

NCERT SOLUTIONS

CLASS - 11th



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Exercise 5.1

Q1 Express each of the complex number in the form $a + ib$.

$$(5i) \left(-\frac{3}{5}i \right)$$

Answer.

$$\begin{aligned}(5i) \left(-\frac{3}{5}i \right) &= -5 \times \frac{3}{5} \times i \times i \\ &= -3i^2 \\ &= -3(-1) \\ &= 3\end{aligned}$$

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Q2 Express each of the complex number in the form $a + ib$.

$$i^9 + i^{19}$$

Answer.

$$\begin{aligned}i^9 + i^{19} &= i^{4 \times 2 + 1} + i^{4 \times 4 + 3} \\ &= (i^4)^2 \cdot i + (i^4)^4 \cdot i^3 \\ &= 1 \times i + 1 \times (-i) \\ &= i + (-i) \\ &= 0\end{aligned}$$

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Q3 Express each of the complex number in the form $a + ib$.

$$i^{-39}$$

Answer.

$$i^{-39} = i^{-1 \times 4 - 3} = (i^4)^{-9} \cdot i^{-3}$$

$$\begin{aligned}
&= (1)^{-9} - F^{-3} \\
&= \frac{1}{p^3} = \frac{1}{-i} \\
&= \frac{-1}{i} \times \frac{j}{i} \\
&= \frac{-i}{i^2} = \frac{-i}{-1} = i
\end{aligned}$$

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Q4 Express each of the complex number in the form a + ib.

$$3(7 + i7) + i(7 + i7)$$

Answer.

$$\begin{aligned}
3(7 + i7) + i(7 + i7) &= 21 + 21i + 7i + 7i^2 \\
&= 21 + 28i + 7 \times (-1) \\
&= 14 + 28i
\end{aligned}$$

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Q5 Express each of the complex number in the form a + ib.

$$(1 - i) - (-1 + i6)$$

Answer.

$$\begin{aligned}
(1 - i) - (-1 + i6) &= 1 - j + 1 - 6i \\
&= 2 - 7i
\end{aligned}$$

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Q6 Express each of the complex number in the form a + ib.

$$\left(\frac{1}{5} + i\frac{2}{5}\right) - \left(4 + i\frac{5}{2}\right)$$

Answer.

$$\begin{aligned}
&\left(\frac{1}{5} + i\frac{2}{5}\right) - \left(4 + i\frac{5}{2}\right) \\
&= \frac{1}{5} + \frac{2}{5}i - 4 - \frac{5}{2}i \\
&= \left(\frac{1}{5} - 4\right) + i\left(\frac{2}{5} - \frac{5}{2}\right) \\
&= \frac{-19}{5} + i\left(\frac{-21}{10}\right) \\
&= \frac{-19}{5} - \frac{21}{10}i
\end{aligned}$$

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Q7 Express each of the complex number in the form a + ib.

$$\left[\left(\frac{1}{3} + i\frac{7}{3}\right) + \left(4 + i\frac{1}{3}\right)\right] - \left(-\frac{4}{3} + i\right)$$

Answer.

$$\begin{aligned}& \left[\left(\frac{1}{3} + i\frac{7}{3} \right) + \left(4 + i\frac{1}{3} \right) \right] - \left(\frac{-4}{3} + i \right) \\&= \frac{1}{3} + \frac{7}{3}i + 4 + \frac{1}{3}i + \frac{4}{3} - i \\&= \left(\frac{1}{3} + 4 + \frac{4}{3} \right) + \left(\frac{7}{3} + \frac{1}{3} - 1 \right) \\&= \frac{17}{3} + i\frac{5}{3}\end{aligned}$$

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Q8 Express each of the complex number in the form $a + ib$.

$$(1 - i)^4$$

Answer.

$$\begin{aligned}(1 - i)^4 &= [(1 - i)^2]^2 \\&= [1^2 + i^2 - 2i]^2 \\&= [1 - 1 - 2i]^2 \\&= (-2i)^2 \\&= (-2i) \times (-2i) \\&= 4i^2 = -4\end{aligned}$$

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Q9 Express each of the complex number in the form $a + ib$.

$$\left(\frac{1}{3} + 3i \right)^3$$

Answer.

$$\begin{aligned}\left(\frac{1}{3} + 3i \right)^3 &= \left(\frac{1}{3} \right)^3 + (3i)^3 + 3 \left(\frac{1}{3} \right) (3i) \left(\frac{1}{3} + 3i \right) \\&= \frac{1}{27} + 27i^3 + 3i \left(\frac{1}{3} + 3i \right) \\&= \frac{1}{27} + 27(-i) + i + 9i^2 \\&= \frac{1}{27} - 27i + i - 9 \\&= \left(\frac{1}{27} - 9 \right) + i(-27 + 1) \\&= \frac{-242}{27} - 26i\end{aligned}$$

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Q10 Express each of the complex number in the form $a + ib$.

$$\left(-2 - \frac{1}{3}i \right)^3$$

Answer.

$$\begin{aligned}\left(-2 - \frac{1}{3}i\right)^3 &= (-1)^3 \left(2 + \frac{1}{3}i\right)^3 \\&= - \left[2^3 + \left(\frac{i}{3}\right)^2 + 3(2) \left(\frac{i}{3}\right) \left(2 + \frac{i}{3}\right) \right] \\&= - \left[8 + \frac{i^3}{27} + 2i \left(2 + \frac{i}{3}\right) \right] \\&= - \left[8 - \frac{i}{27} + 4i + \frac{2i^2}{3} \right] \\&= - \left[8 - \frac{i}{27} + 4i - \frac{2}{3} \right] \\&= - \left[\frac{22}{3} + \frac{107i}{27} \right] \\&= -\frac{22}{3} - \frac{107}{27}i\end{aligned}$$

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Q11 Find the multiplicative inverse of each of the complex numbers given
 $4 - 3i$

Answer.

