

Exercise 5.5

Q.1 $AP = (8x+4), (6x-2), (2x+7)$

ATQ,

$$(6x-2) - (8x+4) = (2x+7) - (6x-2)$$

$$6x-2-8x-4 = 2x+7-6x+2$$

$$-2x-6 = -4x+9$$

$$-2x+4x = 9+6$$

$$2x = 15$$

$$\boxed{x = \frac{15}{2}}$$

Q.2 $AP = (x+1), (3x), (4x+2)$

ATQ,

$$(3x) - (x+1) = (4x+2) - (3x)$$

$$3x - x - 1 = 4x + 2 - 3x$$

$$2x - 1 = x + 2$$

$$2x - x = 2 + 1$$

$$\boxed{x = 3}$$

Q.3 $(a-b)^2, (a^2+b^2)$ and ~~(a^2+b^2)~~ $(a+b)^2$

$$(a^2+b^2) - (a-b)^2 = \cancel{a^2} + \cancel{b^2} - \cancel{a^2} + 2ab - \cancel{b^2} = 2ab$$

$$\cancel{a^2} + \cancel{b^2} (a+b)^2 - (a^2+b^2) = \cancel{a^2} + \cancel{b^2} + 2ab - \cancel{a^2} - \cancel{b^2} = 2ab$$

As the difference is same,
 $\therefore$ It is an AP.

Q.4 Let the three no. in AP be $a, a+d, a+2d$

ATQ,

$$a + a + d + a + 2d = 27$$

$$3a + 3d = 27$$

$$a = \frac{27 - 3d}{3}$$

$$a \times (a+d) \times (a+2d) = 648$$

$$(a^2 + ad) \times (a+2d) = 648$$

$$a^3 + 2a^2d + a^2d + 2ad^2 = 648$$

Q.4 Let the three no. in AP be $a-d, a, a+d$.

ATQ,

$$a-d + a + a+d = 27$$

$$3a = 27$$

$$\boxed{a = 9} \quad \text{--- (1)}$$

$$(a-d)(a)(a+d) = 648$$

$$(a)(a^2 - d^2) = 648$$

$$9(9^2 - d^2) = 648 \quad (\text{Putting (1)})$$

$$9^2 - d^2 = \frac{648}{9}$$

$$81 - d^2 = 72$$

$$d^2 = 81 - 72$$

$$d^2 = 9$$

$$d = \sqrt{9}$$

$$\boxed{d = 3}$$

$$a - d = 9 - 3 = 6$$

$$a = 9$$

$$a + d = 9 + 3 = 12$$

$\therefore$ The no. are 6, 9 & 12.

Q. 5 Let the three no. in AP be $a-d$, a , $a+d$

ATQ,

$$a-d + a + a+d = 21$$

$$\begin{array}{l} 3a = 21 \\ \boxed{a = 7} \end{array} \quad \text{--- ①}$$

$$(a-d)(a+d) = a+6$$

$$a^2 - d^2 = a+6$$

$$7^2 - d^2 = 7+6 \quad (\text{Putting ①})$$

$$49 - 13 = d^2$$

$$d^2 = 36$$

$$d = \sqrt{36}$$

$$\boxed{d = 6}$$

$$a-d = 7-6 = 1$$

$$a = 7$$

$$a+d = 7+6 = 13$$

$\therefore$ The no. are 1, 7, 13.

Q. 6 Let the four terms be $a-3d$, $a-d$, $a+d$, $a+3d$

ATQ,

$$a-3d + a-d + a+d + a+3d = 50$$

$$4a = 50$$

$$a = \frac{50}{4}$$

$$\boxed{a = 12.5}$$

greatest no. = 4 (Least no.)

$$a + 3d = 4(a - 3d)$$

$$a + 3d = 4a - 12d$$

$$4a - a = 12d + 3d$$

$$3a = 15d$$

$$a = \frac{15d}{3}$$

$$a = 5d$$

$$\frac{12.5}{5} = d$$

$$\boxed{d = 2.5}$$

$$a - 3d = 12.5 - 3(2.5) = 5$$

$$a - d = 12.5 - 2.5 = 10$$

$$a + d = 12.5 + 2.5 = 15$$

$$a + 3d = 12.5 + 3(2.5) = 20$$

$\therefore$ The no. are 5, 10, 15, 20.

