending by commercial banks.

(C) Increase in repo rate will decrease cost of borrowing and decrease lending by commercial banks.

(D) Decrease in repo rate will decrease cost of borrowing and increase lending by commercial banks.

Q.No. 7 P, Q, R, S, T, U, V, and W are seated around a circular table.

I. S is seated opposite to W.
II. U is seated at the second place to the right of R.
III. T is seated at the third place to the left of R.
IV. V is a neighbour of S.

Which of the following must be true?

(A) P is a neighbour of R.

(B) Q is a neighbour of R.

(C) P is not seated opposite to Q.

(D) R is the left neighbour of S.

Q.No. 8 The distance between Delhi and Agra is 233 km. A car P started travelling from Delhi to Agra and another car Q started from Agra to Delhi along the same road 1 hour after the car P started. The two cars crossed each other 75 minutes after the car Q started. Both cars were travelling at constant speed. The speed of car P was 10 km/hr more than the speed of car Q. How many kilometers the car Q had travelled when the cars crossed each other?

(A) 66.6

(B) 75.2

(C) 88.2

(D) 116.5

Q.No. 9 For a matrix $M = [m_{ij}];\ i,j = 1,2,3,4$, the diagonal elements are all zero and $m_{ij} = -m_{ji}$. The minimum number of elements required to fully specify the matrix is_____.

(A) 0

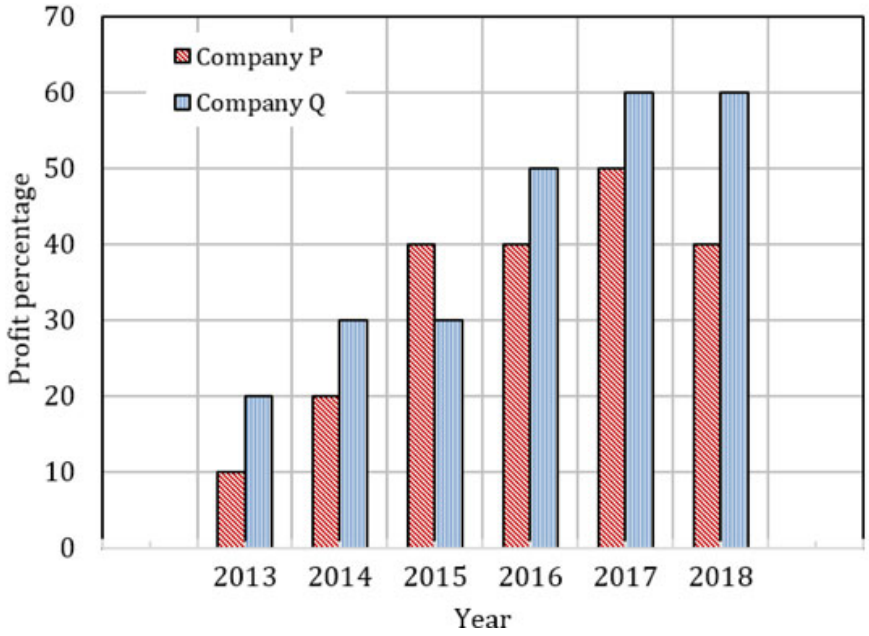
(B) 6

(C) 12

(D) 16

Q.No. 10

The profit shares of two companies P and Q are shown in the figure. If the two companies have invested a fixed and equal amount every year, then the ratio of the total revenue of company P to the total revenue of company Q, during 2013 - 2018 is_____.



(A) 15 : 17
(B) 16 : 17
(C) 17 : 15
(D) 17 : 16

ST: Statistics

Q.No. 1 Let M be a 3×3 non-zero idempotent matrix and let I_3 denote the 3×3 identity matrix. Then which of the following statements is **FALSE**?

(A) The eigenvalues of M are 0 and 1
(B) $\text{Rank}(M) = \text{Trace}(M)$
(C) $I_3 - M$ is idempotent
(D) $(I_3 + M)^{-1} = I_3 - 2M$

Q.No. 2 Let $\mathbb{C}$ denote the set of all complex numbers. Consider the vector space

$$V = \{(a, b, c) : a, b, c \in \mathbb{C}, \quad a + \bar{b} = 0, \quad b + \bar{c} = 0\},$$

over the field of real numbers, where for any complex number z, $\bar{z}$ denotes its complex conjugate. If $i = \sqrt{-1}$, then a basis of V is

(A) $\{(1, -1, 1), (i, i, i)\}$
(B) $\{(1, -1, 1), (i, -i, i)\}$
(C) $\{(1, -i, 1), (i, 1, i)\}$
(D) $\{(1, -i, 1), (i, 1, -i)\}$

Q.No. 3 Let $S = \{(x, y) \in \mathbb{R} \times \mathbb{R}: x^2 - y^2 = 4\}$ and $f: S \to \mathbb{R}$ be defined by $f(x, y) = 6x + y^2$, where $\mathbb{R}$ denotes the set of all real numbers. Then

(A) f is bounded on S
(B) the maximum value of f on S is 13
(C) the minimum value of f on S is -14
(D) the minimum value of f on S is -13

Q.No. 4 Let $f: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} \dfrac{4x^2 + 9y^2}{\sqrt{4x^2 + 9y^2 + 64} - 8}, & (x,y) \neq (0,0) \\ c, & (x,y) = (0,0) \end{cases},$$

where $\mathbb{R}$ denotes the set of all real numbers and $c \in \mathbb{R}$ is a fixed constant. Then, which of the following statements is TRUE?