Let $z = 4 - 3i$

Then, $\bar{z} = 4 + 3i$ and $|z|^2 = 4^2 + (-3)^2 = 16 + 9 = 25$

Therefore, the multiplicative inverse of $4 - 3i$ is given by

$$z^{-1} = \frac{z}{|z|^2} = \frac{4+3i}{25} = \frac{4}{25} + \frac{3}{25}i$$

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Q12 Find the multiplicative inverse of each of the complex numbers given
 $\sqrt{5} + 3i$

Answer.

Let $z = \sqrt{5} + 3i$

Then, $\bar{z} = \sqrt{5} - 3i$ and $|z|^2 = (\sqrt{5})^2 + 3^2 = 5 + 9 = 14$

Therefore, the multiplicative inverse of $\sqrt{5} + 3i$ is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{\sqrt{5}-3i}{14} = \frac{\sqrt{5}}{14} - \frac{3i}{14}$$

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Q13 Find the multiplicative inverse of each of the complex numbers given
 $-i$

Answer.

$$\text{Let } z = -i$$

$$\text{Then, } \bar{z} = i \text{ and } |z|^2 = 1^2 = 1$$

Therefore, the multiplicative inverse of $-i$ is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{i}{1} = i$$

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Q14 Express the following expression in the form of $a + ib$

$$\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-i\sqrt{2})}$$

Answer.

$$\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-i\sqrt{2})}$$

$$= \frac{(3)^2 - (i\sqrt{5})^2}{\sqrt{3}+\sqrt{2}i-\sqrt{3}+i\sqrt{2}}$$

$$= \frac{9-5i^2}{2\sqrt{2}i}$$

$$= \frac{9-5(-1)}{2\sqrt{2}i}$$

$$= \frac{9+5}{2\sqrt{2}i} \times \frac{i}{i}$$

$$= \frac{14i}{2\sqrt{2}i^2}$$

$$= \frac{14i}{2\sqrt{2}(-1)}$$

$$= \frac{-7i}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{-7\sqrt{2}i}{2}$$

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Exercise 5.2

Q1 Find the modulus and the arguments of each of the complex numbers

$$z = -1 - i\sqrt{3}$$

Answer.

$$z = -1 - i\sqrt{3}$$

$$\text{Let } r \cos \theta = -1 \text{ and } r \sin \theta = -\sqrt{3}$$

On squaring and adding, we obtain

$$(r \cos \theta)^2 + (r \sin \theta)^2 = (-1)^2 + (-\sqrt{3})^2$$

$$\Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 1 + 3$$

$$\Rightarrow \text{Modulus} = 2$$

$$\therefore 2 \cos \theta = -1 \text{ and } 2 \sin \theta = -\sqrt{3}$$

$$\Rightarrow \cos \theta = \frac{-1}{2} \text{ and } \sin \theta = \frac{-\sqrt{3}}{2}$$

since both the values of $\sin \theta$ and $\cos \theta$ are negative and $\sin \theta$ and $\cos \theta$ are negative in III quadrant,

$$\text{Argument} = -\left(\pi - \frac{\pi}{3}\right) = \frac{-2\pi}{3}$$

Thus, the modulus and argument of the complex number $-1 - \sqrt{3}i$ are 2 and $\frac{-2\pi}{3}$ respectively.

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Q2 Find the modulus and the arguments of each of the complex numbers $z = -\sqrt{3} + i$

Answer.

$$z = -\sqrt{3} + i$$

$$\text{Let } r \cos \theta = -\sqrt{3} \text{ and } r \sin \theta = 1$$

On squaring and adding, we obtain

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta = (-\sqrt{3})^2 + 1^2$$

$$\Rightarrow r^2 = 3 + 1 = 4$$

$$\Rightarrow r = \sqrt{4} = 2$$

$$\therefore \text{Modulus} = 2$$

$$\therefore 2 \cos \theta = -\sqrt{3} \text{ and } 2 \sin \theta = 1$$

$$\Rightarrow \cos \theta = \frac{-\sqrt{3}}{2} \text{ and } \sin \theta = \frac{1}{2}$$

$$\therefore \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

Thus, the modulus and argument of the complex number $-\sqrt{3} + i$ are 2 and $\frac{5\pi}{6}$ respectively.