Q.7

Let the three terms be $a-d$, a , $a+d$

ATQ,

$$a-d + a + a+d = 12$$

$$3a = 12$$

$$\boxed{a = 4} \quad \text{--- ①}$$

$$(a-d)^3 + (a)^3 + (a+d)^3 = 288$$

$$a^3 - d^3 - 3a^2d + 3ad^2 + a^3 + a^3 + d^3 + 3a^2d + 3ad^2 = 288$$

$$3a^3 + 6ad^2 = 288$$

$$3(4)^3 + 6(4)d^2 = 288 \text{ (Putting ①)}$$

$$192 + 24d^2 = 288$$

$$24d^2 = 288 - 192$$

$$d^2 = \frac{96}{2}$$

$$d = \sqrt{4}$$

$$\boxed{d = \pm 2}$$

$$\text{If } d = +2$$

$$\therefore a - d = 4 - 2 = 2$$

$$a = 4$$

$$a + d = 4 + 2 = 6$$

$\therefore$ The no. are 2, 4, 6

$$\text{If } d = -2$$

$$\therefore a - d = 4 + 2 = 6$$

$$a = 4$$

$$a + d = 4 - 2 = 2$$

$\therefore$ The no. are 6, 4, 2

Q.8 Let the four terms be $a-3d, a-d, a+d, a+3d$.

ATQ,

$$a-3d + a+d + a+d + a+3d = 56$$

$$4a = 56$$

$$\boxed{a = 14} \text{ ————— ①}$$

$$\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{5}{6}$$

$$\frac{a^2 + 3ad - 3da - 9d^2}{a^2 - d^2} = \frac{5}{6}$$

$$\frac{a^2 - 9d^2}{a^2 - d^2} = \frac{5}{6}$$

$$6(a^2 - 9d^2) = 5(a^2 - d^2)$$

$$\cancel{6a^2} - \cancel{54d^2} = 5a^2 - 5d^2$$

$$\cancel{6a^2} - 5a^2 = 54d^2 - 5d^2$$

$$6a^2 - 54d^2 = 5a^2 - 5d^2$$

$$6a^2 - 5a^2 = 54d^2 - 5d^2$$

$$a^2 = 49d^2$$

$$a = 7d$$

$$14 = 7d \quad (\text{Putting } ①)$$

$$d = \frac{14}{7}$$

$$\boxed{d = 2}$$

$$a - 3d = 14 - 3(2) = 8$$

$$a - d = 14 - 2 = 12$$

$$a + d = 14 + 2 = 16$$

$$a + 3d = 14 + 3(2) = 20$$

∴ The no. are 8, 12, 16, 20

Q.9 Let the three terms be $a-d$, a , $a+d$.

ATQ,

$$a-d + a + a+d = 18$$

$$3a = 18$$

$$\boxed{a = 6} \quad \text{--- } ①$$

$$(a-d)(a+d) = 5d$$

$$① \text{ --- } (6-d)(6+d) = 5d$$

$$a^2 - d^2 = 5d$$

$$(6)^2 - d^2 = 5d \quad (\text{Putting } 6)$$

$$36 - d^2 = 5d$$

$$d^2 + 5d - 36 = 0$$

$$d^2 + 9d - 4d - 36 = 0$$

$$d(d+9) - 4(d+9) = 0$$

$$(d+9)(d-4) = 0$$

$$d + 9 = 0$$

$$\boxed{d = -9}$$

$$d - 4 = 0$$

$$\boxed{d = 4}$$

$$\text{If } d = -9,$$

$$\therefore a - d = 6 + 9 = 15$$

$$a = 6$$

$$a + d = 6 - 9 = -3$$

$$\text{If } d = 4$$

$$\therefore a - d = 6 - 4 = 2$$

$$a = 6$$

$$a + d = 6 + 4 = 10$$

$\therefore$ The no. are 15, 6, -3 or 2, 6, 10.

