[This question paper contains 12 printed pages.]

4259 *Your Roll No.* ....

B.A. (Prog.) / III **G-I**

Paper Code : C-155

MATHEMATICS – Paper III

(Selected Topics in Mathematics)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :– The maximum marks printed on the question paper are applicable for the students of the regular colleges (Category 'A'). These marks will, however, be scaled up proportionately in respect of the students of SOL at the time of posting of awards for compilation of result.

*Attempt **six** question in all selecting **two** parts from each question.*

***Unit I** and **Unit II** are compulsory and contain **four** questions.*

*In **Unit III** choose any of the options and attempt **two** questions from the same.*

Marks are indicated against each questions.

*Use of **scientific calculator** is allowed.*

P.T.O.

Unit I

(Real Analysis)

1. (a) State the order completeness property. Prove that the set Q of rational numbers is not order complete. (6)

   (b) Define limit point of a set. Determine the limit points of the following sets:

   (i) The set Z of integers

   (ii) The set N of natural numbers. (6)

   (c) Prove that $f(x) = \sin x$ is uniformly continuous on $[0,\infty]$ (6)

2. (a) Prove that the sequence $< a_n >$ defined by

   $$a_1 = \sqrt{7}, a_{n+1} = \sqrt{7 + a_n}, \quad n \geq 1$$

   converges to the positive root of $x^2 - x - 7 = 0$ (7)

   (b) Test the convergence of the following infinite series:

   (i) $\sum \left(\frac{1}{n}\right)^{1/n}$

   (ii) $\sum \frac{n!}{n^n}$ (7)

(c) State and prove Cauchy's n^{th} root test for infinite series. (7)

3. (a) Test for uniform convergence of the sequence of functions $\{f_n\}$, where

$$f_n(x) = nxe^{-nx^2} \qquad x \in [0, 1]$$

State clearly any result you are using. (6)

(b) Prove that

$$\sqrt{\pi}\Gamma(2m) = 2^{2m-1}\Gamma(m)\Gamma\left(m+\tfrac{1}{2}\right) \qquad (6)$$

(c) Prove that a bounded function f is integrable on $[a, b]$ if for every $\varepsilon > 0$ there exists a partition P of $[a, b]$ such that

$U(P, f) - L(P, f) < \varepsilon$ (6)

Unit II

(Computer programming)

4. (a) Write a program to find roots of a quadratic equation. (6.5)

(b) Write the general form of the for loop. Explain how it works, with an example. (6.5)

(c) Find the invalid conditional statement from the following. Explain why they are invalid:

(i) if a>=b; x=y: else x=z;

(ii) if (x=y) i=1; else if(x!=y) i=2;

(iii) if (z>=a-+c) {x=x+1; y=y+2;} (6.5)

Unit III (1)

(Numerical Analysis)

5. (a) Find the root of the equation $f(x) = x^3 - 3x + 5 = 0$ by the Newton-Raphson Method corrected up to three decimal places. (6)

(b) Compare the Bisection method with Newton Raphson method for solving an equation. Also do the compare their order of convergence, mentioning the advantages of one method over the other. (6)

(c) Calculates $\int_1^2 \frac{dx}{x}$ using Simpson's one-third rule with 10 sub-intervals. (6)

6. (a) Use Gauss-Jordan to solve

$$\begin{pmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \\ 4 \end{pmatrix}$$ (6)

(b) For the following system of equations:

$20x+y-2z = 17, \quad 3x+20y-z = -18, \quad 2x-3y+20z = 25.$

Starting with X=(0, 0, 0), using Gauss Jacobi method, find the solution after performing three iterations. (6)

(c) Find the unique interpolating polynomial P (x) of degree 2 or less which interpolates f(x) at the points x = 1, 3, 4 such that

f (1) = 1, f (3) = 27, f (4) = 64 by Newton's divided difference formula. Hence evaluate P (1.5). (6)

Unit III (2)

(Discrete Mathematics)

5. (a) Define the following:

(i) The directed multigraph,

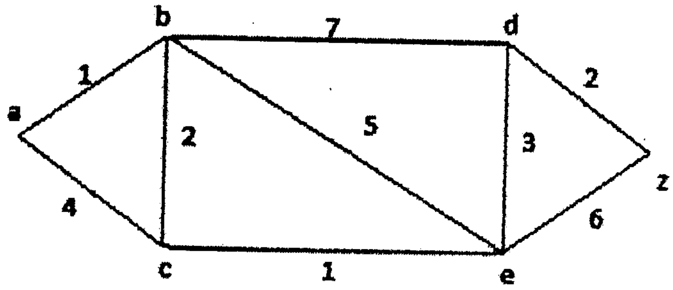
(ii) A path in a graph,

(iii) A connected graph. (6)

(b) For any planar connected graph, prove that

$n - e + r = 2$ where n, e and r are the number of vertices, edges and regions of the graph respectively. (6)

(c) Obtain a shortest path from the vertex a to vertex z in the weighted graph.



(6)

6. (a) Write the truth tables for $p \to q$ and $p \leftrightarrow q$ and show that:

$$p \leftrightarrow q = (p \to q) \wedge (q \to p).$$ (6)

(b) Show that the following statement is a tautology:

$$(A \to B) \to (\bar{A} \to B) \to B$$

Where $\bar{A}$ denotes the negation of A. (6)

(c) Define and illustrate the following:

(i) amaxterm

(ii) conjunctive normal form

(iii) disjunctive normal form

Write $(x_1 \wedge x_2) \vee (x_1 \wedge x_3) \vee (\bar{x}_2 \wedge x_3)$ in disjunctive normal form. (6)

Unit III (3)

Mathematical Statistics)

5. (a) Show that the standard deviation is independent of change of origin but not of scale. (6)

(b) The contents of urns I, II and III are as follows:

1 white, 2 red and 3 black balls.