Page : 108 , Block Name : Exercise 5.2

Q3 Convert each of the complex numbers given in the polar form:

$$1 - i$$

Answer.

$$1 - i$$

$$\text{Let } r \cos \theta = 1 \text{ and } r \sin \theta = -1$$

On squaring and adding, we obtain

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta = 1^2 + (-1)^2$$

$$\Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 1 + 1$$

$$\Rightarrow r^2 = 2$$

$$\Rightarrow r = \sqrt{2}$$

$$\therefore \sqrt{2} \cos \theta = 1 \text{ and } \sqrt{2} \sin \theta = -1$$

$$\Rightarrow \cos \theta = \frac{1}{\sqrt{2}} \text{ and } \sin \theta = -\frac{1}{\sqrt{2}}$$

$\therefore 1 - i = r \cos \theta + r \sin \theta = \sqrt{2} \cos\left(-\frac{\pi}{4}\right) + i\sqrt{2} \sin\left(-\frac{\pi}{4}\right) = \sqrt{2} \left[\cos\left(-\frac{\pi}{4}\right) + i \sin\left(-\frac{\pi}{4}\right) \right]$ This is the required polar form.

Page : 108 , Block Name : Exercise 5.2

Q4 Convert each of the complex numbers given in the polar form:

$$-1 + i$$

Answer.

$$-1 + i$$

$$\text{Let } r \cos \theta = -1 \text{ and } r \sin \theta = 1$$

On squaring and adding, we obtain

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta = (-1)^2 + 1^2$$

$$\Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 1 + 1$$

$$\Rightarrow r = \sqrt{2} \quad [\text{Conventionally, } r > 0]$$

$$\therefore \sqrt{2} \cos \theta = -1 \text{ and } \sqrt{2} \sin \theta = 1$$

$$\Rightarrow \cos \theta = -\frac{1}{\sqrt{2}} \text{ and } \sin \theta = \frac{1}{\sqrt{2}}$$

$$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \quad [\text{As } \theta \text{ lies in the II quadrant}]$$

It can be written,

$$\therefore -1 + i = r \cos \theta + ir \sin \theta = \sqrt{2} \cos \frac{3\pi}{4} + i\sqrt{2} \sin \frac{3\pi}{4} = \sqrt{2} \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)$$

This is the required polar form.

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Q5 Convert each of the complex numbers given in the polar form:

$$-1 - i$$

Answer.

$$-1 - i$$

$$\text{Let } r \cos \theta = -1 \text{ and } r \sin \theta = -1$$

On squaring and adding, we obtain

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta = (-1)^2 + (-1)^2$$

$$\Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 1 + 1$$

$$\Rightarrow r^2 = 2$$

$$\Rightarrow r = \sqrt{2}$$

$$\therefore \sqrt{2} \cos \theta = -1 \text{ and } \sqrt{2} \sin \theta = -1$$

$$\Rightarrow \cos \theta = -\frac{1}{\sqrt{2}} \text{ and } \sin \theta = -\frac{1}{\sqrt{2}}$$

$$\therefore \theta = -\left(\pi - \frac{\pi}{4}\right) = -\frac{3\pi}{4} \quad [\text{As } \theta \text{ lies in the III quadrant}]$$

$$\therefore -1 - i = r \cos \theta + ir \sin \theta = \sqrt{2} \cos \frac{-3\pi}{4} + i\sqrt{2} \sin \frac{-3\pi}{4} = \sqrt{2} \left(\cos \frac{-3\pi}{4} + i \sin \frac{-3\pi}{4} \right) \text{ This is the required polar form.}$$

Q6 Convert each of the complex numbers given in the polar form:

$$-3$$

Answer.

$$\text{Let } r \cos \theta = -3 \text{ and } r \sin \theta = 0$$

On squaring and adding, we obtain

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta = (-3)^2$$

$$\Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 9$$

$$\Rightarrow r^2 = 9$$

$$\Rightarrow r = \sqrt{9} = 3$$

$$\therefore 3 \cos \theta = -3 \text{ and } 3 \sin \theta = 0$$

$$\Rightarrow \theta = \pi$$

$$\therefore -3 = r \cos \theta + ir \sin \theta = 3 \cos \pi + 3i \sin \pi = 3(\cos \pi + i \sin \pi)$$

This is the required polar form.

Q7 Convert each of the complex numbers given in the polar form:

$$\sqrt{3} + i$$

Answer.

$$\sqrt{3} + i$$

$$\text{Let } r \cos \theta = \sqrt{3} \text{ and } r \sin \theta = 1$$

On squaring and adding, we obtain

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta = (\sqrt{3})^2 + 1^2$$

$$\Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 3 + 1$$

$$\Rightarrow r^2 = 4$$

$$\Rightarrow r = \sqrt{4} = 2 \quad [\text{Conventionally, } r > 0]$$

$$\therefore 2 \cos \theta = \sqrt{3} \text{ and } 2 \sin \theta = 1$$

$$\Rightarrow \cos \theta = \frac{\sqrt{3}}{2} \text{ and } \sin \theta = \frac{1}{2}$$

$$\therefore \theta = \frac{\pi}{6}$$

$$\therefore \sqrt{3} + i = r \cos \theta + ir \sin \theta = 2 \cos \frac{\pi}{6} + i 2 \sin \frac{\pi}{6} = 2 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$$

Q8 Convert each of the complex numbers given in the polar form:

$$i$$

Answer.

