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FOR ISI ADMISSION TEST EXAM PREPARATION

ISI Admission Test 2016

Sample Paper · JRF
Physics, JRF QROR,
MS(QMBA),

Notes · Sample Papers · Previous Year Papers · Mock Tests

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M.Tech(QROR) MMA

Note: For each question there are four suggested answers of which only one is correct.

1. Let $\{f_n(x)\}$ be a sequence of polynomials defined inductively as

$$\begin{aligned} f_1(x) &= (x-2)^2 \\ f_{n+1}(x) &= (f_n(x)-2)^2, \quad n \geq 1. \end{aligned}$$

Let a_n and b_n respectively denote the constant term and the coefficient of x in $f_n(x)$. Then

- (A) $a_n = 4, b_n = -4^n$ (B) $a_n = 4, b_n = -4n^2$
 (C) $a_n = 4^{(n-1)!}, b_n = -4^n$ (D) $a_n = 4^{(n-1)!}, b_n = -4n^2$.
2. If a, b are positive real variables whose sum is a constant λ , then the minimum value of $\sqrt{(1+1/a)(1+1/b)}$ is

- (A) $\lambda - 1/\lambda$ (B) $\lambda + 2/\lambda$ (C) $\lambda + 1/\lambda$ (D) none of the above.

3. Let x be a positive real number. Then
- (A) $x^2 + \pi^2 + x^{2\pi} > x\pi + (\pi + x)x^\pi$
 (B) $x^\pi + \pi^x > x^{2\pi} + \pi^{2x}$
 (C) $\pi x + (\pi + x)x^\pi > x^2 + \pi^2 + x^{2\pi}$
 (D) none of the above.
4. Suppose in a competition 11 matches are to be played, each having one of 3 distinct outcomes as possibilities. The number of ways one can predict the outcomes of all 11 matches such that exactly 6 of the predictions turn out to be correct is
- (A) $\binom{11}{6} \times 2^5$ (B) $\binom{11}{6}$ (C) 3^6 (D) none of the above.
5. A set contains $2n+1$ elements. The number of subsets of the set which contain at most n elements is
- (A) 2^n (B) 2^{n+1} (C) 2^{n-1} (D) 2^{2n} .
6. A club with x members is organized into four committees such that
- (a) each member is in exactly two committees,
 (b) any two committees have exactly one member in common.
- Then x has
- (A) exactly two values both between 4 and 8
 (B) exactly one value and this lies between 4 and 8
 (C) exactly two values both between 8 and 16
 (D) exactly one value and this lies between 8 and 16.
7. Let X be the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Define the set $\mathcal{R}$ by
- $$\mathcal{R} = \{(x, y) \in X \times X : x \text{ and } y \text{ have the same remainder when divided by } 3\}.$$
- Then the number of elements in $\mathcal{R}$ is
- (A) 40 (B) 36 (C) 34 (D) 33.
8. Let A be a set of n elements. The number of ways, we can choose an ordered pair (B, C) , where B, C are disjoint subsets of A , equals
- (A) n^2 (B) n^3 (C) 2^n (D) 3^n .

9. Let $(1+x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$, n being a positive integer. The value of

$$\left(1 + \frac{C_0}{C_1}\right) \left(1 + \frac{C_1}{C_2}\right) \dots \left(1 + \frac{C_{n-1}}{C_n}\right)$$

is

- (A) $\left(\frac{n+1}{n+2}\right)^n$ (B) $\frac{n^n}{n!}$ (C) $\left(\frac{n}{n+1}\right)^n$ (D) $\frac{(n+1)^n}{n!}$.

10. The value of the infinite product

$$P = \frac{7}{9} \times \frac{26}{28} \times \frac{63}{65} \times \dots \times \frac{n^3 - 1}{n^3 + 1} \times \dots$$

is

- (A) 1 (B) $\frac{2}{3}$ (C) $\frac{7}{3}$ (D) none of the above.

11. The number of positive integers which are less than or equal to 1000 and are divisible by none of 17, 19 and 23 equals

- (A) 854 (B) 153 (C) 160 (D) none of the above.

12. Consider the polynomial $x^5 + ax^4 + bx^3 + cx^2 + dx + 4$ where a, b, c, d are real numbers. If $(1 + 2i)$ and $(3 - 2i)$ are two roots of this polynomial then the value of a is

- (A) $-524/65$ (B) $524/65$ (C) $-1/65$ (D) $1/65$.

13. The number of real roots of the equation

$$2 \cos \left(\frac{x^2 + x}{6} \right) = 2^x + 2^{-x}$$

is

- (A) 0 (B) 1 (C) 2 (D) infinitely many.

