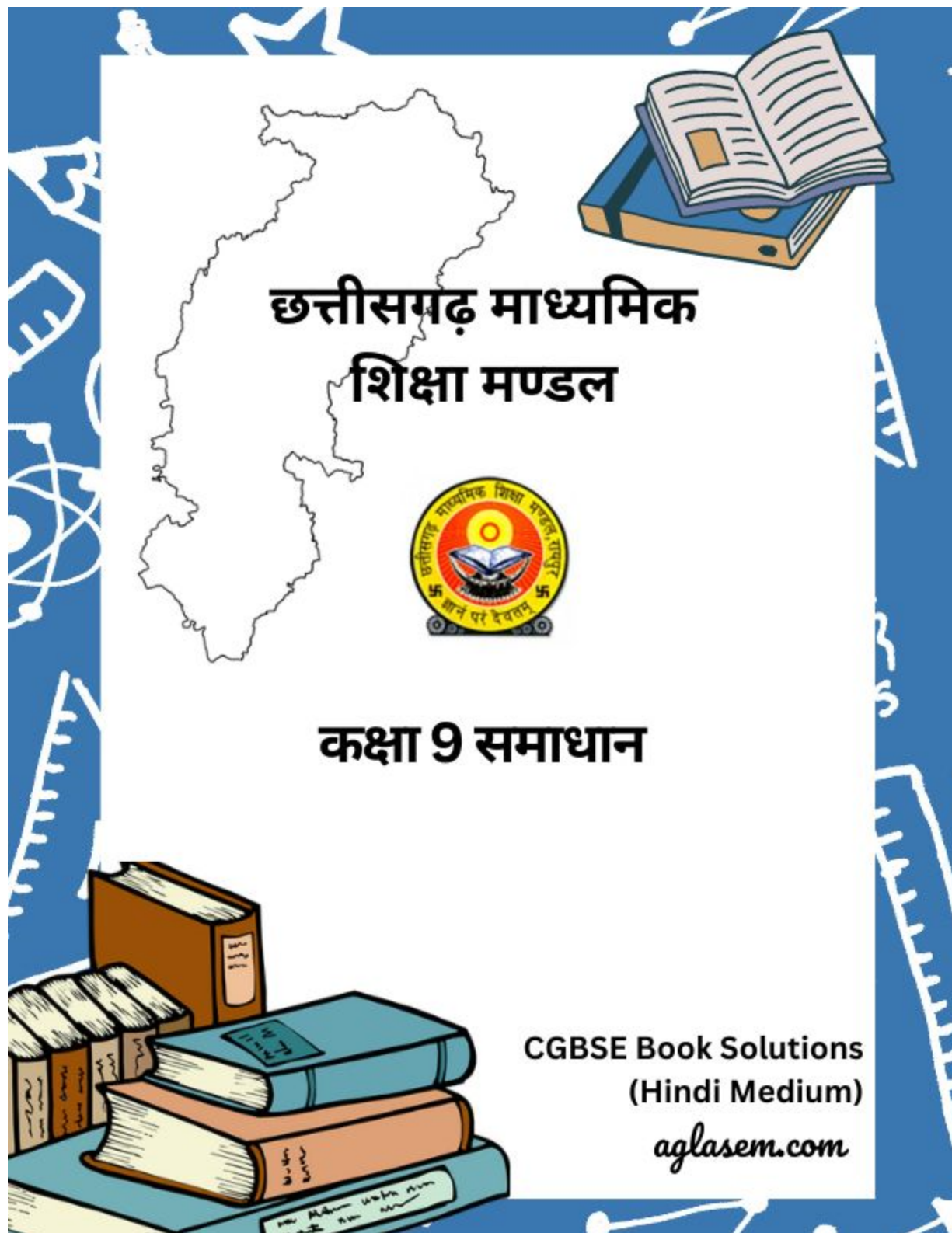


CG Board Class 9 Maths Book Solutions



CGBSE Class 9 Math 1 Solutions

Unit - 4

Chapter - 8

त्रिकोणमितीय अनुपात एवं सर्वसमिकाएँ

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प्रश्न 1 - यदि समकोण त्रिभुज ABC में $\angle B$ समकोण है तो निम्नलिखित में $\sin A$, $\cos C$, $\tan A$ का मान ज्ञात कीजिए जबकि -