Let $r \cos \theta = 0$ and $r \sin \theta = 1$

On squaring and adding, we obtain

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta = 0^2 + 1^2$$

$$\Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 1$$

$$\Rightarrow r^2 = 1$$

$$\Rightarrow r = \sqrt{1} = 1$$

$$\therefore \cos \theta = 0 \text{ and } \sin \theta = 1$$

$$\therefore \theta = \frac{\pi}{2}$$

$$\therefore i = r \cos \theta + ir \sin \theta = \cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$$

This is the required polar form.

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Exercise 5.3

Q1 Solve each of the following equations:

$$x^2 + 3 = 0$$

Answer.

The given quadratic equation is $x^2 + 3 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$$a = 1, b = 0, \text{ and } c = 3$$

Therefore, the discriminant of the given equation is

$$D = b^2 - 4ac = 0^2 - 4 \times 1 \times 3 = -12$$

Therefore, the required solutions are

$$\begin{aligned} \frac{-b \pm \sqrt{D}}{2a} &= \frac{\pm \sqrt{-12}}{2 \times 1} = \frac{\pm \sqrt{12}i}{2} \\ &= \frac{\pm 2\sqrt{3}i}{2} = \pm \sqrt{3}i \end{aligned}$$

Page : 109 , Block Name : Exercise 5.3

Q2 Solve each of the following equations:

$$2x^2 + x + 1 = 0$$

Answer.

The given quadratic equation is $2x^2 + x + 1 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$$a = 2, b = 1, \text{ and } c = 1$$

Therefore, the discriminant of the given equation is

$$D = b^2 - 4ac = 1^2 - 4 \times 2 \times 1 = 1 - 8 = -7$$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{-7}}{2 \times 2} = \frac{-1 \pm \sqrt{7}i}{4}$$

Page : 109 , Block Name : Exercise 5.3

Q3 Solve each of the following equations:

$$x^2 + 3x + 9 = 0$$

Answer.

The given quadratic equation is $x^2 + 3x + 9 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$$a = 1, b = 3, \text{ and } c = 9$$

Therefore, the discriminant of the given equation is

$$D = b^2 - 4ac = 3^2 - 4 \times 1 \times 9 = 9 - 36 = -27$$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-3 \pm \sqrt{-27}}{2(1)} = \frac{-3 \pm 3\sqrt{-3}}{2} = \frac{-3 \pm 3\sqrt{3}i}{2}$$

Page : 109 , Block Name : Exercise 5.3

Q4 Solve each of the following equations:

$$-x^2 + x - 2 = 0$$

Answer.

The given quadratic equation is $-x^2 + x - 2 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$$a = -1, b = 1, \text{ and } c = -2$$

Therefore, the discriminant of the given equation is

$$D = b^2 - 4ac = 1^2 - 4 \times (-1) \times (-2) = 1 - 8 = -7$$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{-7}}{2 \times (-1)} = \frac{-1 \pm \sqrt{7}i}{-2}$$

Page : 109 , Block Name : Exercise 5.3

Q5 Solve each of the following equations:

$$x^2 + 3x + 5 = 0$$

Answer.

The given quadratic equation is $x^2 + 3x + 5 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$$a = 1, b = 3, \text{ and } c = 5$$

Therefore, the discriminant of the given equation is

$$D = b^2 - 4ac = 3^2 - 4 \times 1 \times 5 = 9 - 20 = -11$$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-3 \pm \sqrt{-11}}{2 \times 1} = \frac{-3 \pm \sqrt{11}i}{2}$$

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Q6 Solve each of the following equations:

$$x^2 - x + 2 = 0$$

Answer.

The given quadratic equation is $x^2 - x + 2 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$$a = 1, b = -1, \text{ and } c = 2$$

Therefore, the discriminant of the given equation is

$$D = b^2 - 4ac = (-1)^2 - 4 \times 1 \times 2 = 1 - 8 = -7$$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-(-1) \pm \sqrt{-7}}{2 \times 1} = \frac{1 \pm \sqrt{7}i}{2}$$

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Q7 Solve each of the following equations:

$$\sqrt{2}x^2 + x + \sqrt{2} = 0$$

Answer.

The given quadratic equation is $\sqrt{2}x^2 + x + \sqrt{2} = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$$a = \sqrt{2}, b = 1, \text{ and } c = \sqrt{2}$$

Therefore, the discriminant of the given equation is

$$D = b^2 - 4ac = 1^2 - 4 \times \sqrt{2} \times \sqrt{2} = 1 - 8 = -7$$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{-7}}{2 \times \sqrt{2}} = \frac{-1 \pm \sqrt{7}i}{2\sqrt{2}}$$

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Q8 Solve each of the following equations:

$$\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$$

Answer.