14. Consider the following system of equivalences of integers.

$$x \equiv 2 \pmod{15}$$

$$x \equiv 4 \pmod{21}.$$

The number of solutions in x , where $1 \leq x \leq 315$, to the above system of equivalences is

- (A) 0 (B) 1 (C) 2 (D) 3.

15. The number of real solutions of the equation $(9/10)^x = -3 + x - x^2$ is
- (A) 2 (B) 0 (C) 1 (D) none of the above.
16. If two real polynomials $f(x)$ and $g(x)$ of degrees $m (\geq 2)$ and $n (\geq 1)$ respectively, satisfy
- $$f(x^2 + 1) = f(x)g(x),$$
- for every $x \in \mathbb{R}$, then
- (A) f has exactly one real root x_0 such that $f'(x_0) \neq 0$
(B) f has exactly one real root x_0 such that $f'(x_0) = 0$
(C) f has m distinct real roots
(D) f has no real root.
17. Let $X = \frac{1}{1001} + \frac{1}{1002} + \frac{1}{1003} + \cdots + \frac{1}{3001}$. Then,
- (A) $X < 1$ (B) $X > 3/2$
(C) $1 < X < 3/2$ (D) none of the above holds.
18. The set of complex numbers z satisfying the equation
- $$(3 + 7i)z + (10 - 2i)\bar{z} + 100 = 0$$
- represents, in the complex plane,
- (A) a straight line
(B) a pair of intersecting straight lines
(C) a point
(D) a pair of distinct parallel straight lines.
19. The limit $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left| e^{\frac{2\pi i k}{n}} - e^{\frac{2\pi i (k-1)}{n}} \right|$ is
- (A) 2 (B) $2e$ (C) 2π (D) $2i$.
20. The limit $\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n^2} \right)^n$ equals
- (A) e^{-1} (B) $e^{-1/2}$ (C) e^{-2} (D) 1.

21. Let ω denote a complex fifth root of unity. Define

$$b_k = \sum_{j=0}^4 j\omega^{-kj},$$

for $0 \leq k \leq 4$. Then $\sum_{k=0}^4 b_k \omega^k$ is equal to

- (A) 5 (B) 5ω (C) $5(1 + \omega)$ (D) 0.

22. Let $a_n = \left(1 - \frac{1}{\sqrt{2}}\right) \cdots \left(1 - \frac{1}{\sqrt{n+1}}\right)$, $n \geq 1$. Then $\lim_{n \rightarrow \infty} a_n$

- (A) equals 1 (B) does not exist (C) equals $\frac{1}{\sqrt{\pi}}$ (D) equals 0.

23. Let X be a nonempty set and let $\mathcal{P}(X)$ denote the collection of all subsets of X . Define $f : X \times \mathcal{P}(X) \rightarrow \mathbb{R}$ by

$$f(x, A) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A. \end{cases}$$

Then $f(x, A \cup B)$ equals

- (A) $f(x, A) + f(x, B)$
 (B) $f(x, A) + f(x, B) - 1$
 (C) $f(x, A) + f(x, B) - f(x, A) \cdot f(x, B)$
 (D) $f(x, A) + |f(x, A) - f(x, B)|$

24. The series $\sum_{k=2}^{\infty} \frac{1}{k(k-1)}$ converges to

- (A) -1 (B) 1 (C) 0 (D) does not converge.

25. The limit $\lim_{x \rightarrow \infty} \left(\frac{3x-1}{3x+1}\right)^{4x}$ equals

- (A) 1 (B) 0 (C) $e^{-8/3}$ (D) $e^{4/9}$

26. $\lim_{n \rightarrow \infty} \frac{1}{n} \left(\frac{n}{n+1} + \frac{n}{n+2} + \cdots + \frac{n}{2n} \right)$ is equal to

- (A) ∞ (B) 0 (C) $\log_e 2$ (D) 1

27. Let $\cos^6 \theta = a_6 \cos 6\theta + a_5 \cos 5\theta + a_4 \cos 4\theta + a_3 \cos 3\theta + a_2 \cos 2\theta + a_1 \cos \theta + a_0$.
Then a_0 is

(A) 0 (B) $1/32$. (C) $15/32$. (D) $10/32$.

28. In a triangle ABC , AD is the median. If length of AB is 7, length of AC is 15 and length of BC is 10 then length of AD equals

(A) $\sqrt{125}$ (B) $69/5$ (C) $\sqrt{112}$ (D) $\sqrt{864}/5$.