2 white, 3 red and 1 black balls, and

3 white, 1 red and 2 black balls.

One urn is chosen at random and two balls drawn. They happen to be white and red. What is the probability that they come from urns I, II and III? (6)

(c) Two variables are correlated by the equation $aX + bY + c = 0$. Show that the Correlation coefficient between them is +1 or −1, according as a and b have unlike and like signs. (6)

6. (a) Determine the binomial distribution, when

$$\beta_1 = \frac{1}{5}, \beta_2 = \frac{89}{30}.$$ (6)

(b) In a normal distribution 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. Given that if

$f(t) = \frac{1}{\sqrt{2\pi}}\int_0^t \exp\left(-\frac{x^2}{2}\right) dx$, Then $f(0.496) = 0.9$ and $f(1.405) = 0.42$. (6)

(c) For two variables X and Y, the two regression lines are

3X+2Y=26 and 6X+Y=31.

Calculate

(i) the mean values of X and Y

(ii) the correlation coefficient between X and Y. (6)

Unit III (4)

(Mechanics)

5. (a) Three forces P,Q,R in one plane act on a particle,the angles between Q and R,R and P,P and Q being α, β, and γ respectively. Show that their resultant is

$$\sqrt{P^2 + Q^2 + R^2 - 2QRcos\alpha - 2RP\cos\beta - 2PQ\cos\gamma} \qquad (6)$$

(b) A particle of weight W rest on a rough horizontal plane. If the angle of friction be, Prove that the least force which will just make it move along the plane is $P = W\ sin\mu$. (6)

(c) Find the mass centre of a wire bent into the form of an isosceles right-angled triangle. (6)

6. (a) The speed 'v' of a particle along the x-axis is given by the relation

$v^2 = p^2(8bx - x^2 - 12b^2)$. Show that the motion is simple harmonic motion with centre at $x = 4b$, and amplitude is $2b$. Find also the time from $x = 5b$ to $x = 6b$. (6)

(b) Derive the expression $\frac{d^2}{d\theta^2} + u = \frac{p}{h^2u^2}$ for motion of a particle describing central orbit under an attraction 'p' per unit mass where $\mathrm{p} = \frac{1}{r}, \theta = hu^2$ (6)

(c) A projectile just goes over a wall of height 'h' metres and at a distance 'd' metres apart from the point of projection, and later it hits a mark at a height 'h' metres and distance '$2d$' metres. Show that the velocity of projection 'v' is given by

$$\frac{4v^2}{g} = \frac{4d^2 + 9h^2}{h}$$ (6)

6. (a) A particle is performing a S.H.M. of period T about a centre 'O' and it passes through a point P, where OP = b with velocity v in the direction OP. Prove that the time which elapses before its return to P is

$$\left(\frac{T}{\pi}\right)\tan^{-1}\left(\frac{vT}{2\pi b}\right)$$ (6)

(b) A particle moves under the influence of a centre which attracts with a force:

$$\left(\frac{b}{r^2}+\frac{c}{r^4}\right)$$

'b' and 'c' being positive constants and 'r' the distance from the centre. The particle moves in a circular orbit of radius 'a'. Prove that the motion is stable if and only if, $a^2b > c$. (6)

(c) If v_1, and v_2, be the velocities at the ends of a focal chord of a projectile's path and 'u', the horizontal component of velocity, show that

$$\frac{1}{v_1^2}+\frac{1}{v_2^2}=\frac{1}{u^2}$$ (6)

Unit III (5)

(Theory of Games)

5. (a) Solve graphically the following LPP

Maximize $Z = 8x_2 - 2x_1$

Subject to: $x_1 - x_2 \geq 0$, $-x_1 + 5x_2 \geq 4$, $x_1, x_2 \geq 0$ (6)

(b) Use Big-M method to solve following problem:

Minimize $Z = 3x_1 + x_2$

Subject to:

$$2x_1 + x_2 = 3$$

$3x_1 + 3x_2 \geq 6$, $x_1 + x_2 \leq 3$

x_1 , $x_2 \geq 0$

(c) Find the dual of the LPP:

Maximize : $Z = x_1 - x_2 + 4x_3 + 3x_4$

subject to:

$x_1 + x_2 \geq -2$

$x_1 - 3x_2 - x_3 \leq 6$

$x_1 + x_3 - 3x_4 = -1$

x_1, $x_2 \geq 0$, x_2, x_3 are unrestricted. (6)

6. (a) Explain the max-Min and Min-Max principle used in game theory. Determine the saddle point of a game whose pay-off matrix is:

$$\begin{array}{cc} & \text{Player B} \\ & \begin{array}{cc} B_1 & B_2 \end{array} \\ \text{Player } A \begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} & \begin{bmatrix} -1 & 6 \\ 2 & 4 \\ -2 & 6 \end{bmatrix} \end{array}$$ (6)

(b) Solve graphically the rectangular game whose pay-off matrix is:

$$\begin{bmatrix} 0 & 4 & -8 & -5 & 1 \\ 1 & 5 & 8 & -4 & 0 \end{bmatrix}$$ (6)

(c) Transform the matrix game:

$$\begin{bmatrix} 1 & -1 & 3 \\ 3 & 5 & -3 \\ 6 & 2 & -2 \end{bmatrix}$$

into its corresponding primal and dual LPP and solve. (6)