The given quadratic equation is $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$$a = \sqrt{3}, b = -\sqrt{2}, \text{ and } c = 3\sqrt{3}$$

Therefore, the discriminant of the given equation is

$$D = b^2 - 4ac = (-\sqrt{2})^2 - 4(\sqrt{3})(3\sqrt{3}) = 2 - 36 = -34$$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-(-\sqrt{2}) \pm \sqrt{-34}}{2 \times \sqrt{3}} = \frac{\sqrt{2} \pm \sqrt{34}i}{2\sqrt{3}} \quad [\sqrt{-1} = i]$$

Page : 109 , Block Name : Exercise 5.3

Q9 Solve each of the following equations:

$$x^2 + x + \frac{1}{\sqrt{2}} = 0$$

Answer.

The given quadratic equation is

$$\text{This equation can also be written as } \sqrt{2}x^2 + \sqrt{2}x + 1 = 0$$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$$a = \sqrt{2}, b = \sqrt{2}, \text{ and } c = 1$$

$$\therefore \text{ Discriminant (D)} = b^2 - 4ac = (\sqrt{2})^2 - 4 \times (\sqrt{2}) \times 1 = 2 - 4\sqrt{2}$$

Therefore, the required solutions are

$$\begin{aligned} \frac{-b \pm \sqrt{D}}{2a} &= \frac{-\sqrt{2} \pm \sqrt{2 - 4\sqrt{2}}}{2 \times \sqrt{2}} = \frac{-\sqrt{2} \pm \sqrt{2(1 - 2\sqrt{2})}}{2\sqrt{2}} = \frac{-1 \pm (\sqrt{2\sqrt{2}-1})i}{2} \\ &= \left(\frac{-\sqrt{2} \pm \sqrt{2}(\sqrt{2\sqrt{2}-1})}{2\sqrt{2}} \right) \end{aligned}$$

Page : 109 , Block Name : Exercise 5.3

Q10 Solve each of the following equations:

$$x^2 + \frac{x}{\sqrt{2}} + 1 = 0$$

Answer.

The given quadratic equation is

$$\text{This equation can also be written as } \sqrt{2}x^2 + x + \sqrt{2} = 0$$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$$a = \sqrt{2}, b = 1, \text{ and } c = \sqrt{2}$$

$$\therefore \text{ Discriminant (D)} = b^2 - 4ac = 1^2 - 4 \times \sqrt{2} \times \sqrt{2} = 1 - 8 = -7$$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{-7}}{2\sqrt{2}} = \frac{-1 \pm \sqrt{7}i}{2\sqrt{2}} \quad [\sqrt{-1} = i]$$

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Miscellaneous Exercise

Q1 Evaluate

$$\left[i^{18} + \left(\frac{1}{i} \right)^{25} \right]^3$$

Answer.

$$\begin{aligned} & \left[i^{18} + \left(\frac{1}{i} \right)^{25} \right]^3 \\ &= \left[i^{4 \times 4 + 2} + \frac{1}{i^{4 \times 6 + 1}} \right]^3 \\ &= \left[(i^4)^4 - i^2 + \frac{1}{(i^4)^6 \cdot i} \right]^3 \\ &= \left[1 - i^2 + \frac{1}{i} \right]^3 \\ &= \left[-1 + \frac{1}{i} \times \frac{i}{i} \right]^3 \\ &= \left[-1 + \frac{j}{i^2} \right]^3 \\ &= \left[-1 + \frac{j}{j^2} \right]^3 \\ &= [-1 - i]^3 \\ &= (-1)^3 [1 + i]^3 \\ &= -[1 + i^5 + 3i + 3i^2] \\ &= [1 - i + 3i - 3] \\ &= -[-2 + 2i] \\ &= 2 - 2i \end{aligned}$$

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Q2

For any two complex numbers z_1 and z_2 , prove that

$$\operatorname{Re}(z_1 z_2) = \operatorname{Re} z_1 \operatorname{Re} z_2 - \operatorname{Im} z_1 \operatorname{Im} z_2$$

Answer.

$$\begin{aligned} & \text{Let } z_1 = x_1 + iy_1 \text{ and } z_2 = x_2 + iy_2 \\ \therefore z_1 z_2 &= (x_1 + iy_1)(x_2 + iy_2) \\ &= x_1(x_2 + iy_2) + iy_1(x_2 + iy_2) \\ &= x_1 x_2 + ix_1 y_2 + iy_1 x_2 + i^2 y_1 y_2 \\ &= x_1 x_2 + ix_1 y_2 + iy_1 x_2 - y_1 y_2 \\ &= (x_1 x_2 - y_1 y_2) + i(x_2 y_2 + y_1 x_2) \end{aligned}$$

$$\Rightarrow \operatorname{Re}(z_1 z_2) = x_1 x_2 - y_1 y_2$$

$$\Rightarrow \operatorname{Re}(z_1 z_2) = \operatorname{Re} z_1 \operatorname{Re} z_2 - \operatorname{Im} z_1 \operatorname{Im} z_2$$

Hence, proved.