(A) There does NOT exist a value of c for which f is continuous at $(0,0)$

(B) f is continuous at $(0,0)$ if $c = 0$

(C) f is continuous at $(0,0)$ if $c = 10$

(D) f is continuous at $(0,0)$ if $c = 16$

Q.No. 5 The moment generating function of a random variable X is given by

$$M_X(t) = \frac{1}{6} + \frac{1}{3}e^t + \frac{1}{3}e^{2t} + \frac{1}{6}e^{3t}, \quad -\infty < t < \infty.$$

Then $P(X \leq 2)$ equals

(A) $\frac{1}{3}$

(B) $\frac{1}{6}$

(C) $\frac{1}{2}$

(D) $\frac{5}{6}$

Q.No. 6 Consider the following two-way fixed effects analysis of variance model

$$Y_{ijk} = \mu + \alpha_i + \beta_j + \epsilon_{ijk}, \qquad i = 1,2; j = 1,2,3; k = 1,2,3;$$

where ϵ_{ijk}'s are independently and identically distributed $N(0, \sigma^2)$ random variables, $\sigma \in (0, \infty)$, $\alpha_1 + \alpha_2 = 0$ and $\beta_1 + \beta_2 + \beta_3 = 0$. Let SSE denote the sum of squares due to error. For any positive integer ν and any $\alpha \in (0,1)$, let $\chi^2_{\nu,\alpha}$ denote the $(1-\alpha)$-th quantile of the central chi-square distribution with ν degrees of freedom. Then a 95% confidence interval for σ^2 is given by

(A) $\left(0, \dfrac{SSE}{\chi^2_{13,0.95}}\right)$

(B) $\left(0, \dfrac{SSE}{\chi^2_{13,0.05}}\right)$

(C) $\left(0, \dfrac{SSE}{\chi^2_{14,0.05}}\right)$

(D) $\left(0, \dfrac{SSE}{\chi^2_{14,0.95}}\right)$

Q.No. 7

Let $X_1, \ldots, X_{20}$ be independent and identically distributed random variables with the common probability density function

$$f(x) = \frac{1}{6} e^{-\frac{|x-2|}{3}}, \qquad x \in (-\infty, \infty).$$

Then the distribution of the random variable

$$W = \frac{2}{3} \sum_{i=1}^{20} |X_i - 2|$$

is

(A) central chi-square with 10 degrees of freedom

(B) central chi-square with 20 degrees of freedom

(C) central chi-square with 30 degrees of freedom

(D) central chi-square with 40 degrees of freedom

Q.No. 8 Let $X_1, \ldots, X_{10}$ be a random sample from a Weibull distribution with the probability density function

$$f(x;\theta) = \begin{cases} 3\,\theta x^2 e^{-\theta x^3}, & x > 0 \\ 0, & \text{otherwise} \end{cases},$$

where $\theta \in (0, \infty)$. For any positive integer ν and any $\alpha \in (0,1)$, let $\chi^2_{\nu,\alpha}$ denote the $(1-\alpha)$-th quantile of the central chi-square distribution with ν degrees of freedom. Then, a 90% confidence interval for θ is

(A) $\left(\frac{\chi^2_{20,0.95}}{2\sum_{i=1}^{10} X_i^3}, \frac{\chi^2_{20,0.05}}{2\sum_{i=1}^{10} X_i^3} \right)$

(B) $\left(\frac{\chi^2_{20,0.90}}{2\sum_{i=1}^{10} X_i^4}, \frac{\chi^2_{20,0.10}}{2\sum_{i=1}^{10} X_i^4} \right)$

(C) $\left(\frac{\chi^2_{10,0.90}}{4\sum_{i=1}^{10} X_i^3}, \frac{\chi^2_{10,0.10}}{4\sum_{i=1}^{10} X_i^3} \right)$

(D) $\left(\frac{\chi^2_{10,0.95}}{3\sum_{i=1}^{10} X_i^2}, \frac{\chi^2_{10,0.05}}{3\sum_{i=1}^{10} X_i^2} \right)$

Q.No. 9 Let $X_1, \ldots, X_n$ be a random sample of size n (≥ 2) from a uniform distribution on the interval $[-\theta, \theta]$, where $\theta \in (0, \infty)$. A minimal sufficient statistic for θ is

(A) $\max\limits_{1 \leq i \leq n} X_i$

(B) $\left(-\min\limits_{1 \leq i \leq n} X_i, \max\limits_{1 \leq i \leq n} X_i \right)$

(C) $\max\limits_{1 \leq i \leq n} |X_i|$

(D) $\min\limits_{1 \leq i \leq n} |X_i|$

Q.No. 10 Let $X_1, \ldots, X_n$ be a random sample of size n (≥ 2) from $N(\theta, 2\theta^2)$ distribution, where $\theta \in (0, \infty)$. Which of the following statements is TRUE?