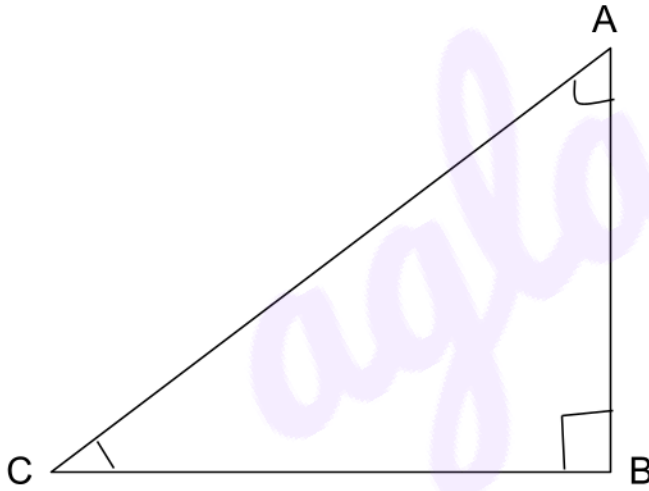
(i) $AC = 5, AB = 3, BC = 4$

(ii) $AB = 12, BC = 5, AC = 13$

(iii) $AB = 5, AC = 13, BC = 12$

(iv) $BC = 12, AB = 9, AC = 15$

हल :-



(i) $AC = 5, AB = 3, BC = 4$

$$\therefore \sin A = \frac{\text{लंब } BC}{\text{कर्ण } AC} = \frac{4}{5}$$

$$\cos C = \frac{\text{आधार } BC}{\text{कर्ण } AC} = \frac{4}{5}$$

$$\tan A = \frac{\text{लंब } BC}{\text{आधार } AB} = \frac{4}{3}.$$

उत्तर

(ii) $AB = 12, BC = 5, AC = 13$

$$\therefore \sin A = \frac{\text{लंब } BC}{\text{कर्ण } AC} = \frac{5}{13}$$

$$\cos C = \frac{\text{आधार } BC}{\text{कर्ण } AC} = \frac{5}{13}$$

$$\tan A = \frac{\text{लंब } BC}{\text{आधार } AB} = \frac{5}{12}.$$

उत्तर

(iii) AB=5, AC=13, BC=12

$$\therefore \sin A = \frac{\text{लंब } BC}{\text{कर्ण } AC} = \frac{12}{13}$$

$$\cos C = \frac{\text{आधार } BC}{\text{कर्ण } AC} = \frac{12}{13}$$

$$\tan A = \frac{\text{लंब } BC}{\text{आधार } AB} = \frac{12}{5}.$$

उत्तर

(iv) BC=12, AB=9, AC=15

$$\therefore \sin A = \frac{\text{लंब } BC}{\text{कर्ण } AC} = \frac{12}{15} = \frac{4}{5}$$

$$\cos C = \frac{\text{आधार } BC}{\text{कर्ण } AC} = \frac{12}{15} = \frac{4}{5}$$

$$\tan A = \frac{\text{लंब } BC}{\text{आधार } AB} = \frac{12}{9} = \frac{4}{3}.$$

उत्तर

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प्रश्न 1 - निम्नलिखित में कोई एक त्रिकोणमितीय अनुपात दिया गया है। शेष त्रिकोणमितीय अनुपात ज्ञात कीजिए -

$$(i) \tan \theta = \frac{3}{4} \quad (ii) \sin \theta = \frac{5}{13} \quad (iii) \cos \alpha = \frac{1}{3}$$

$$(iv) \cot \theta = 1 \quad (v) \operatorname{cosec} A = \frac{5}{4} \quad (vi) \sec \beta = 2$$

$$(vii) \operatorname{cosec} A = \sqrt{10}.$$

$$\text{हल :- (i) } \tan \theta = \frac{3}{4}.$$

हम जानते हैं,

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\therefore 1 + \left(\frac{3}{4}\right)^2 = \sec^2 \theta$$

$$\Rightarrow \sec^2 \theta = 1 + \frac{9}{16} = \frac{16+9}{16} = \frac{25}{16}$$

$$\therefore \sec \theta = \frac{5}{4}.$$

$$\cos \theta = \frac{1}{\sec \theta} = \frac{1}{\frac{5}{4}} = \frac{4}{5}.$$

$$\sin^2 \theta = 1 - \cos^2 \theta \text{ से,}$$

$$= 1 - \left(\frac{4}{5}\right)^2 = 1 - \frac{16}{25}$$

$$= \frac{25-16}{25} = \frac{9}{25}$$

$$\therefore \sin\theta = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{3}{5}} = \frac{5}{3}.$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{3}{4}} = \frac{4}{3}.$$

$$(ii) \sin\theta = \frac{5}{13}$$

$$\therefore \operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{5}{13}} = \frac{13}{5}.$$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$= 1 - \left(\frac{5}{13}\right)^2 = 1 - \frac{25}{169}$$