29. The set $\{x : \left|x + \frac{1}{x}\right| > 6\}$ equals the set

(A) $(0, 3 - 2\sqrt{2}) \cup (3 + 2\sqrt{2}, \infty)$
 (B) $(-\infty, -3 - 2\sqrt{2}) \cup (-3 + 2\sqrt{2}, \infty)$
 (C) $(-\infty, 3 - 2\sqrt{2}) \cup (3 + 2\sqrt{2}, \infty)$
 (D) $(-\infty, -3 - 2\sqrt{2}) \cup (-3 + 2\sqrt{2}, 3 - 2\sqrt{2}) \cup (3 + 2\sqrt{2}, \infty)$

30. Suppose that a function f defined on $\mathbb{R}^2$ satisfies the following conditions:

$$\begin{aligned} f(x+t, y) &= f(x, y) + ty, \\ f(x, t+y) &= f(x, y) + tx \text{ and} \\ f(0, 0) &= K, \text{ a constant.} \end{aligned}$$

Then for all $x, y \in \mathbb{R}$, $f(x, y)$ is equal to

(A) $K(x+y)$. (B) $K - xy$. (C) $K + xy$. (D) none of the above.

31. Consider the sets defined by the real solutions of the inequalities

$$A = \{(x, y) : x^2 + y^4 \leq 1\} \quad B = \{(x, y) : x^4 + y^6 \leq 1\}.$$

Then

(A) $B \subseteq A$
 (B) $A \subseteq B$
 (C) Each of the sets $A - B$, $B - A$ and $A \cap B$ is non-empty
 (D) none of the above.

32. If a square of side a and an equilateral triangle of side b are inscribed in a circle then a/b equals

(A) $\sqrt{2/3}$ (B) $\sqrt{3/2}$ (C) $3/\sqrt{2}$ (D) $\sqrt{2}/3$.

33. If $f(x)$ is a real valued function such that

$$2f(x) + 3f(-x) = 15 - 4x,$$

for every $x \in \mathbb{R}$, then $f(2)$ is

- (A) -15 (B) 22 (C) 11 (D) 0 .

34. If $f(x) = \frac{\sqrt{3} \sin x}{2 + \cos x}$, then the range of $f(x)$ is

- (A) the interval $[-1, \sqrt{3}/2]$ (B) the interval $[-\sqrt{3}/2, 1]$
(C) the interval $[-1, 1]$ (D) none of the above.

35. If $f(x) = x^2$ and $g(x) = x \sin x + \cos x$ then

- (A) f and g agree at no points
(B) f and g agree at exactly one point
(C) f and g agree at exactly two points
(D) f and g agree at more than two points.

36. For non-negative integers m, n define a function as follows

$$f(m, n) = \begin{cases} n + 1 & \text{if } m = 0 \\ f(m - 1, 1) & \text{if } m \neq 0, n = 0 \\ f(m - 1, f(m, n - 1)) & \text{if } m \neq 0, n \neq 0 \end{cases}$$

Then the value of $f(1, 1)$ is

- (A) 4 (B) 3 (C) 2 (D) 1 .

37. Let a be a nonzero real number. Define

$$f(x) = \begin{vmatrix} x & a & a & a \\ a & x & a & a \\ a & a & x & a \\ a & a & a & x \end{vmatrix}$$

for $x \in \mathbb{R}$. Then, the number of distinct real roots of $f(x) = 0$ is

- (A) 1 (B) 2 (C) 3 (D) 4 .

38. A real 2×2 matrix M such that

$$M^2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 - \varepsilon \end{pmatrix}$$

- (A) exists for all $\varepsilon > 0$
- (B) does not exist for any $\varepsilon > 0$
- (C) exists for some $\varepsilon > 0$
- (D) none of the above is true

39. The eigenvalues of the matrix $X = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$ are

- (A) 1, 1, 4
- (B) 1, 4, 4
- (C) 0, 1, 4
- (D) 0, 4, 4.

40. Let $x_1, x_2, x_3, x_4, y_1, y_2, y_3$ and y_4 be fixed real numbers, not all of them equal to zero. Define a 4×4 matrix $\mathbf{A}$ by

$$\mathbf{A} = \begin{pmatrix} x_1^2 + y_1^2 & x_1x_2 + y_1y_2 & x_1x_3 + y_1y_3 & x_1x_4 + y_1y_4 \\ x_2x_1 + y_2y_1 & x_2^2 + y_2^2 & x_2x_3 + y_2y_3 & x_2x_4 + y_2y_4 \\ x_3x_1 + y_3y_1 & x_3x_2 + y_3y_2 & x_3^2 + y_3^2 & x_3x_4 + y_3y_4 \\ x_4x_1 + y_4y_1 & x_4x_2 + y_4y_2 & x_4x_3 + y_4y_3 & x_4^2 + y_4^2 \end{pmatrix}.$$

Then $\text{rank}(\mathbf{A})$ equals

- (A) 1 or 2.
- (B) 0.
- (C) 4.
- (D) 2 or 3.