(A) $\frac{1}{n(n+2)} (\sum_{i=1}^{n} X_i)^2$ is the unique unbiased estimator of θ^2 that is a function of minimal sufficient statistic

(B) $\frac{1}{(3n+1)} \sum_{i=1}^{n} X_i^2$ is an unbiased estimator of θ^2

(C) There exist infinite number of unbiased estimators of θ^2 which are functions of minimal sufficient statistic

(D) There does **NOT** exist any unbiased estimator of $\theta(\theta+1)$ that is a function of minimal sufficient statistic

Q.No. 11 Let $\{N(t),\ t \geq 0\}$ be a Poisson process with rate $\lambda = 2$. Given that $N(3) = 1$, the expected arrival time of the first event of the process is

(A) 1

(B) $\frac{3}{2}$

(C) $\frac{2}{3}$

(D) 3

Q.No. 12 Consider the regression model

$$Y_i = \beta_0 + \beta_1 x_i^2 + \epsilon_i, i = 1, 2, \ldots, n\ (n \geq 2)\ ;$$

where β_0 and β_1 are unknown parameters and ϵ_i's are random errors. Let y_i be the observed value of $Y_i,\ i = 1, \ldots, n$. Using the method of ordinary least squares, the estimate of β_1 is

(A) $\frac{1}{n}\left(\frac{n\sum x_i^2 y_i - (\sum y_i)(\sum x_i^2)}{\sum x_i^4 - (\sum x_i^2)^2}\right)$

(B) $\frac{n\sum x_i^2 y_i - (\sum y_i)(\sum x_i^2)}{n\sum x_i^4 - (\sum x_i^2)^2}$

(C) $\frac{\sum x_i^2 y_i - (\sum y_i)(\sum x_i^2)}{\sum x_i^4 - (\sum x_i^2)^2}$

(D) $\frac{\sum x_i^2 y_i - n(\sum y_i)(\sum x_i^2)}{\sum x_i^4 - n(\sum x_i^2)^2}$

Q.No. 13 Let $\underline{X}_1, \ldots, \underline{X}_n$ be a random sample of size $n\ (\geq 2)$ from $N_p(\underline{0}, \Sigma)$ distribution, where $1 \leq p \leq n-1$ and Σ is a positive definite matrix. Define

$$\bar{\underline{X}} = \frac{1}{n}\sum_{i=1}^{n} \underline{X}_i \ \text{ and } (n-1)S = \sum_{i=1}^{n} (\underline{X}_i - \bar{\underline{X}})(\underline{X}_i - \bar{\underline{X}})^t,$$

where for any column vector $\underline{U}, \underline{U}^t$ denotes its transpose. Then the distribution of the statistic $T^2 = n\,\bar{\underline{X}}^t S^{-1} \bar{\underline{X}}$ is

(A) χ_p^2, the central chi-square distribution with p degrees of freedom

(B) $F_{p,n-p}$, the central F distribution with p and $n-p$ degrees of freedom

(C) $\frac{p(n-1)}{n-p} F_{p,n-p}$, where $F_{p,n-p}$ is the central F distribution with p and $n-p$ degrees of freedom

(D) $\frac{n-p}{p(n-1)} F_{n-p,p}$, where $F_{n-p,p}$ is the central F distribution with $n-p$ and p degrees of freedom

Q.No. 14 Consider a two-way fixed effects analysis of variance model without interaction effect and one observation per cell. If there are 5 factors and 4 columns, then the degrees of freedom for the error sum of squares is

(A) 20

(B) 19

(C) 12

(D) 11

Q.No. 15 Let $X_1, \ldots, X_n$ be a random sample of size n (≥ 2) from an exponential distribution with the probability density function

$$f(x;\theta) = \begin{cases} \frac{1}{\theta} e^{-x/\theta}, & x > 0 \\ 0, & \text{otherwise} \end{cases},$$

where $\theta \in \{1,2\}$. Consider the problem of testing $H_0: \theta = 1$ against $H_1: \theta = 2$, based on $X_1, \ldots, X_n$. Which of the following statements is TRUE?

(A) Likelihood ratio test at level α $(0 < \alpha < 1)$ leads to the same critical region as the corresponding most powerful test at the same level.

(B) Critical region of level α $(0 < \alpha < 1)$ likelihood ratio test is $\{(x_1, \ldots, x_n): \sum_{i=1}^{n} x_i < 0.5\, \chi^2_{2n,1-\alpha}\}$, where $\chi^2_{2n,1-\alpha}$ is the α-th quantile of the central chi-square distribution with $2n$ degrees of freedom

(C) Likelihood ratio test for testing H_0 against H_1 does not exist

(D) At any fixed level α $(0 < \alpha < 1)$, the power of the likelihood ratio test is lower than that of the most powerful test

Q.No. 16 The characteristic function of a random variable X is given by

$$\phi_X(t) = \begin{cases} \frac{\sin t \ \cos t}{t}, & \text{for } t \neq 0 \\ 1, & \text{for } t = 0 \end{cases}.$$

Then $P\left(|X| \leq \frac{3}{2}\right) =$ _____ (correct up to two decimal places).