$$= \frac{169-25}{169} = \frac{144}{169}$$

$$\therefore \cos\theta = \frac{12}{13}.$$

$$\therefore \sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{12}{13}} = \frac{13}{12}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{5}{13}}{\frac{12}{13}} = \frac{5}{12}.$$

$$\therefore \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{5}{12}} = \frac{12}{5}.$$

$$(iii) \cos\alpha = \frac{1}{3}$$

$$\therefore \sec\alpha = 3$$

$$\sin\alpha = \sqrt{1 - \cos^2\alpha}$$

$$= \sqrt{1 - \frac{1}{9}} = \sqrt{\frac{9-1}{9}} = \frac{\sqrt{8}}{3} = \frac{2\sqrt{2}}{3}.$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\frac{2\sqrt{2}}{3}}{\frac{1}{3}} = 2\sqrt{2}.$$

$$\therefore \operatorname{cosec}\alpha = \frac{1}{\sin\alpha} = \frac{1}{\frac{2\sqrt{2}}{3}} = \frac{3}{2\sqrt{2}}$$

$$= \frac{3\sqrt{2}}{4}.$$

$$(iv) \cot\theta = 1$$

$$\therefore \tan\theta = \frac{1}{\cot\theta} = 1.$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + 1 = \sec^2\theta$$

$$\therefore \sec\theta = \sqrt{2}$$

$$\cos\theta = \frac{1}{\sqrt{2}}$$

$$\sin\theta = \sqrt{1 - \cos^2\theta}$$

$$\sin\theta = \sqrt{1 - \frac{1}{2}} = \frac{1}{\sqrt{2}}$$

$$\therefore \operatorname{cosec}\theta = \sqrt{2}.$$

$$(v) \operatorname{cosec}A = \frac{5}{4}$$

$$\therefore \sin A = \frac{4}{5}$$

$$\therefore \cos A = \sqrt{1 - \sin^2 A}$$

$$= \sqrt{1 - \frac{16}{25}} = \sqrt{\frac{25-16}{25}}$$

$$= \sqrt{\frac{9}{25}} = \frac{3}{5}$$

$$\therefore \sec A = \frac{5}{3}$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}$$

$$\therefore \cot A = \frac{3}{4}.$$

$$(vi) \sec\beta = 2$$

$$\therefore \cos\beta = \frac{1}{2}$$

$$\sin\beta = \sqrt{1 - \cos^2\beta}$$

$$\sin\beta = \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{4-1}{4}} = \frac{\sqrt{3}}{2}$$

$$\therefore \operatorname{cosec}\beta = \frac{2}{\sqrt{3}}$$

$$\tan\beta = \frac{\sin\beta}{\cos\beta} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

$$\therefore \cot \beta = \frac{1}{\sqrt{3}}.$$

$$(vii) \operatorname{cosec} A = \sqrt{10}$$

$$\therefore \sin A = \frac{1}{\sqrt{10}}$$

$$\begin{aligned} \cos A &= \sqrt{1 - \sin^2 A} \\ &= \sqrt{1 - \frac{1}{10}} = \sqrt{\frac{10-1}{10}} \\ &= \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}} \end{aligned}$$

$$\therefore \sec A = \frac{\sqrt{10}}{3}$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$= \frac{1}{\frac{\frac{3}{\sqrt{10}}}{\frac{3}{\sqrt{10}}}} = \frac{1}{3}$$

$$\cot A = 3.$$

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प्रश्न 2 - यदि $\cot \theta = \frac{21}{20}$ हो तो $\sin \theta \times \cos \theta$ का मान ज्ञात कीजिए।

हल :- माना $\triangle ABC$ में $\angle B$ समकोण है तथा $\angle A = \theta$

$$\text{तब } \cos \theta = \frac{21}{20}$$

$\therefore$ आधार $AB = 21$ तथा लम्ब $BC = 20$

$\therefore$ पाइथागोरस प्रमेय से ,

$$AB^2 + BC^2 = AC^2$$

$$\Rightarrow 21^2 + 20^2 = AC^2$$

$$\Rightarrow 441 + 400 = AC^2$$

$$\Rightarrow 841 = AC^2$$

$$\therefore AC = \sqrt{841} = 29$$

$$\sin \theta = \frac{BC}{AC} = \frac{20}{29}$$

$$\cos \theta = \frac{AB}{AC} = \frac{21}{29}$$

$$\therefore \sin \theta \times \cos \theta = \frac{20}{29} \times \frac{21}{29}$$

$$= \frac{420}{841}.$$

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प्रश्न 3 - यदि $\cos A = \frac{4}{5}$ हो। तो $\frac{\cot A - \sin A}{2 \tan A}$ का मान ज्ञात कीजिए।