41. Let k and n be integers greater than 1. Then $(kn)!$ is not necessarily divisible by

- (A) $(n!)^k$.
- (B) $(k!)^n$.
- (C) $n!.k!$.
- (D) 2^{kn} .

42. Let $\lambda_1, \lambda_2, \lambda_3$ denote the eigenvalues of the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos t & \sin t \\ 0 & -\sin t & \cos t \end{pmatrix}.$$

If $\lambda_1 + \lambda_2 + \lambda_3 = \sqrt{2} + 1$, then the set of possible values of t , $-\pi \leq t < \pi$, is

- (A) Empty set (B) $\left\{\frac{\pi}{4}\right\}$ (C) $\left\{-\frac{\pi}{4}, \frac{\pi}{4}\right\}$ (D) $\left\{-\frac{\pi}{3}, \frac{\pi}{3}\right\}$.

43. The values of η for which the following system of equations

$$\begin{aligned} x + y + z &= 1 \\ x + 2y + 4z &= \eta \\ x + 4y + 10z &= \eta^2 \end{aligned}$$

has a solution are

- (A) $\eta = 1, -2$ (B) $\eta = -1, -2$ (C) $\eta = 3, -3$ (D) $\eta = 1, 2$.

44. Let P_1, P_2 and P_3 denote, respectively, the planes defined by

$$\begin{aligned} a_1x + b_1y + c_1z &= \alpha_1 \\ a_2x + b_2y + c_2z &= \alpha_2 \\ a_3x + b_3y + c_3z &= \alpha_3. \end{aligned}$$

It is given that P_1, P_2 and P_3 intersect exactly at one point when $\alpha_1 = \alpha_2 = \alpha_3 = 1$. If now $\alpha_1 = 2, \alpha_2 = 3$ and $\alpha_3 = 4$ then the planes

- (A) do not have any common point of intersection
(B) intersect at a unique point
(C) intersect along a straight line
(D) intersect along a plane.

45. Angles between any pair of 4 main diagonals of a cube are

- (A) $\cos^{-1} 1/\sqrt{3}, \pi - \cos^{-1} 1/\sqrt{3}$ (B) $\cos^{-1} 1/3, \pi - \cos^{-1} 1/3$
(C) $\pi/2$ (D) none of the above.

46. If the tangent at the point P with co-ordinates (h, k) on the curve $y^2 = 2x^3$ is perpendicular to the straight line $4x = 3y$, then

- (A) $(h, k) = (0, 0)$
 (B) $(h, k) = (1/8, -1/16)$
 (C) $(h, k) = (0, 0)$ or $(h, k) = (1/8, -1/16)$
 (D) no such point (h, k) exists.

47. Consider the family $\mathcal{F}$ of curves in the plane given by $x = cy^2$, where c is a real parameter. Let $\mathcal{G}$ be the family of curves having the following property: every member of $\mathcal{G}$ intersects each member of $\mathcal{F}$ orthogonally. Then $\mathcal{G}$ is given by

- (A) $xy = k$ (B) $x^2 + y^2 = k^2$
 (C) $y^2 + 2x^2 = k^2$ (D) $x^2 - y^2 + 2yk = k^2$

48. Suppose the circle with equation $x^2 + y^2 + 2fx + 2gy + c = 0$ cuts the parabola $y^2 = 4ax$, ($a > 0$) at four distinct points. If d denotes the sum of ordinates of these four points, then the set of possible values of d is

- (A) $\{0\}$ (B) $(-4a, 4a)$ (C) $(-a, a)$ (D) $(-\infty, \infty)$.

49. The polar equation $r = a \cos \theta$ represents

- (A) a spiral (B) a parabola (C) a circle (D) none of the above.