Q.No. 17 Let the random vector $\underline{X} = (X_1, X_2, X_3, X_4)$ follow $N_4(\underline{\mu}, \Sigma)$ distribution, where

$$\underline{\mu} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{and} \quad \Sigma = \begin{bmatrix} 1 & 0.7 & 0.6 & 0.4 \\ 0.7 & 1 & 0.5 & 0.8 \\ 0.6 & 0.5 & 1 & 0.7 \\ 0.4 & 0.8 & 0.7 & 1 \end{bmatrix}.$$

Then

$P(X_1 + X_2 + X_3 + X_4 > 0) =$ _____ (correct up to one decimal place).

Q.No. 18 Let $\{X_n\}_{n\geq 0}$ be a homogeneous Markov chain with state space $\{0,1\}$ and one-step transition probability matrix $P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$. If $P(X_0 = 0) = \frac{1}{3}$, then

$27 \times E(X_2) =$ _____ (correct up to two decimal places).

Q.No. 19 Let E, F and G be mutually independent events with $P(E) = \frac{1}{2}, P(F) = \frac{1}{3}$ and $P(G) = \frac{1}{4}$. Let p be the probability that at least two of the events among E, F and G occur. Then $12 \times p =$ _____(correct up to one decimal place).

Q.No. 20 Let the joint probability mass function of (X, Y, Z) be

$$\mathrm{P}(X = x, Y = y, Z = z) = \frac{10!}{x!\, y!\, z!\, k!}\,(0.2)^x (0.3)^y (0.4)^z (0.1)^k,$$

where $k = 10 - x - y - z;\ x, y, z = 0,1, \ldots, 10;\ x + y + z \leq 10$.

Then the variance of the random variable $Y + Z$ equals _____

(correct up to one decimal place).

Q.No. 21 The total number of standard 4×4 Latin squares is _____

Q.No. 22 Let $\underline{X}$ be a 4×1 random vector with $E(\underline{X}) = \underline{0}$ and variance-covariance matrix

$$\Sigma = \begin{bmatrix} 1 & 0.5 & 0.5 & 0.5 \\ 0.5 & 1 & 0.5 & 0.5 \\ 0.5 & 0.5 & 1 & 0.5 \\ 0.5 & 0.5 & 0.5 & 1 \end{bmatrix}.$$

Let $\underline{Y}$ be the 4×1 random vector of principal components derived from Σ. The proportion of total variation explained by the first two principal components equals _____ (correct up to two decimal places).

Q.No. 23 Let $X_1, \ldots, X_n$ be a random sample of size $n\ (\geq 2)$ from an exponential distribution with the probability density function

$$f(x; \theta) = \begin{cases} e^{-(x-2\theta)}, & x > 2\,\theta \\ 0, & \text{otherwise} \end{cases},$$

where $\theta \in (0, \infty)$. If $X_{(1)} = \min\{X_1, \ldots, X_n\}$ then the conditional expectation

$$E\left[\frac{1}{\theta}\left(X_{(1)} - \frac{1}{n}\right) \mid \quad X_1 - X_2 = 2\right] = _____$$

Q.No. 24 Let $Y_i = \alpha + \beta x_i + \epsilon_i,\ i = 1, 2, \ldots, 7$, where x_i's are fixed covariates and ϵ_i's are independent and identically distributed random variables with mean zero and finite variance. Suppose that $\hat{\alpha}$ and $\hat{\beta}$ are the least squares estimators of α and β, respectively. Given the following data:

$$\sum_{i=1}^{7} x_i = 0, \quad \sum_{i=1}^{7} x_i^2 = 28, \quad \sum_{i=1}^{7} x_i y_i = 28, \quad \sum_{i=1}^{7} y_i = 21 \text{ and } \sum_{i=1}^{7} y_i^2 = 91,$$

where y_i is the observed value of $Y_i,\ i = 1, \ldots, 7$. Then the correlation coefficient between $\hat{\alpha}$ and $\hat{\beta}$ equals _____

Q.No. 25 Let $\{0,1,2,3\}$ be an observed sample of size 4 from $N(\theta, 5)$ distribution, where $\theta \in [2, \infty)$. Then the maximum likelihood estimate of θ based on the observed sample is _____.

Q.No. 26 Let $f: \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x, y) = x^4 - 2\,x^3 y + 16\,y + 17,$$

where $\mathbb{R}$ denotes the set of all real numbers. Then

(A) f has a local minimum at $\left(2,\frac{4}{3}\right)$

(B) f has a local maximum at $\left(2,\frac{4}{3}\right)$

(C) f has a saddle point at $\left(2,\frac{4}{3}\right)$

(D) f is bounded

Q.No. 27 Consider the linear transformation $T\colon \mathbb{C}^3 \to \mathbb{C}^3$ defined by

$$T\big((x,y,z)\big) = \left(x, \quad \frac{\sqrt{3}}{2}y - \frac{1}{2}z, \quad \frac{1}{2}y + \frac{\sqrt{3}}{2}z\right),$$

where $\mathbb{C}$ is the set of all complex numbers and $\mathbb{C}^3 = \mathbb{C} \times \mathbb{C} \times \mathbb{C}$. Which of the following statements is TRUE?