Q.10 Let the four angles be $a - 3d$, $a - d$, $a + d$, $a + 3d$.

$$d = 10^\circ$$

ATQ,

$$a - 3d + a - d + a + d + a + 3d = 360^\circ$$

$$4a = 360^\circ$$

$$\boxed{a = 90^\circ} \quad \text{--- (1)}$$

$$(a - d) - (a - 3d) = 10^\circ$$

$$a - d - a + 3d = 10^\circ$$

$$2d = 10^\circ$$

$$\boxed{d = 5^\circ}$$

$$a - 3d = 90^\circ - 3(15^\circ) = \boxed{75^\circ}$$

$$a-d = 90^\circ - 5^\circ = 85^\circ$$

$$a+d = 90^\circ + 5^\circ = 95^\circ$$

$$a+3d = 90^\circ + 3(5^\circ) = 105^\circ$$

$\therefore$ The angles are $75^\circ, 85^\circ, 95^\circ, 105^\circ$.

Q.11 Let the three terms be $a-d, a, a+d$

ATQ,

$$a-d + a + a+d = 207$$

$$3a = 207$$

$$\boxed{a = 69} \quad \text{--- (1)}$$

$$(a-d)(a) = 4623$$

~~$$a^2 - da = 4623$$~~

~~$$(69)^2 - 69d = 4623$$~~

$$(69-d)(69) = 4623$$

$$69-d = \frac{4623}{69}$$

$$69$$

$$d = 69 - 67$$

$$\boxed{d = 2}$$

$$a-d = 69 - 2 = 67$$

$$a = 69$$

$$a+d = 69 + 2 = 71$$

Q.12 Let the angles be $a-d, a, a+d$

ATQ,

$$a-d + a + a+d = 180^\circ$$

$$3a = 180^\circ$$

$$\boxed{a = 60^\circ} \quad \text{--- (1)}$$

$$\text{Largest angle} = 2(\text{Least angle})$$

$$a+d = 2(a-d)$$

$$a+d = 2a - 2d$$

$$2d+d = 2a-a$$

$$3d = a$$

$$3d = 60 \quad (\text{Putting } \textcircled{1})$$

$$d = \frac{60}{3}$$

$$\boxed{d = 20^\circ}$$

$$a-d = 60^\circ - 20^\circ = 40^\circ$$

$$a = 60^\circ$$

$$a+d = 60^\circ + 20^\circ = 80^\circ$$

$\therefore$ The angles are $40^\circ, 60^\circ, 80^\circ$.

Q. 13 Let the four terms be $a-3d, a-d, a+d, a+3d$

ATQ,

$$a-3d + a-d + a+d + a+3d = 32$$

$$4a = 32$$

$$\boxed{a = 8} \quad \text{--- } \textcircled{1}$$

$$\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$$

$$\frac{(a)^2 - (3d)^2}{(a)^2 - (d)^2} = \frac{7}{15}$$

$$\frac{(a)^2 - (3d)^2}{(a)^2 - (d)^2} = \frac{7}{15}$$

$$15(a^2 - 9d^2) = 7(a^2 - d^2)$$

$$15a^2 - 135d^2 = 7a^2 - 7d^2$$

$$15a^2 - 7a^2 = 135d^2 - 7d^2$$

$$8a^2 = 128d^2$$

$$8a^2 = 128d^2$$

$$d^2 = \frac{8a^2}{128}$$

$$d^2 = \frac{8 \times 8 \times 8 \times 4}{128 \times 16 \times 2}$$

$$d = \sqrt{4}$$

$$d = \pm 2$$

$$\text{If } d = +2$$

$$\therefore a - 3d = 8 - 3(2) = 2$$

$$a - d = 8 - 2 = 6$$

$$a + d = 8 + 2 = 10$$

$$a + 3d = 8 + 3(2) = 14$$

$$\text{If } d = -2$$

$$\therefore a - 3d = 8 - 3(-2) = 14$$

$$a - d = 8 + 2 = 10$$

$$a + d = 8 - 2 = 6$$

$$a + 3d = 8 + 3(-2) = 2$$

$\therefore$ The no. are 2, 6, 10, 14.