50. Let

$$\begin{aligned} V_1 &= \frac{7^2 + 8^2 + 15^2 + 23^2}{4} - \left(\frac{7 + 8 + 15 + 23}{4} \right)^2, \\ V_2 &= \frac{6^2 + 8^2 + 15^2 + 24^2}{4} - \left(\frac{6 + 8 + 15 + 24}{4} \right)^2, \\ V_3 &= \frac{5^2 + 8^2 + 15^2 + 25^2}{4} - \left(\frac{5 + 8 + 15 + 25}{4} \right)^2. \end{aligned}$$

Then

- (A) $V_3 < V_2 < V_1$ (B) $V_3 < V_1 < V_2$
 (C) $V_1 < V_2 < V_3$ (D) $V_2 < V_3 < V_1$.

51. A permutation of $1, 2, \dots, n$ is chosen at random. Then the probability that the numbers 1 and 2 appear as neighbour equals

(A) $\frac{1}{n}$ (B) $\frac{2}{n}$ (C) $\frac{1}{n-1}$ (D) $\frac{1}{n-2}$.

52. Two coins are tossed independently where $P(\text{head occurs when coin } i \text{ is tossed}) = p_i, i = 1, 2$. Given that at least one head has occurred, the probability that coins produced different outcomes is

(A) $\frac{2p_1p_2}{p_1 + p_2 - 2p_1p_2}$ (B) $\frac{p_1 + p_2 - 2p_1p_2}{p_1 + p_2 - p_1p_2}$ (C) $\frac{2}{3}$ (D) none of the above.

53. The number of cars (X) arriving at a service station per day follows a Poisson distribution with mean 4. The service station can provide service to a maximum of 4 cars per day. Then the expected number of cars that do not get service per day equals

(A) 4 (B) 0 (C) $\sum_{i=0}^{\infty} iP(X = i + 4)$ (D) $\sum_{i=4}^{\infty} iP(X = i - 4)$.

54. If $0 < x < 1$, then the sum of the infinite series $\frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{3}{4}x^4 + \dots$ is

(A) $\log \frac{1+x}{1-x}$ (B) $\frac{x}{1-x} + \log(1+x)$
 (C) $\frac{1}{1-x} + \log(1-x)$ (D) $\frac{x}{1-x} + \log(1-x)$.

55. Let $\{a_n\}$ be a sequence of real numbers. Then $\lim_{n \rightarrow \infty} a_n$ exists if and only if

- (A) $\lim_{n \rightarrow \infty} a_{2n}$ and $\lim_{n \rightarrow \infty} a_{2n+2}$ exists
 (B) $\lim_{n \rightarrow \infty} a_{2n}$ and $\lim_{n \rightarrow \infty} a_{2n+1}$ exist
 (C) $\lim_{n \rightarrow \infty} a_{2n}$, $\lim_{n \rightarrow \infty} a_{2n+1}$ and $\lim_{n \rightarrow \infty} a_{3n}$ exist
 (D) none of the above.

56. Let $\{a_n\}$ be a sequence of non-negative real numbers such that the series $\sum_{n=1}^{\infty} a_n$ is convergent. If p is a real number such that the series $\sum \frac{\sqrt{a_n}}{n^p}$ diverges, then

- (A) p must be strictly less than $\frac{1}{2}$
 (B) p must be strictly less than or equal to $\frac{1}{2}$
 (C) p must be strictly less than or equal to 1 but can be greater than $\frac{1}{2}$
 (D) p must be strictly less than 1 but can be greater than or equal to $\frac{1}{2}$.

57. Suppose $a > 0$. Consider the sequence

$$a_n = n\{\sqrt[n]{ea} - \sqrt[n]{a}\}, \quad n \geq 1.$$

Then

- (A) $\lim_{n \rightarrow \infty} a_n$ does not exist
 (B) $\lim_{n \rightarrow \infty} a_n = e$
 (C) $\lim_{n \rightarrow \infty} a_n = 0$
 (D) none of the above.

58. Let $\{a_n\}$, $n \geq 1$, be a sequence of real numbers satisfying $|a_n| \leq 1$ for all n . Define

$$A_n = \frac{1}{n}(a_1 + a_2 + \cdots + a_n),$$

for $n \geq 1$. Then $\lim_{n \rightarrow \infty} \sqrt{n}(A_{n+1} - A_n)$ is equal to

- (A) 0 (B) -1 (C) 1 (D) none of these.

59. In the Taylor expansion of the function $f(x) = e^{x/2}$ about $x = 3$, the coefficient of $(x - 3)^5$ is

- (A) $e^{3/2} \frac{1}{5!}$ (B) $e^{3/2} \frac{1}{2^5 5!}$ (C) $e^{-3/2} \frac{1}{2^5 5!}$ (D) none of the above.