हल :- $\cos A = \frac{4}{5}$

$\therefore \sec A = \frac{5}{4}$

अब, $1 + \tan^2 A = \sec^2 A$ से,

$$1 + \tan^2 A = \left(\frac{5}{4}\right)^2$$

$$\Rightarrow \tan^2 A = \frac{25}{16} - 1 \\ = \frac{25-16}{16} = \frac{9}{16}$$

$\therefore \tan A = \frac{3}{4}$

$\therefore \cot A = \frac{4}{3}$

$\cos A = \frac{4}{5}$

$$\therefore \sin A = \sqrt{1 - \cos^2 A} \\ = \sqrt{1 - \left(\frac{4}{5}\right)^2} = \sqrt{1 - \frac{16}{25}} \\ = \sqrt{\frac{25-16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

अब ,
$$\frac{\cot A - \sin A}{2 \tan A} = \frac{\frac{4}{3} - \frac{3}{5}}{2 \cdot \frac{3}{4}} \\ = \frac{\frac{20-9}{15}}{\frac{3}{2}} = \frac{11}{15} \times \frac{2}{3} = \frac{22}{45} \text{ उत्तर}$$

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प्रश्न 4 - यदि $\sec \theta = \frac{5}{3}$ हो, तो $\frac{\tan \theta - \sin \theta}{1 + \tan \theta \sin \theta}$ का मान ज्ञात कीजिए।

हल :- $\sec \theta = \frac{5}{3}$

$\therefore \cos \theta = \frac{3}{5}$

$$\sin \theta = \sqrt{1 - \cos^2 \theta} \\ = \sqrt{1 - \left(\frac{3}{5}\right)^2}$$

$$= \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{25-9}{25}}$$

$$= \sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$\therefore \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}$$

$$\text{अतः, } \frac{\tan \theta - \sin \theta}{1 + \tan \theta \cdot \sin \theta}$$

$$= \frac{\frac{4}{3} - \frac{4}{5}}{1 + \frac{4}{3} \cdot \frac{4}{5}} = \frac{\frac{20-12}{15}}{1 + \frac{16}{15}}$$

$$= \frac{\frac{8}{15}}{\frac{15+16}{15}} = \frac{8}{15} \times \frac{15}{31} = \frac{8}{31}. \quad \text{उत्तर}$$

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प्रश्न 5 - यदि $\sin A = \frac{1}{3}$ हो, तो $\cos A, \operatorname{cosec} A + \tan A, \sec A$ का मान ज्ञात कीजिए।

$$\text{हल :- } \sin A = \frac{1}{3}$$

$$\begin{aligned} \therefore \cos A &= \sqrt{1 - \sin^2 A} \\ &= \sqrt{1 - \frac{1}{9}} = \sqrt{\frac{9-1}{9}} \\ &= \frac{\sqrt{8}}{3} = \frac{2\sqrt{2}}{3} \end{aligned}$$

$$\sin A = \frac{1}{3}$$

$$\therefore \operatorname{cosec} A = 3$$

$$\begin{aligned} \tan A &= \frac{\sin A}{\cos A} \\ &= \frac{\frac{1}{3}}{\frac{2\sqrt{2}}{3}} = \frac{1}{2\sqrt{2}} \end{aligned}$$

$$\therefore \operatorname{cosec} A + \tan A$$

$$= 3 + \frac{1}{2\sqrt{2}} = \frac{6\sqrt{2}+1}{2\sqrt{2}}$$

$$= \frac{(6\sqrt{2}+1)\sqrt{2}}{2\sqrt{2} \times \sqrt{2}}$$

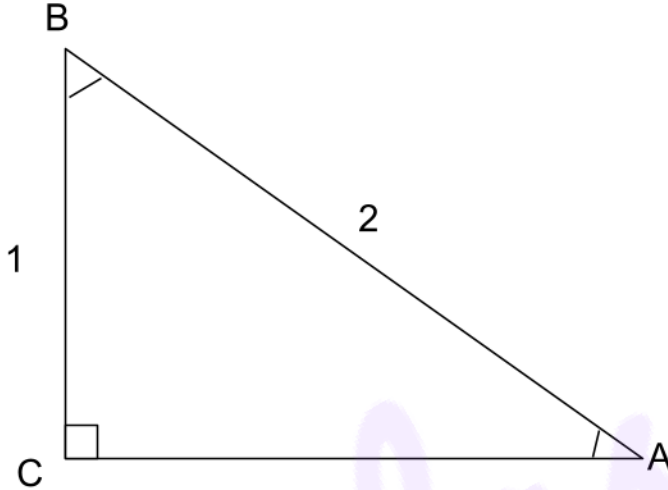
$$= \frac{(6\sqrt{2}+1)\sqrt{2}}{4}$$

$$\cos A = \frac{2\sqrt{2}}{3}$$

$$\therefore \sec A = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4} \quad \text{उत्तर}$$