(A) There exists a non-zero vector X such that $T(X) = -X$

(B) There exist a non-zero vector Y and a real number $\lambda \neq 1$ such that $T(Y) = \lambda Y$

(C) T is diagonalizable

(D) $T^2 = I_3$, where I_3 is the 3×3 identity matrix

Q.No. 28 For real numbers a, b and c, let

$$M = \begin{bmatrix} a & ac & 0 \\ 1 & c & 0 \\ b & bc & 1 \end{bmatrix}.$$

Then, which of the following statements is TRUE?

(A) $\text{Rank}(M) = 3$ for every $a, b, c \in \mathbb{R}$

(B) If $a + c = 0$ then M is diagonalizable for every $b \in \mathbb{R}$

(C) M has a pair of orthogonal eigenvectors for every $a, b, c \in \mathbb{R}$

(D) If $b = 0$ and $a + c = 1$ then M is **NOT** idempotent

Q.No. 29 Let M be a 4×4 matrix with $(x-1)^2(x-3)^2$ as its minimal polynomial. Then, which of the following statements is **FALSE**?

(A) The eigenvalues of M are 1 and 3

(B) The algebraic multiplicity of the eigenvalue 1 is 3

(C) M is **NOT** diagonalizable

(D) $\text{Trace}(M) = 8$

Q.No. 30 Let $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x,y) = |y-2|\sqrt{|x-1|}, \qquad (x,y) \in \mathbb{R} \times \mathbb{R},$$

where $\mathbb{R}$ denotes the set of all real numbers. Then which of the following statements is TRUE?

(A) f is differentiable at $(1,2)$

(B) f is continuous at $(1,2)$ but **NOT** differentiable at $(1,2)$

(C) The partial derivative of f, with respect to x, at $(1,2)$ does **NOT** exist

(D) The directional derivative of f at $(1,2)$ along $\vec{u} = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ equals 1

Q.No. 31 Which of the following functions is uniformly continuous on the specified domain?

(A) $f_1(x) = e^{x^2}, -\infty < x < \infty$

(B)

$$f_2(x) = \begin{cases} \frac{1}{x}, & 0 < x \leq 1 \\ 0, & x = 0 \end{cases}$$

(C) $$f_3(x) = \begin{cases} x^2, & |x| \leq 1 \\ \frac{2}{1+x^2}, & |x| > 1 \end{cases}$$

(D) $$f_4(x) = \begin{cases} x, & |x| \leq 1 \\ x^2, & |x| > 1 \end{cases}$$

Q.No. 32 Let the random vector $\underline{X} = (X_1, X_2, X_3)$ have the joint probability density function

$$f_{\underline{X}}(x_1, x_2, x_3) = \begin{cases} \frac{1 - \sin x_1 \sin x_2 \sin x_3}{8\pi^3}, & 0 \leq x_1, x_2, x_3 \leq 2\pi \\ 0, & \text{otherwise} \end{cases}.$$

Which of the following statements is TRUE?

(A) X_1, X_2 and X_3 are mutually independent

(B) X_1, X_2 and X_3 are pairwise independent

(C) (X_1, X_2) and X_3 are independently distributed

(D) Variance of $X_1 + X_2$ is π^2

Q.No. 33 Suppose that P_1 and P_2 are two populations having bivariate normal distributions with mean vectors $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, respectively, and the same variance-covariance matrix $\begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}$. Let $Z_1 = \begin{pmatrix} 0.75 \\ 0.75 \end{pmatrix}$ and $Z_2 = \begin{pmatrix} 0.25 \\ 0.25 \end{pmatrix}$ be two new observations. If the prior probabilities for P_1 and P_2 are assumed to be equal and the misclassification costs are also assumed to be equal then, according to linear discriminant rule,

(A) Z_1 is assigned to P_1 and Z_2 is assigned to P_2

(B) Z_1 is assigned to P_2 and Z_2 is assigned to P_1

(C) both Z_1 and Z_2 are assigned to P_1

(D) both Z_1 and Z_2 are assigned to P_2

Q.No. 34 Let $X_1, \ldots, X_n$ be a random sample of size n (≥ 2) from an exponential distribution with the probability density function

$$f(x;\theta) = \begin{cases} \frac{1}{\theta} e^{-(x-\theta)/\theta}, & x > \theta \\ 0, & \text{otherwise} \end{cases},$$

where $\theta \in (0, \infty)$. Which of the following statements is TRUE?

(A) $\min\limits_{1 \leq i \leq n} X_i$ is a minimal sufficient statistic

(B) $\sum_{i=1}^{n} X_i$ is a minimal sufficient statistic

(C) Any minimal sufficient statistic is complete

(D) $\left(\min\limits_{1 \leq i \leq n} X_i, \frac{1}{n} \sum_{i=1}^{n} X_i \right)$ is minimal sufficient statistic

Q.No. 35

Let the joint distribution of (X, Y) be bivariate normal with mean vector $\begin{pmatrix}0\\0\end{pmatrix}$ and variance-covariance matrix $\begin{pmatrix}1 & \rho\\ \rho & 1\end{pmatrix}$, where $-1 < \rho < 1$. Then $\mathrm{E}\left[\max(X,Y)\right]$ equals