60. Let σ be the permutation:

$$\begin{array}{cccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & \\ 3 & 5 & 6 & 2 & 4 & 9 & 8 & 7 & 1, & \end{array}$$

I be the identity permutation and m be the order of σ i.e.

$$m = \min\{\text{positive integers } n : \sigma^n = I\}.$$

Then m is

- (A) 8 (B) 12 (C) 360 (D) 2520.

61. Let

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}.$$

Then

- (A) there exists a matrix C such that $A = BC = CB$
 (B) there is no matrix C such that $A = BC$
 (C) there exists a matrix C such that $A = BC$, but $A \neq CB$
 (D) there is no matrix C such that $A = CB$.

62. If the matrix

$$A = \begin{bmatrix} a & 1 \\ 2 & 3 \end{bmatrix}$$

has 1 as an eigenvalue, then $\text{trace}(A)$ is

- (A) 4 (B) 5 (C) 6 (D) 7.

63. Let $\theta = 2\pi/67$. Now consider the matrix

$$A = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}.$$

Then the matrix A^{2010} is

- (A) $\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$ (B) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
 (C) $\begin{pmatrix} \cos^{30} \theta & \sin^{30} \theta \\ -\sin^{30} \theta & \cos^{30} \theta \end{pmatrix}$ (D) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$

64. Let the position of a particle in three dimensional space at time t be $(t, \cos t, \sin t)$. Then the length of the path traversed by the particle between the times $t = 0$ and $t = 2\pi$ is

(A) 2π . (B) $2\sqrt{2}\pi$. (C) $\sqrt{2}\pi$ (D) none of the above.

65. Let n be a positive real number and p be a positive integer. Which of the following inequalities is true?

(A) $n^p > \frac{(n+1)^{p+1} - n^{p+1}}{p+1}$ (B) $n^p < \frac{(n+1)^{p+1} - n^{p+1}}{p+1}$
 (C) $(n+1)^p < \frac{(n+1)^{p+1} - n^{p+1}}{p+1}$ (D) none of the above.

66. The smallest positive number K for which the inequality

$$|\sin^2 x - \sin^2 y| \leq K|x - y|$$

holds for all x and y is

(A) 2 (B) 1 (C) $\frac{\pi}{2}$ (D) there is no smallest positive value of K ; any $K > 0$ will make the inequality hold.

67. Given two real numbers $a < b$, let

$$d(x, [a, b]) = \min\{|x - y| : a \leq y \leq b\} \quad \text{for } -\infty < x < \infty.$$

Then the function

$$f(x) = \frac{d(x, [0, 1])}{d(x, [0, 1]) + d(x, [2, 3])}$$

satisfies

(A) $0 \leq f(x) < \frac{1}{2}$ for every x
 (B) $0 < f(x) < 1$ for every x
 (C) $f(x) = 0$ if $2 \leq x \leq 3$ and $f(x) = 1$ if $0 \leq x \leq 1$
 (D) $f(x) = 0$ if $0 \leq x \leq 1$ and $f(x) = 1$ if $2 \leq x \leq 3$.

68. Let

$$f(x, y) = \begin{cases} e^{-1/(x^2+y^2)} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$$

Then $f(x, y)$ is

(A) not continuous at $(0, 0)$
 (B) continuous at $(0, 0)$ but does not have first order partial derivatives

- (C) continuous at $(0, 0)$ and has first order partial derivatives, but not differentiable at $(0, 0)$
 (D) differentiable at $(0, 0)$

69. Consider the function

$$f(x) = \begin{cases} \int_0^x \{5 + |1 - y|\} dy & \text{if } x > 2 \\ 5x + 2 & \text{if } x \leq 2 \end{cases}$$

Then

- (A) f is not continuous at $x = 2$
 (B) f is continuous and differentiable everywhere
 (C) f is continuous everywhere but not differentiable at $x = 1$
 (D) f is continuous everywhere but not differentiable at $x = 2$.

70. Let $w = \log(u^2 + v^2)$ where $u = e^{(x^2+y)}$ and $v = e^{(x+y^2)}$. Then

$$\left. \frac{\partial w}{\partial x} \right|_{x=0, y=0}$$

is

- (A) 0 (B) 1 (C) 2 (D) 4

71. Let

$$f(x, y) = \begin{cases} 1, & \text{if } xy = 0, \\ xy, & \text{if } xy \neq 0. \end{cases}$$

Then

- (A) f is continuous at $(0, 0)$ and $\frac{\partial f}{\partial x}(0, 0)$ exists
 (B) f is not continuous at $(0, 0)$ and $\frac{\partial f}{\partial x}(0, 0)$ exists
 (C) f is continuous at $(0, 0)$ and $\frac{\partial f}{\partial x}(0, 0)$ does not exist
 (D) f is not continuous at $(0, 0)$ and $\frac{\partial f}{\partial x}(0, 0)$ does not exist.