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प्रश्न 6- किसी समकोण त्रिभुज ABC में $\angle C$ समकोण हो तथा $\tan A = \frac{1}{\sqrt{3}}$ हो, तो $\sin A \cos B + \cos A \sin B$ का मान ज्ञात कीजिये।



हल :-

$\triangle ABC$ में $\angle C$ समकोण है।

$$\tan A = \frac{1}{\sqrt{3}}$$

$\therefore$ लम्ब $BC = 1$ तथा आधार $AC = \sqrt{3}$

$\therefore$ पाइथागोरस प्रमेय से ,

$$AC^2 + BC^2 = AB^2$$

$$\Rightarrow (\sqrt{3})^2 + 1^2 = AB^2$$

$$\Rightarrow 3 + 1 = AB^2$$

$$\Rightarrow 4 = AB^2$$

$$\therefore \sin A = \frac{BC}{AB} = \frac{1}{2}$$

$$\cos B = \frac{BC}{AB} = \frac{1}{2}$$

$$\cos A = \frac{AC}{AB} = \frac{\sqrt{3}}{2}$$

$$\sin B = \frac{AC}{AB} = \frac{\sqrt{3}}{2}$$

$$\therefore \sin A \cos B + \cos A \sin B$$

$$\begin{aligned}
 &= \frac{1}{2} \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} \\
 &= \frac{1}{4} + \frac{3}{4} \\
 &= \frac{1+3}{4} = \frac{4}{4} = 1. \quad \text{उत्तर}
 \end{aligned}$$

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प्रश्न 7 - यदि $\cot A = \frac{3}{4}$ हो तो $\frac{\sin A + \cos A}{\sin A - \cos A}$ का मान ज्ञात कीजिए।

हल :- $\cot A = \frac{3}{4}$

$$\Rightarrow \frac{\cos A}{\sin A} = \frac{3}{4}$$

$$\Rightarrow \frac{\sin A}{\cos A} = \frac{4}{3} \quad \text{-----(1)}$$

समी (1) के दोनों पक्षों में 1 जोड़ने पर ,

$$\frac{\sin A}{\cos A} + 1 = \frac{4}{3} + 1$$

$$\Rightarrow \frac{\sin A + \cos A}{\cos A} = \frac{4+3}{3} = \frac{7}{3} \quad \text{-----(2)}$$

पुनः समी (1) के दोनों पक्षों में 1 घटाने पर ,

$$\frac{\sin A}{\cos A} - 1 = \frac{4}{3} - 1$$

$$\Rightarrow \frac{\sin A - \cos A}{\cos A} = \frac{4-3}{3} = \frac{1}{3} \quad \text{-----(3)}$$

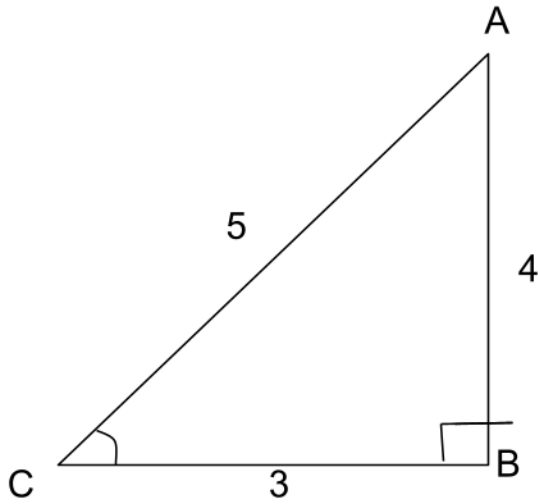
समी (2) को समी (3) से भाग देने पर ,

$$\frac{\frac{\sin A + \cos A}{\cos A}}{\frac{\sin A - \cos A}{\cos A}} = \frac{\frac{7}{3}}{\frac{1}{3}}$$

$$\Rightarrow \frac{\sin A + \cos A}{\sin A - \cos A} = 7. \quad \text{उत्तर}$$

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प्रश्न 8 - यदि $\sin \theta = \frac{4}{5}$ हो , तो $\frac{4 \tan \theta - 5 \cos \theta}{\sec \theta + 4 \cot \theta}$ का मान ज्ञात कीजिए।