(A) $\frac{\sqrt{1-\rho}}{\pi}$

(B) $\sqrt{\frac{1-\rho}{\pi}}$

(C) 0

(D) $\frac{1}{2}$

Q.No. 36 Let $\underline{X}_1, \underline{X}_2, \ldots, \underline{X}_{10}$ be independent and identically distributed $N_3(\underline{0}, I_3)$ random vectors, where I_3 is the 3×3 identity matrix. Let

$$T = \sum_{i=1}^{10} \left(\underline{X}_i^t \left(I_3 - \frac{1}{3} J_3\right) \underline{X}_i\right),$$

where J_3 is the 3×3 matrix with each entry 1 and for any column vector $\underline{U}, \underline{U}^t$ denotes its transpose. Then the distribution of T is

(A) central chi-square with 5 degrees of freedom

(B) central chi-square with 10 degrees of freedom

(C) central chi-square with 20 degrees of freedom

(D) central chi-square with 30 degrees of freedom

Q.No. 37 Let $\underline{X}_1, \underline{X}_2$ and $\underline{X}_3$ be independent and identically distributed $N_4(\underline{0}, \Sigma)$ random vectors, where Σ is a positive definite matrix. Further, let $\boldsymbol{X} = \begin{pmatrix}\underline{X}_1^t\\ \underline{X}_2^t\\ \underline{X}_3^t\end{pmatrix}$ be a 3×4 matrix, where for any matrix M, M^t denotes its transpose. If $W_m(n, \Sigma)$ denotes a Wishart distribution of order m with n degrees of freedom and variance-covariance matrix Σ, then which of the following statements is TRUE?

(A) $\Sigma^{-1/2}\, \boldsymbol{X}^t \boldsymbol{X}\, \Sigma^{-1/2}$ follows $W_4(3, \mathrm{I}_4)$ distribution

(B) $\Sigma^{-1/2}\, \boldsymbol{X}^t \boldsymbol{X}\, \Sigma^{-1/2}$ follows $W_3(4, \mathrm{I}_3)$ distribution

(C) $\mathrm{Trace}(\boldsymbol{X}\, \Sigma^{-1} \boldsymbol{X}^t)$ follows χ_4^2 distribution

(D) $\boldsymbol{X}^t \boldsymbol{X}$ follows $W_3(4, \Sigma)$ distribution

Q.No. 38 Let the joint distribution of the random variables X_1, X_2 and X_3 be $N_3(\underline{\mu}, \Sigma)$, where

$$\underline{\mu} = \begin{pmatrix}1\\2\\3\end{pmatrix} \text{ and } \Sigma = \begin{bmatrix}1 & 0.5 & 0\\ 0.5 & 1 & 0\\ 0 & 0 & 5\end{bmatrix}.$$

Then which of the following statements is TRUE?

(A) $X_1 - X_2 + X_3$ and X_1 are independent

(B) $X_1 + X_2$ and $X_3 - X_1$ are independent

(C) $X_1 - X_2 + X_3$ and $X_1 + X_2$ are independent

(D) $X_1 - 2X_2$ and $2X_1 + X_2$ are independent

Q.No. 39 Consider the following one-way fixed effects analysis of variance model

$$Y_{ij} = \mu + \tau_i + \epsilon_{ij}, i = 1,2,3; j = 1,2,3,4;$$

where ϵ_{ij}'s are independent and identically distributed $N(0, \sigma^2)$ random variables, $\sigma \in (0, \infty)$ and $\tau_1 + \tau_2 + \tau_3 = 0$. Let MST and MSE denote the mean sum of squares due to treatment and the mean sum of squares due to error, respectively. For testing $H_0: \tau_1 = \tau_2 = \tau_3 = 0$ against $H_1: \tau_i \neq 0$, for some $i = 1,2,3$, consider the test based on the statistic $\frac{MST}{MSE}$. For positive integers ν_1 and ν_2, let F_{ν_1,ν_2} be a random variable having the central F-distribution with ν_1 and ν_2 degrees of freedom. If the observed value of $\frac{MST}{MSE}$ is given to be 104.45, then the p-value of this test equals

(A) $P(F_{2,9} > 104.45)$

(B) $P(F_{9,2} < 104.45)$

(C) $P(F_{3,11} < 104.45)$

(D) $P(F_{2,6} > 104.45)$

Q.No. 40 Let $X_1, \ldots, X_n$ be a random sample of size n (≥ 2) from $N(\theta, 1)$ distribution, where $\theta \in (-\infty, \infty)$. Consider the problem of testing $H_0: \theta \in [1, 2]$ against $H_1: \theta < 1$ or $\theta > 2$, based on $X_1, \ldots, X_n$. Which of the following statements is TRUE?