72. The map $f(x) = a_0 \cos |x| + a_1 \sin |x| + a_2 |x|^3$ is differentiable at $x = 0$ if and only if

- (A) $a_1 = 0$ and $a_2 = 0$ (B) $a_0 = 0$ and $a_1 = 0$
 (C) $a_1 = 0$ (D) a_0, a_1, a_2 can take any real value.

73. $f(x)$ is a differentiable function on the real line such that $\lim_{x \rightarrow \infty} f(x) = 1$ and $\lim_{x \rightarrow \infty} f'(x) = \alpha$. Then

- (A) α must be 0 (B) α need not be 0, but $|\alpha| < 1$
 (C) $\alpha > 1$ (D) $\alpha < -1$.

74. Let f and g be two differentiable functions such that $f'(x) \leq g'(x)$ for all $x < 1$ and $f'(x) \geq g'(x)$ for all $x > 1$. Then

- (A) if $f(1) \geq g(1)$, then $f(x) \geq g(x)$ for all x
 (B) if $f(1) \leq g(1)$, then $f(x) \leq g(x)$ for all x
 (C) $f(1) \leq g(1)$
 (D) $f(1) \geq g(1)$.

75. The length of the curve $x = t^3$, $y = 3t^2$ from $t = 0$ to $t = 4$ is

- (A) $5\sqrt{5} + 1$ (B) $8(5\sqrt{5} + 1)$
 (C) $5\sqrt{5} - 1$ (D) $8(5\sqrt{5} - 1)$.

76. Given that $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$, the value of

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2+xy+y^2)} dx dy$$

is

- (A) $\sqrt{\pi/3}$ (B) $\pi/\sqrt{3}$ (C) $\sqrt{2\pi/3}$ (D) $2\pi/\sqrt{3}$.

77. Let R be the triangle in the xy -plane bounded by the x -axis, the line $y = x$, and the line $x = 1$. The value of the double integral

$$\int \int_R \frac{\sin x}{x} dx dy$$

is

- (A) $1 - \cos 1$ (B) $\cos 1$ (C) $\frac{\pi}{2}$ (D) π .

78. The value of

$$\lim_{n \rightarrow \infty} \left[(n+1) \int_0^1 x^n \ln(1+x) dx \right]$$

is

- (A) 0 (B) $\ln 2$ (C) $\ln 3$ (D) ∞ .

79. Let $g(x, y) = \max\{12 - x, 8 - y\}$. Then the minimum value of $g(x, y)$ as (x, y) varies over the line $x + y = 10$ is

- (A) 5 (B) 7 (C) 1 (D) 3.

80. Let $0 < \alpha < \beta < 1$. Then

$$\sum_{k=1}^{\infty} \int_{1/(k+\beta)}^{1/(k+\alpha)} \frac{1}{1+x} dx$$

is equal to

- (A) $\log_e \frac{\beta}{\alpha}$ (B) $\log_e \frac{1+\beta}{1+\alpha}$ (C) $\log_e \frac{1+\alpha}{1+\beta}$ (D) ∞ .

81. If f is continuous in $[0, 1]$ then

$$\lim_{n \rightarrow \infty} \sum_{j=0}^{[n/2]} \frac{1}{n} f\left(\frac{j}{n}\right)$$

(where $[y]$ is the largest integer less than or equal to y)

- (A) does not exist
 (B) exists and is equal to $\frac{1}{2} \int_0^1 f(x) dx$
 (C) exists and is equal to $\int_0^1 f(x) dx$
 (D) exists and is equal to $\int_0^{1/2} f(x) dx$.

82. The volume of the solid, generated by revolving about the horizontal line $y = 2$ the region bounded by $y^2 \leq 2x$, $x \leq 8$ and $y \geq 2$, is

- (A) $2\sqrt{2}\pi$ (B) $28\pi/3$ (C) 84π (D) none of the above.

83. If α, β are complex numbers then the maximum value of $\frac{\alpha\bar{\beta} + \bar{\alpha}\beta}{|\alpha\beta|}$ is

- (A) 2
 (B) 1
 (C) the expression may not always be a real number and hence maximum does not make sense
 (D) none of the above.