हल :-

माना ΔABC में, $\angle B = 90^\circ$ तथा $\angle C = \theta$ तब $\sin \theta = \frac{4}{5}$

$\therefore \sin \theta = \frac{\text{लम्ब}}{\text{कर्ण}}$ से,

लम्ब $AB = 4$; कर्ण $AC = 5$

अब, ΔABC में पाइथागोरस प्रमेय से,

$$AB^2 + BC^2 = AC^2$$

$$\Rightarrow 4^2 + BC^2 = AC^2$$

$$\Rightarrow BC^2 = 25 - 16 = 9$$

$$\therefore BC = 3$$

$$\text{अतः } \tan \theta = \frac{\text{लम्ब}}{\text{आधार}} = \frac{AB}{BC}$$

$$= \frac{4}{3}$$

$$\therefore \cot \theta = \frac{3}{4}$$

$$\cos \theta = \frac{\text{आधार}}{\text{कर्ण}} = \frac{BC}{AC} = \frac{3}{5}$$

$$\sec \theta = \frac{5}{3}$$

$$\frac{4 \tan \theta - 5 \cos \theta}{\sec \theta + 4 \cot \theta}$$

$$= \frac{4 \cdot \frac{4}{3} - 5 \cdot \frac{3}{5}}{\frac{5}{3} + 4 \cdot \frac{3}{4}} = \frac{16 - 9}{\frac{5 + 9}{3}}$$

$$= \frac{7}{14} = \frac{1}{2}. \text{ उत्तर}$$

प्रश्न 1 . निम्न में से सही विकल्प चुनिए -

(i) $\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ}$

(a) 1 (b) $\tan 90^\circ$ (c) 0 (d) $\sin 45^\circ$

$$\begin{aligned}\text{हल :- } & \frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} \\ & = \frac{1 - (1)^2}{1 + (1)^2} = \frac{0}{2} = 0.\end{aligned}$$

∴ उत्तर -(c)

(ii) $\frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} =$

(a) $\sin 60^\circ$ (b) $\sin 30^\circ$ (c) $\tan 60^\circ$ (d) $\cos 60^\circ$

$$\begin{aligned}\text{हल :- } & \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} \\ & = \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{2}{1 - \frac{1}{3}} \\ & = \frac{\frac{2}{\sqrt{3}}}{\frac{3-1}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{2} = \sqrt{3} \\ & = \tan 60^\circ.\end{aligned}$$

∴ उत्तर -(c)

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प्रश्न 2 - निम्नलिखित त्रिकोणमितीय व्यंजकों के मान ज्ञात कीजिये

(i) $\cos 30^\circ \cos 45^\circ - \sin 30^\circ \sin 45^\circ$

$$\begin{aligned}\text{हल :- } & \cos 30^\circ \cos 45^\circ - \sin 30^\circ \sin 45^\circ \\ & = \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} - \frac{1}{2} \cdot \frac{1}{\sqrt{2}} \\ & = \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}} \\ & = \frac{\sqrt{2}(\sqrt{3}-1)}{4} \quad \text{उत्तर}\end{aligned}$$

(ii) $\tan 30^\circ \sec 45^\circ + \tan 60^\circ \sec 30^\circ$

$$\text{हल :- } \tan 30^\circ \sec 45^\circ + \tan 60^\circ \sec 30^\circ.$$

$$= \frac{1}{\sqrt{3}} \cdot \sqrt{2} + \sqrt{3} \cdot \frac{2}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} + 2$$

$$= \frac{\sqrt{2} + 2\sqrt{3}}{\sqrt{3}}$$

$$= \frac{\sqrt{3}(\sqrt{2} + 2\sqrt{3})}{3} = \frac{\sqrt{6} + 6}{3}. \quad \text{उत्तर}$$

$$(iii) \operatorname{cosec} 30^\circ + \cot 45^\circ$$

$$\text{हल :- } \operatorname{cosec} 30^\circ + \cot 45^\circ \\ = 2 + 1 = 3.$$

$$(iv) \frac{\cot 60^\circ}{\sec 30^\circ - \tan 45^\circ}$$

$$\text{हल :- } \frac{\cot 60^\circ}{\sec 30^\circ - \tan 45^\circ}$$

$$= \frac{1}{\frac{2}{\sqrt{3}} - 1} = \frac{\frac{1}{\sqrt{3}}}{\frac{2 - \sqrt{3}}{\sqrt{3}}}$$