(A) Critical region, of level α $(0 < \alpha < 1)$ of uniformly most powerful test for H_0 against H_1 is of the form $\{(x_1, \ldots, x_n): c_1 \leq \sum_{i=1}^n x_i \leq c_2\}$, where c_1 and c_2 are such that the test is of level α

(B) Critical region, of level α $(0 < \alpha < 1)$ of uniformly most powerful test for H_0 against H_1 is of the form $\{(x_1, \ldots, x_n): \sum_{i=1}^n x_i > c$ or $\sum_{i=1}^n x_i < d\}$, where c and d are such that the test is of level α

(C) At any level $\alpha \in (0, 1)$, uniformly most powerful test for H_0 against H_1 does **NOT** exist

(D) At any level $\alpha \in (0, 1)$, the power of uniformly most powerful test for H_0 against H_1 is less than α

Q.No. 41 In a pure birth process with birth rates $\lambda_n = 2^n, n \geq 0$, let the random variable T denote the time taken for the population size to grow from 0 to 5. If $\mathrm{Var}(T)$ denotes the variance of the random variable T, then

$$256 \times \mathrm{Var}(T) = _____$$

Q.No. 42

Let $\{X_n\}_{n\geq 0}$ be a homogeneous Markov chain whose state space is $\{0,1,2\}$ and whose one-step transition probability matrix is $P = \begin{bmatrix} 0 & 1 & 0 \\ 0.3 & 0 & 0.7 \\ 0 & 1 & 0 \end{bmatrix}$. Then $\lim_{n\to\infty} P(X_{2n} = 2 \mid X_0 = 2) =$ _____ (correct up to one decimal place).

Q.No. 43 Let (X, Y) be a random vector such that, for any $y > 0$, the conditional probability density function of X given $Y = y$ is

$$f_{X|Y=y}(x) = y\, e^{-yx}, x > 0.$$

If the marginal probability density function of Y is

$$g(y) = ye^{-y}, y > 0$$

then $E(Y|X = 1) =$ _____ (correct up to one decimal place).

Q.No. 44 Let (X, Y) be a random vector with the joint moment generating function

$$M_{X,Y}(s,t) = e^{2\,s^2+t}, -\infty < s, t < \infty.$$

Let $\Phi(\cdot)$ denote the distribution function of the standard normal distribution and $p = P(X + 2\,Y < 1)$. If $\Phi(0) = 0.5, \Phi(0.5) = 0.6915, \Phi(1) = 0.8413$ and $\Phi(1.5) = 0.9332$ then the value of $2\,p + 1$ (round off to two decimal places) equals _____

Q.No. 45 Consider a homogeneous Markov chain $\{X_n\}_{n\geq 0}$ with state space $\{0,1,2,3\}$ and one-step transition probability matrix

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & \frac{3}{4} \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Assume that $P(X_0 = 1) = 1$. Let p be the probability that state 0 will be visited before state 3. Then $6 \times p =$ _____

Q.No. 46 Let (X, Y) be a random vector with joint probability mass function

$$f_{X,Y}(x,y) = \begin{cases} {}^{x}C_y \left(\frac{1}{4}\right)^x, & y = 0,1,2,\ldots,x; \quad x = 1,2,\ldots, \\ 0, & \text{otherwise} \end{cases}$$

where ${}^{x}C_y = \frac{x!}{y!(x-y)!}$. Then the variance of Y equals _____

Q.No. 47

Let X be a discrete random variable with probability mass function $f \in \{f_0, f_1\}$, where

	$x=1$	$x=2$	$x=3$	$x=4$	$x=5$
$f_0(x)$	0.10	0.10	0.10	0.10	0.60
$f_1(x)$	0.05	0.06	0.08	0.09	0.72

The power of the most powerful level $\alpha = 0.1$ test for testing $H_0: X \sim f_0$ against $H_1: X \sim f_1$, based on X, equals _____ (correct up to two decimal places).

Q.No. 48 Let $\underline{X} = (X_1, X_2, X_3)$ be a random vector following $N_3(\underline{0}, \Sigma)$ distribution, where $\Sigma = \begin{bmatrix} 1 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & 1 & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & 1 \end{bmatrix}$. Then the partial correlation coefficient between X_2 and X_3, with fixed X_1, equals _____(correct up to two decimal places).

Q.No. 49 Let X_1, X_2, X_3 and X_4 be a random sample from a population having probability density function $f_\theta(x) = f(x-\theta), -\infty < x < \infty$, where $\theta \in (-\infty, \infty)$ and $f(-x) = f(x)$, for all $x \in (-\infty, \infty)$. For testing $H_0: \theta = 0$ against $H_1: \theta < 0$, let T^+ denote the Wilcoxon Signed-rank statistic. Then under H_0,

$$32 \times P(T^+ \leq 5) = _____$$

Q.No. 50 A simple linear regression model with unknown intercept and unknown slope is fitted to the following data

x	-2	-1	0	1	2
y	3	5	8	9	10

using the method of ordinary least squares. Then the predicted value of y corresponding to $x = 5$ is _____

Q.No. 51 Let $D = \{(x, y, z) \in \mathbb{R} \times \mathbb{R} \times \mathbb{R} : 0 \leq x, y, z \leq 1, x + y + z \leq 1\}$, where $\mathbb{R}$ denotes the set of all real numbers. If

$$I = \iiint_D (x + y)\, dx\, dy\, dz,$$

then $84 \times I =$ _____

Q.No. 52

Let the random vector (X, Y) have the joint distribution function

$$F(x,y) = \begin{cases} 0, & x < 0 \text{ or } y < 0 \\ \dfrac{1-e^{-x}}{4}, & x \geq 0, 0 \leq y < 1 \\ 1-e^{-x}, & x \geq 0, y \geq 1 \end{cases}.$$

