

Factorise each of the following expressions.

1. $qr - pr + qs - ps$

Sol: $qr - pr + qs - ps$

By regrouping terms

$$= (qr + qs) - (pr + ps)$$

$$= q(r+s) - p(r+s)$$

$$= (r+s)(q-p)$$

2. $p^2q - p^2r - pq + r^2$

Sol: By regrouping terms

$$(p^2q - pq) + (-p^2r + r^2)$$

$$= pq(p-1) - r^2(p-1)$$

$$= (p-1)(pq - r^2)$$

3. $1 + x + xy + x^2y$

Sol: $(1+x) + (xy + x^2y)$

$$= (1+x) + xy(1+x)$$

$$= (1+x)(1+xy)$$

$$\begin{aligned}
 & \underline{4} \quad ax + ay - bx - by \\
 &= a(x+y) - b(x+y) \\
 &= (x+y)(a-b)
 \end{aligned}$$

$$\begin{aligned}
 & \underline{5} \quad xa^2 + xb^2 - ya^2 - yb^2 \\
 &= x(a^2 + b^2) - y(a^2 + b^2) \\
 &= (a^2 + b^2)(x-y)
 \end{aligned}$$

$$\begin{aligned}
 & \underline{6} \quad x^2 + xy + xz + yz \\
 &= x(x+y) + z(x+y) \\
 &= (x+y)(x+z)
 \end{aligned}$$

$$\begin{aligned}
 & \underline{7} \quad 2ax + bx + 2ay + by \\
 &= x(2a+b) + y(2a+b) \\
 &= (2a+b)(x+y)
 \end{aligned}$$

$$\begin{aligned}
 & \underline{8} \quad ab - by - ay + y^2 \\
 &= b(a-y) - y(a-y) \\
 &= (a-y)(b-y)
 \end{aligned}$$

$$\begin{aligned}
 & \underline{9} \quad axy + bcy - az - bcz \\
 &= xy(a+bc) - z(a+bc) \\
 &= (a+bc)(xy-z)
 \end{aligned}$$

10] $m^2 - mn^2 - m + n^2$

Sol: $m^2 - m - mn^2 + n^2$ (By rearranging terms)

$$= m(m-1) - n^2(m-1)$$

$$= (m-1)(m-n^2)$$

11] $x^3 - y^2 + x - x^2y^2$

Sol: $x^3 + x - y^2 - x^2y^2$ (By rearranging terms)

$$= x(x^2+1) - y^2(x^2+1)$$

$$= (x^2+1)(x-y^2)$$

12] $6xy + 6 - 9y - 4x$

Sol: $6xy - 9y + 6 - 4x$ (By rearranging terms)

$$= 3y(2x-3) + 2(3-2x)$$

$$= 3y(2x-3) - 2(2x-3)$$

$$= (2x-3)(3y-2)$$

13] $x^2 - 2ax - 2ab + bx$

$$= x(x-2a) + b(-2a+x)$$

$$= (x-2a)(x+b)$$

$$\begin{aligned}
 \underline{14)} \quad & x^3 - 2x^2y + 3xy^2 - 6y^3 \\
 &= x^2(x-2y) + 3y^2(x-2y) \\
 &= (x-2y)(x^2+3y^2)
 \end{aligned}$$

$$\begin{aligned}
 \underline{15)} \quad & abx^2 + (ay-b)x - y \\
 &= abx^2 + ayx - bx - y \\
 &= ax(bx+y) - 1(bx+y) \\
 &= (bx+y)(ax-1)
 \end{aligned}$$

$$\underline{16)} \quad (ax+by)^2 + (bx-ay)^2$$

$$\underline{\text{Sol:}} \quad \{ (ax)^2 + (by)^2 + 2(ax)(by) \} + \{ (bx)^2 + (ay)^2 - 2(bx)(ay) \}$$

$$= a^2x^2 + b^2y^2 + 2abxy + b^2x^2 + a^2y^2 - 2abxy$$

$$= a^2x^2 + b^2y^2 + b^2x^2 + a^2y^2$$

$$= \text{By rearranging terms.}$$

$$(a^2x^2 + a^2y^2) + (b^2y^2 + b^2x^2)$$

$$= a^2(x^2+y^2) + b^2(y^2+x^2)$$

$$= (x^2+y^2)(a^2+b^2)$$

$$\begin{aligned}
 17] \quad & 16(a-b)^3 - 24(a-b)^2 \\
 &= 8(a-b)^2 \{ 2(a-b) - 3 \} \\
 &= 8(a-b)^2 (2a-2b-3)
 \end{aligned}$$

$$\begin{aligned}
 18] \quad & ab(x^2+1) + x(a^2+b^2) \\
 &= abx^2 + ab + xa^2 + xb^2 \quad (\text{By rearranging terms}) \\
 &= abx^2 + xb^2 + ab + xa^2 \\
 &= bx(ax+b) + a(b+ax) \\
 &= bx(ax+b) + a(ax+b) \\
 &= (ax+b)(bx+a)
 \end{aligned}$$

$$\begin{aligned}
 19] \quad & a^2x^2 + (ax^2+1)x + a \\
 &= a^2x^2 + ax^3 + x + a \quad (\text{By rearranging terms}) \\
 &= (a^2x^2 + a) + (ax^3 + x) \\
 &= a(ax^2+1) + x(ax^2+1) \\
 &= (ax^2+1)(a+x)
 \end{aligned}$$

$$\begin{aligned}
 20] \quad & a(a-2b-c) + 2bc \\
 &= a^2 - 2ab - ac + 2bc \\
 &= (a^2 - ac) + (-2ab + 2bc) \quad (\text{By regrouping terms}) \\
 &= a(a-c) - 2b(a-c) \\
 &= (a-c)(a-2b)
 \end{aligned}$$

$$\begin{aligned}
 \underline{21} \quad & a(a+b-c)-bc \\
 &= a^2+ab-ac-bc \\
 &= a(a+b)-c(a+b) \\
 &= (a+b)(a-c)
 \end{aligned}$$

$$\begin{aligned}
 \underline{22} \quad & x^2-11xy-x+11y \\
 &= x(x-11y)-1(x-11y) \\
 &= (x-11y)(x-1)
 \end{aligned}$$

$$\begin{aligned}
 \underline{23} \quad & ab-a-b+1 \\
 &= a(b-1)-1(b-1) \\
 &= (b-1)(a-1)
 \end{aligned}$$

$$\begin{aligned}
 \underline{24} \quad & x^2+y-xy-x \quad (\text{By regrouping terms}) \\
 &= x^2-xy+y-x \\
 &= x(x-y)-1(x-y) \\
 &= (x-y)(x-1)
 \end{aligned}$$