$$= \frac{1}{2 - \sqrt{3}} = \frac{2 + \sqrt{3}}{(2 - \sqrt{3})(2 + \sqrt{3})}$$

$$= \frac{2 + \sqrt{3}}{4 - 3} = 2 + \sqrt{3}. \quad \text{उत्तर}$$

$$(V) \tan^2 60^\circ + \tan^2 45^\circ$$

$$\text{हल :- } \tan^2 60^\circ + \tan^2 45^\circ$$

$$= (\sqrt{3})^2 + (1)^2 = 3 + 1 = 4. \quad \text{उत्तर}$$

$$(vi) \frac{\cos 30^\circ + \sin 60^\circ}{1 + \cos 60^\circ + \sin 30^\circ}$$

$$\text{हल :- } \frac{\cos 30^\circ + \sin 60^\circ}{1 + \cos 60^\circ + \sin 30^\circ}$$

$$= \frac{\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}}{1 + \frac{1}{2} + \frac{1}{2}}$$

$$= \frac{\frac{2\sqrt{3}}{2}}{\frac{2+1+1}{2}} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}. \quad \text{उत्तर}$$

$$(vii) \frac{\sin^2 45^\circ + \cos^2 45^\circ}{\tan^2 60^\circ}$$

$$\begin{aligned} \text{हल :- } & \frac{\sin^2 45^\circ + \cos^2 45^\circ}{\tan^2 60^\circ} \\ &= \frac{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}{(\sqrt{3})^2} \\ &= \frac{\frac{1}{2} + \frac{1}{2}}{3} = \frac{1}{3}. \quad \text{उत्तर} \end{aligned}$$

$$(viii) \frac{\sin 30^\circ - \sin 90^\circ + 2 \cos 0^\circ}{\tan 30^\circ \tan 60^\circ}$$

$$\begin{aligned} \text{हल :- } & \frac{\sin 30^\circ - \sin 90^\circ + 2 \cos 0^\circ}{\tan 30^\circ \tan 60^\circ} \\ &= \frac{\frac{1}{2} - 1 + 2 \cdot 1}{\frac{1}{\sqrt{3}} \cdot \sqrt{3}} \\ &= \frac{1 - 2 + 4}{2} = \frac{3}{2}. \quad \text{उत्तर} \end{aligned}$$

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प्रश्न 3 . जाँचिए सत्य या असत्य —

$$(i) \sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ = \tan 90^\circ$$

$$\begin{aligned} \text{हल :- L.H.S.} &= \sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ \\ &= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} \\ &= \frac{1}{4} + \frac{3}{4} = \frac{1+3}{4} = \frac{4}{4} = 1 \\ &= \tan 45^\circ = R.H.S. \quad \text{असत्य} \end{aligned}$$

$$(ii) 1 - 2\sin^2 30^\circ = \cos^2 60^\circ$$

$$\begin{aligned} \text{हल :- L.H.S.} &= 1 - 2\sin^2 30^\circ \\ &= 1 - 2\left(\frac{1}{2}\right)^2 = 1 - 2 \cdot \frac{1}{4} \\ &= 1 - \frac{1}{2} = \frac{1}{2} \\ &= \cos^2 60^\circ = R.H.S. \quad \text{असत्य} \end{aligned}$$

$$(iii) 2\cos^2 45^\circ - 1 = \cos 90^\circ.$$

$$\begin{aligned}
 \text{हल :- L.H.S.} &= 2\cos^2 45^0 - 1 \\
 &= 2\left(\frac{1}{\sqrt{2}}\right)^2 - 1 = 2 \cdot \frac{1}{2} - 1 \\
 &= 1 - 1 = 0 \\
 &= \cos 90^0 = R.H.S. \quad \text{सत्य}
 \end{aligned}$$

$$(iv) \sin^2 45^0 = 1 - \cos^2 45^0.$$

$$\begin{aligned}
 \text{हल :-} \sin^2 45^0 &= \left(\frac{1}{\sqrt{2}}\right)^2 \\
 &= \frac{1}{2}. \quad \text{-----}(1)
 \end{aligned}$$

$$\begin{aligned}
 1 - \cos^2 45^0 \\
 &= 1 - \left(\frac{1}{\sqrt{2}}\right)^2 = 1 - \frac{1}{2} \\
 &= \frac{1}{2} \quad \text{-----}(2)
 \end{aligned}$$

समी (1) और (2) से

$$\sin^2 45^0 = 1 - \cos^2 45^0. \quad \text{सत्य}$$

$$(v) \sin^2 60^0 + \cos^2 60^0 = 1.$$

$$\begin{aligned}
 \text{हल :- L.H.S.} \sin^2 60^0 + \cos^2 60^0 \\
 &= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 \\
 &= \frac{3}{4} + \frac{1}{4} = \frac{3+1}{4} \\
 &= \frac{4}{4} = 1 = R.H.S. \quad \text{सत्य}
 \end{aligned}$$