Let $\mathrm{Var}(X)$ and $\mathrm{Var}(Y)$ denote the variances of random variables X and Y, respectively. Then

$$16\,\mathrm{Var}(X) + 32\,\mathrm{Var}(Y) = _____$$

Q.No. 53 Let $\{X_n\}_{n\geq 1}$ be a sequence of independent and identically distributed random variables with $E(X_1) = 0, E(X_1^2) = 1$ and $E(X_1^4) = 3$. Further, let

$$Y_n = \frac{X_1^2 + X_2^2 + \cdots + X_n^2}{n}.$$

If

$$\lim_{n\to\infty} P\left(Y_n + \frac{\sqrt{n}(Y_n - 1)}{\sqrt{3}} \leq 2\right) = \Phi(c),$$

where $\Phi(\cdot)$ denotes the cumulative distribution function of the standard normal distribution, then $c^2 =$ _____(correct up to one decimal place).

Q.No. 54 Let the random vector $\underline{X} = (X_1, X_2, X_3)$ have the joint probability density function

$$f_{\underline{X}}(x_1, x_2, x_3) = \begin{cases} \dfrac{81}{4} x_1^2 x_2^2 x_3^2, & -1 \leq x_1 \leq x_2 \leq x_3 \leq 1 \\ 0, & \text{otherwise} \end{cases}.$$

Then the variance of the random variable $X_1 + X_2 + X_3$ equals _____ (correct up to one decimal place).

Q.No. 55 Let $X_1, \ldots, X_5$ be a random sample from a distribution with the probability density function

$$f(x;\theta) = \frac{1}{2} e^{-|x-\theta|}, x \in (-\infty, \infty),$$

where $\theta \in (-\infty, \infty)$. For testing $H_0: \theta = 0$ against $H_1: \theta < 0$, let $\sum_{i=1}^{5} Y_i$ be the sign test statistic, where

$$Y_i = \begin{cases} 1, & X_i > 0 \\ 0, & \text{otherwise} \end{cases}.$$

Then the size of the test, which rejects H_0 if and only if $\sum_{i=1}^{5} Y_i \leq 2$, equals _____(correct up to one decimal place).

Q.No.	Session	Que.Type	Sec. Name	Key	Marks
1	3	MCQ	GA	C	1
2	3	MCQ	GA	B	1
3	3	MCQ	GA	D	1
4	3	MCQ	GA	D	1
5	3	MCQ	GA	B	1
6	3	MCQ	GA	D	2
7	3	MCQ	GA	C	2
8	3	MCQ	GA	B	2
9	3	MCQ	GA	B	2
10	3	MCQ	GA	B	2
1	3	MCQ	ST	D	1
2	3	MCQ	ST	A	1
3	3	MCQ	ST	D	1
4	3	MCQ	ST	D	1
5	3	MCQ	ST	D	1
6	3	MCQ	ST	D	1
7	3	MCQ	ST	D	1
8	3	MCQ	ST	A	1
9	3	MCQ	ST	C	1
10	3	MCQ	ST	C	1
11	3	MCQ	ST	B	1
12	3	MCQ	ST	B	1
13	3	MCQ	ST	C	1
14	3	MCQ	ST	C	1
15	3	MCQ	ST	A	1
16	3	NAT	ST	0.75 to 0.75	1
17	3	NAT	ST	0.5 to 0.5	1
18	3	NAT	ST	16.25 to 16.25	1
19	3	NAT	ST	3.5 to 3.5	1
20	3	NAT	ST	2.1 to 201	1
21	3	NAT	ST	4 to 4	1
22	3	NAT	ST	0.75 to 0.75 or 75 to 75	1
23	3	NAT	ST	2 to 2	1
24	3	NAT	ST	0 to 0	1
25	3	NAT	ST	2 to 2	1
26	3	MCQ	ST	C	2
27	3	MCQ	ST	C	2
28	3	MCQ	ST	C	2
29	3	MCQ	ST	B	2
30	3	MCQ	ST	A	2
31	3	MCQ	ST	C	2
32	3	MCQ	ST	B	2
33	3	MCQ	ST	B	2
34	3	MCQ	ST	D	2
35	3	MCQ	ST	B	2
36	3	MCQ	ST	C	2
37	3	MCQ	ST	A	2
38	3	MCQ	ST	C	2
39	3	MCQ	ST	A	2
40	3	MCQ	ST	C	2
41	3	NAT	ST	341 to 341	2
42	3	NAT	ST	0.7 to 0.7	2
43	3	NAT	ST	1.5 to 1.5	2
44	3	NAT	ST	1.61 to 1.63	2
45	3	NAT	ST	4 to 4	2
46	3	NAT	ST	1 to 1	2
47	3	NAT	ST	0.12 to 0.12	2
48	3	NAT	ST	0.25 to 0.25	2
49	3	NAT	ST	18 to 18	2
50	3	NAT	ST	16 to 16	2
51	3	NAT	ST	7 to 7	2
52	3	NAT	ST	22 to 22	2
53	3	NAT	ST	1.5 to 1.5	2
54	3	NAT	ST	1.8 to 1.8	2
55	3	NAT	ST	0.5 to 0.5	2