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Q3 Reduce $\left(\frac{1}{1-4i} - \frac{2}{1+i}\right) \left(\frac{3-4i}{5+i}\right)$ to standard form

Answer.

$$\begin{aligned} \left(\frac{1}{1-4i} - \frac{2}{1+i}\right) \left(\frac{3-4i}{5+i}\right) &= \left[\frac{(1+i)-2(1-4i)}{(1-4i)(1+i)}\right] \left[\frac{3-4i}{5+i}\right] \\ &= \left[\frac{1+i-2+8i}{1+i-4i-4i^2}\right] \left[\frac{3-4i}{5+i}\right] = \left[\frac{-1+9i}{5-3i}\right] \left[\frac{3-4i}{5+i}\right] \\ &= \left[\frac{-3+4i+27i-36i^2}{25+5i-15i-3i^2}\right] = \frac{33+31i}{28-10i} = \frac{33+31i}{2(14-5i)} \\ &= \frac{(33+31i)}{2(14-5i)} \times \frac{(14+5i)}{(14+5i)} \quad [\text{On multiplying numerator and denominator by } (14+5i)] \\ &= \frac{462 + 165i + 434i + 155i^2}{2[(14)^2 - (5i)^2]} = \frac{307 + 599i}{2(196 - 25i^2)} \\ &= \frac{307 + 599i}{2(221)} = \frac{307 + 599i}{442} = \frac{307}{442} + \frac{599i}{442} \end{aligned}$$

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Q4 if $x - iy = \sqrt{\frac{a-ib}{c-id}}$ prove that $(x^2 + y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$

Answer.

$$\begin{aligned} x - iy &= \sqrt{\frac{a-ib}{c-id}} \\ &= \sqrt{\frac{a-ib}{c-id} \times \frac{c+id}{c+id}} \quad [\text{On multiplying numerator and denominator by } (c+id)] \\ &= \sqrt{\frac{(ac+bd)+i(ad-bc)}{c^2+d^2}} \\ \therefore (x - iy)^2 &= \frac{(ac+bd)+i(ad-bc)}{c^2+d^2} \\ \Rightarrow x^2 - y^2 - 2ixy &= \frac{(ac+bd)+i(ad-bc)}{c^2+d^2} \\ \therefore (x - iy)^2 &= \frac{(ac+bd)+i(ad-bc)}{c^2+d^2} \\ \Rightarrow x^2 - y^2 - 2ixy &= \frac{(ac+bd)+i(ad-bc)}{c^2+d^2} \\ x^2 - y^2 &= \frac{ac+bd}{c^2+d^2}, -2xy = \frac{ad-bc}{c^2+d^2} \\ (x^2 + y^2)^2 &= (x^2 - y^2)^2 + 4x^2 y^2 \quad [\text{Using (i)}] \\ &= \left(\frac{ac+bd}{c^2+d^2}\right)^2 + \left(\frac{ad-bc}{c^2+d^2}\right)^2 \end{aligned}$$

$$\begin{aligned}
&= \frac{a^2c^2 + b^2d^2 + 2acbd + a^2d^2 + b^2c^2 - 2adbc}{(c^2 + d^2)^2} \\
&= \frac{a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2}{(c^2 + d^2)^2} \\
&= \frac{a^2(c^2 + d^2) + b^2(c^2 + d^2)}{(c^2 + d^2)^2} \\
&= \frac{(c^2 + d^2)(a^2 + b^2)}{(c^2 + d^2)^2} \\
&= \frac{a^2 + b^2}{c^2 + d^2}
\end{aligned}$$

Hence, proved.

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Q5

Convert the following in the polar form:

$$(i) \frac{1+7i}{(2-i)^2}, \quad (ii) \frac{1+3i}{1-2i}$$

Answer.

$$\begin{aligned}
(i) \text{ Here } z &= \frac{1+7i}{(2-i)^2} \\
&= \frac{1+7i}{(2-i)^2} = \frac{1+7i}{4+i^2-4i} = \frac{1+7i}{4-1-4i} \\
&= \frac{1+7i}{3-4i} \times \frac{3+4i}{3+4i} = \frac{3+4i+21i+28i^2}{3^2+4^2} \\
&= \frac{3+4i+21i-28}{3^2+4^2} = \frac{-25+25i}{25} \\
&= -1 + i \\
&= -1 + i
\end{aligned}$$

$$\text{Let } r \cos \theta = -1 \text{ and } r \sin \theta = 1$$

On squaring and adding, we obtain

$$r^2 (\cos^2 \theta + \sin^2 \theta) = 1 + 1$$

$$\Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 2$$

$$\Rightarrow r^2 = 2 \quad [\cos^2 \theta + \sin^2 \theta = 1]$$

$$\Rightarrow r = \sqrt{2} \quad [\text{Conventionally, } r > 0]$$

$$\therefore \sqrt{2} \cos \theta = -1 \text{ and } \sqrt{2} \sin \theta = 1$$

$$\therefore \sqrt{2} \cos \theta = -1 \text{ and } \sqrt{2} \sin \theta = 1$$

$$\Rightarrow \cos \theta = \frac{-1}{\sqrt{2}} \text{ and } \sin \theta = \frac{1}{\sqrt{2}}$$

$$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

$$: z = r \cos \theta + ir \sin \theta$$

$$= \sqrt{2} \cos \frac{3\pi}{4} + i\sqrt{2} \sin \frac{3\pi}{4} = \sqrt{2} \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)$$

This is the required polar form.