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निम्नलिखित त्रिकोणमितीय समीकरण को θ के मान के लिए हल कीजिए ;जबकि

$$0^\circ \leq \theta^\circ \leq 90^\circ \text{ —}$$

$$1. \sin \theta = \cos \theta$$

$$\text{हल :- चूँकि } \sin 45^0 = \cos 45^0 = \frac{1}{\sqrt{2}}$$

$$\therefore \sin \theta = \cos \theta$$

$$\text{तब, } \theta = 45^0. \quad \text{उत्तर}$$

2. $2 \cos \theta = 1$

हल :- $2 \cos \theta = 1$

$$\Rightarrow \cos \theta = \frac{1}{2}$$

$$\Rightarrow \cos \theta = \cos 60^\circ$$

$$\therefore \theta = 60^\circ. \quad \text{उत्तर}$$

3. $2 \sin^2 \theta = \frac{1}{2}$

हल :- $2 \sin^2 \theta = \frac{1}{2}$

$$\Rightarrow \sin^2 \theta = \frac{1}{4}$$

$$\Rightarrow \sqrt{\sin^2 \theta} = \sqrt{\frac{1}{4}}$$

$$\Rightarrow \sin \theta = \frac{1}{2} = \sin 30^\circ$$

$$\therefore \theta = 30^\circ. \quad \text{उत्तर}$$

4. $3 \tan^2 \theta - 1 = 0$

हल :- $3 \tan^2 \theta - 1 = 0$

$$\Rightarrow 3 \tan^2 \theta = 1$$

$$\Rightarrow \tan^2 \theta = \frac{1}{3} = \left(\frac{1}{\sqrt{3}}\right)^2$$

$$\Rightarrow \tan \theta = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

$$\therefore \theta = 30^\circ. \quad \text{उत्तर}$$

5. $2 \sin \theta = \sqrt{3}$

हल :- $2 \sin \theta = \sqrt{3}$

$$\Rightarrow \sin \theta = \frac{\sqrt{3}}{2} = \sin 60^\circ$$

$$\therefore \theta = 60^\circ. \quad \text{उत्तर}$$

6. $\tan \theta = 0$

हल :- $\tan \theta = 0$

$$\tan \theta = \tan 0^\circ$$

$$\theta = 0^\circ \quad \text{उत्तर}$$

$$7. 3\operatorname{cosec}^2\theta = 4$$

$$\text{हल :- } 3\operatorname{cosec}^2\theta = 4$$

$$\Rightarrow \operatorname{cosec}^2\theta = \frac{4}{3}$$

$$\Rightarrow \operatorname{cosec}\theta = \frac{2}{\sqrt{3}} = \operatorname{cosec}60^\circ$$

$$\therefore \theta = 60^\circ. \quad \text{उत्तर}$$

$$8. 2\cos^2\theta = \frac{1}{2}$$

$$\text{हल :- } 2\cos^2\theta = \frac{1}{2}$$

$$\Rightarrow \cos^2\theta = \frac{1}{4} = \left(\frac{1}{2}\right)^2 = \cos^2 60^\circ$$

$$\Rightarrow \cos\theta = \cos 60^\circ$$

$$\therefore \theta = 60^\circ \quad \text{उत्तर}$$

$$9. 4\sin^2\theta - 3 = 0$$

$$\text{हल :- } 4\sin^2\theta - 3 = 0$$

$$\Rightarrow 4\sin^2\theta = 3$$

$$\Rightarrow \sin^2\theta = \frac{3}{4}$$

$$\Rightarrow \sin^2\theta = \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= \sin^2 60^\circ$$

$$\Rightarrow \sin\theta = \sin 60^\circ$$

$$\therefore \theta = 60^\circ. \quad \text{उत्तर}$$

$$10. 4\sec^2\theta - 1 = 3$$

$$\text{हल :- } 4\sec^2\theta - 1 = 3$$

$$\Rightarrow \sec^2\theta = \frac{4}{4} = 1$$

$$= (1)^2 = \sec^2 0^\circ$$

$$\Rightarrow \sec\theta = \sec 0^\circ$$

$$\therefore \theta = 0^\circ \quad \text{उत्तर}$$

$$11. \cot^2 \theta = 3$$

$$\text{हल :-} \cot^2 \theta = 3$$

$$\Rightarrow \cot \theta = \sqrt{3}$$

$$\Rightarrow \cot 30^\circ$$

$$\therefore \theta = 30^\circ \quad \text{उत्तर}$$

aglasem