84. For positive real numbers $a_1, a_2, \dots, a_{100}$, let

$$p = \sum_{i=1}^{100} a_i \quad \text{and} \quad q = \sum_{1 \leq i < j \leq 100} a_i a_j.$$

Then

- (A) $q = \frac{p^2}{2}$ (B) $q^2 \geq \frac{p^2}{2}$ (C) $q < \frac{p^2}{2}$ (D) none of the above.

85. The differential equation of all the ellipses centred at the origin is

- (A) $y^2 + x(y')^2 - yy' = 0$ (B) $xyy'' + x(y')^2 - yy' = 0$
 (C) $yy'' + x(y')^2 - xy' = 0$ (D) none of these.

86. The coordinates of a moving point P satisfy the equations

$$\frac{dx}{dt} = \tan x, \quad \frac{dy}{dt} = -\sin^2 x, \quad t \geq 0.$$

If the curve passes through the point $(\pi/2, 0)$ when $t = 0$, then the equation of the curve in rectangular co-ordinates is

- (A) $y = 1/2 \cos^2 x$ (B) $y = \sin 2x$
 (C) $y = \cos 2x + 1$ (D) $y = \sin^2 x - 1$.

87. If $x(t)$ is a solution of

$$(1 - t^2) dx - tx dt = dt$$

and $x(0) = 1$, then $x(\frac{1}{2})$ is equal to

- (A) $\frac{2}{\sqrt{3}}(\frac{\pi}{6} + 1)$ (B) $\frac{2}{\sqrt{3}}(\frac{\pi}{6} - 1)$ (C) $\frac{\pi}{3\sqrt{3}}$ (D) $\frac{\pi}{\sqrt{3}}$.

88. Let $f(x)$ be a given differentiable function. Consider the following differential equation in y

$$f(x) \frac{dy}{dx} = yf'(x) - y^2.$$

The general solution of this equation is given by

- (A) $y = -\frac{x+c}{f(x)}$ (B) $y^2 = \frac{f(x)}{x+c}$
 (C) $y = \frac{f(x)}{x+c}$ (D) $y = \frac{[f(x)]^2}{x+c}$.

89. Let $y(x)$ be a non-trivial solution of the second order linear differential equation

$$\frac{d^2 y}{dx^2} + 2c \frac{dy}{dx} + ky = 0,$$

where $c < 0$, $k > 0$ and $c^2 > k$. Then

- (A) $|y(x)| \rightarrow \infty$ as $x \rightarrow \infty$
 (B) $|y(x)| \rightarrow 0$ as $x \rightarrow \infty$
 (C) $\lim_{x \rightarrow \pm\infty} |y(x)|$ exists and is finite
 (D) none of the above is true.

90. The differential equation of the system of circles touching the y -axis at the origin is

- (A) $x^2 + y^2 - 2xy \frac{dy}{dx} = 0$ (B) $x^2 + y^2 + 2xy \frac{dy}{dx} = 0$
 (C) $x^2 - y^2 - 2xy \frac{dy}{dx} = 0$ (D) $x^2 - y^2 + 2xy \frac{dy}{dx} = 0$.

91. Suppose a solution of the differential equation

$$(xy^3 + x^2y^7) \frac{dy}{dx} = 1,$$

satisfies the initial condition $y(1/4) = 1$. Then the value of $\frac{dy}{dx}$ when $y = -1$ is

- (A) $\frac{4}{3}$ (B) $-\frac{4}{3}$ (C) $\frac{16}{5}$ (D) $-\frac{16}{5}$.

92. Consider the group

$$G = \left\{ \begin{pmatrix} a & b \\ 0 & a^{-1} \end{pmatrix} : a, b \in \mathbb{R}, a > 0 \right\}$$

with usual matrix multiplication. Let

$$N = \left\{ \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} : b \in \mathbb{R} \right\}.$$

Then,

- (A) N is not a subgroup of G
 (B) N is a subgroup of G but not a normal subgroup
 (C) N is a normal subgroup and the quotient group G/N is of finite order
 (D) N is a normal subgroup and the quotient group is isomorphic to $\mathbb{R}^+$ (the group of positive reals with multiplication).

93. Let G be a group with identity element e . If x and y are elements in G satisfying $x^5y^3 = x^8y^5 = e$, then which of the following conditions is true?

- (A) $x = e, y = e$
 (B) $x \neq e, y = e$
 (C) $x = e, y \neq e$
 (D) $x \neq e, y \neq e$

94. Let G be the group $\{\pm 1, \pm i\}$ with multiplication of complex numbers as composition. Let H be the quotient group $\mathbb{Z}/4\mathbb{Z}$. Then the number of nontrivial group homomorphisms from H to G is
- (A) 4 (B) 1 (C) 2 (D) 3.