$$(ii) \text{ Here } z = \frac{1+3i}{1-2i}$$

$$\begin{aligned}
&= \frac{1+3i}{1-2i} \times \frac{1+2i}{1+2i} \\
&= \frac{1+2i+3i-6}{1+4} \\
&= \frac{-5+5i}{5} = -1+i
\end{aligned}$$

Let $r \cos \theta = -1$ and $r \sin \theta = 1$

On squaring and adding, we obtain

$$r^2 (\cos^2 \theta + \sin^2 \theta) = 1 + 1$$

$$\Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 2$$

$$\Rightarrow r^2 = 2 \quad [\cos^2 \theta + \sin^2 \theta = 1]$$

$$\Rightarrow r = \sqrt{2} \quad [\text{Conventionally, } r > 0]$$

$$\therefore \sqrt{2} \cos \theta = -1 \text{ and } \sqrt{2} \sin \theta = 1$$

$$\Rightarrow \cos \theta = \frac{-1}{\sqrt{2}} \text{ and } \sin \theta = \frac{1}{\sqrt{2}}$$

$$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

$$\therefore z = r \cos \theta + ir \sin \theta$$

$$= \sqrt{2} \cos \frac{3\pi}{4} + i\sqrt{2} \sin \frac{3\pi}{4} = \sqrt{2} \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)$$

This is the required polar form.

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Q6 Solve the equation $3x^2 - 4x + \frac{20}{3} = 0$

Answer.

The given quadratic equation is

$$\text{This equation can also be written as } 9x^2 - 12x + 20 = 0$$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$$a = 9, b = -12, \text{ and } c = 20$$

Therefore, the discriminant of the given equation is

$$D = b^2 - 4ac = (-12)^2 - 4 \times 9 \times 20 = 144 - 720 = -576$$

Therefore, the required solutions are

Therefore, the discriminant of the given equation is

$$D = b^2 - 4ac = (-12)^2 - 4 \times 9 \times 20 = 144 - 720 = -576$$

Therefore, the required solutions are

$$\begin{aligned}
\frac{-b \pm \sqrt{D}}{2a} &= \frac{-(-12) \pm \sqrt{-576}}{2 \times 9} = \frac{12 \pm \sqrt{576}i}{18} \\
&= \frac{12 \pm 24i}{18} = \frac{6(2 \pm 4i)}{18} = \frac{2 \pm 4i}{3} = \frac{2}{3} \pm \frac{4}{3}i
\end{aligned}$$

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Q7 Solve the equation $x^2 - 2x + \frac{3}{2} = 0$

Answer.

$$x^2 - 2x + \frac{3}{2} = 0$$

This equation can also be written as $2x^2 - 4x + 3 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$$a = 2, b = -4, \text{ and } c = 3$$

Therefore, the discriminant of the given equation is

$$D = b^2 - 4ac = (-4)^2 - 4 \times 2 \times 3 = 16 - 24 = -8$$

Therefore, the required solutions are

$$\begin{aligned} \frac{-b \pm \sqrt{D}}{2a} &= \frac{-(-4) \pm \sqrt{-8}}{2 \times 2} = \frac{4 \pm 2\sqrt{2}i}{4} \quad [\sqrt{-1} = i] \\ &= \frac{2 \pm \sqrt{2}i}{2} = 1 \pm \frac{\sqrt{2}}{2}i \end{aligned}$$

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$$\text{Q8 Solve the equation } 27x^2 - 10x + 1 = 0$$

Answer.

The given quadratic equation is $27x^2 - 10x + 1 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$$a = 27, b = -10, \text{ and } c = 1$$

Therefore, the discriminant of the given equation is

$$D = b^2 - 4ac = (-10)^2 - 4 \times 27 \times 1 = 100 - 108 = -8$$

Therefore, the required solutions are

$$\begin{aligned} \frac{-b \pm \sqrt{D}}{2a} &= \frac{-(-10) \pm \sqrt{-8}}{2 \times 27} = \frac{10 \pm 2\sqrt{2}i}{54} \\ &= \frac{5 \pm \sqrt{2}i}{27} = \frac{5}{27} \pm \frac{\sqrt{2}}{27}i \end{aligned}$$

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$$\text{Q9 Solve the equation } 21x^2 - 28x + 10 = 0$$

Answer.

The given quadratic equation is $21x^2 - 28x + 10 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$$a = 21, b = -28, \text{ and } c = 10$$

Therefore, the discriminant of the given equation is

$$D = b^2 - 4ac = (-28)^2 - 4 \times 21 \times 10 = 784 - 840 = -56$$

Therefore, the required solutions are

$$\begin{aligned} \frac{-b \pm \sqrt{D}}{2a} &= \frac{-(-28) \pm \sqrt{-56}}{2 \times 21} = \frac{28 \pm \sqrt{56}i}{42} \\ &= \frac{28 \pm 2\sqrt{14}i}{42} = \frac{28}{42} \pm \frac{2\sqrt{14}}{42}i = \frac{2}{3} \pm \frac{\sqrt{14}}{21}i \end{aligned}$$

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$$\text{Q10 if } z_1 = 2 - i, z_2 = 1 + i, \text{ find } \left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + 1} \right|$$

Answer.

$$\begin{aligned}
 z_1 &= 2 - i, z_2 = 1 + i \\
 \therefore \left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + 1} \right| &= \frac{|(2-i) + (1+i) + 1|}{|(2-i) - (1+i) + 1|} \\
 &= \frac{4}{2-2i} = \frac{4}{2(1-i)} \\
 &= \left| \frac{2}{1-i} \times \frac{1+i}{1+i} \right| = \left| \frac{2(1+i)}{1^2 - i^2} \right| \\
 &= \left| \frac{2(1+i)}{1+1} \right| \quad [i^2 = -1] \\
 &= \left| \frac{2(1+i)}{2} \right| \\
 &= |1+i| = \sqrt{1^2 + 1^2} = \sqrt{2} \\
 \left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + 1} \right| &\text{ is } \sqrt{2}
 \end{aligned}$$

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Q11 if $a + ib = \frac{(x+i)^2}{2x^2+1}$, prove that $a^2 + b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$

Answer.

$$\begin{aligned}
 a + ib &= \frac{(x+i)^2}{2x^2+1} \\
 &= \frac{x^2 + i^2 + 2xi}{2x^2+1} \\
 &= \frac{x^2 - 1 + i2x}{2x^2+1} \\
 &= \frac{x^2 - 1}{2x^2+1} + i \left(\frac{2x}{2x^2+1} \right) \\
 \therefore a^2 + b^2 &= \left(\frac{x^2 - 1}{2x^2+1} \right)^2 + \left(\frac{2x}{2x^2+1} \right)^2 \\
 &= \frac{x^4 + 1 - 2x^2 + 4x^2}{(2x^2+1)^2} \\
 &= \frac{x^4 + 1 + 2x^2}{(2x^2+1)^2} \\
 &= \frac{(x^2+1)^2}{(2x^2+1)^2} \\
 \therefore a^2 + b^2 &= \frac{(x^2+1)^2}{(2x^2+1)^2}
 \end{aligned}$$

Hence, proved.

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Q12

Let $z_1 = 2 - i, z_2 = -2 + i$. Find

$$\begin{aligned}
 \text{(i) } \operatorname{Re} \left(\frac{z_1 z_2}{z_1} \right), & \quad \text{(ii) } \operatorname{Im} \left(\frac{1}{z_1 \bar{z}_1} \right)
 \end{aligned}$$

Answer.

$$z_1 = 2 - i, z_2 = -2 + i$$

$$Z_1 I_2 = (2 - i)(-2 + i) = -4 + 2i + 2i - i^2 = -4 + 4i - (-1) = -3 + 4i$$

$$\bar{Z}_1 = 2 + i$$

$$\therefore \frac{Z_1 Z_2}{\bar{Z}_1} = \frac{-3+4i}{2+i}$$

$$\begin{aligned} \frac{z_1 z_2}{\bar{z}_1} &= \frac{(-3 + 4i)(2 - i)}{(2 + i)(2 - i)} = \frac{-6 + 3i + 8i - 4i^2}{2^2 + 1^2} = \frac{-6 + 11i - 4(-1)}{2^2 + 1^2} \\ &= \frac{-2 + 11i}{5} = \frac{-2}{5} + \frac{11}{5}i \end{aligned}$$

On comparing real parts, we obtain

$$\begin{aligned} \operatorname{Re}\left(\frac{z_1 z_2}{\bar{z}_1}\right) &= \frac{-2}{5} \\ \frac{1}{z_1 \bar{z}_1} &= \frac{1}{(2-i)(2+i)} = \frac{1}{(2)^2 + (1)^2} = \frac{1}{5} \end{aligned}$$

On comparing imaginary parts, we obtain

$$\operatorname{Im}\left(\frac{1}{z_1 \bar{z}_1}\right) = 0$$

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Q13 Find the modulus and argument of the complex number $\frac{1+2i}{1-3i}$

Answer.

$$\begin{aligned} z &= \frac{1+2i}{1-3i}, \\ z &= \frac{1+2i}{1-3i} \times \frac{1+3i}{1+3i} = \frac{1+3i+2i+6i^2}{1^2+3^2} = \frac{1+5i+6(-1)}{1+9} \\ &= \frac{-5+5i}{10} = \frac{-5}{10} + \frac{5i}{10} = \frac{-1}{2} + \frac{1}{2}i \end{aligned}$$

$$\text{Let } z = r \cos \theta + ir \sin \theta$$

$$\text{i.e., } r \cos \theta = \frac{-1}{2} \text{ and } r \sin \theta = \frac{1}{2}$$

$$\text{i.e., } r \cos \theta = \frac{-1}{2} \text{ and } r \sin \theta = \frac{1}{2}$$

On squaring and adding, we obtain

$$r^2 (\cos^2 \theta + \sin^2 \theta) = \left(\frac{-1}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$\Rightarrow r^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$\Rightarrow r = \frac{1}{\sqrt{2}} \quad [\text{Conventionally, } r > 0]$$

$$\therefore \frac{1}{\sqrt{2}} \cos \theta = \frac{-1}{2} \text{ and } \frac{1}{\sqrt{2}} \sin \theta = \frac{1}{2}$$

$$\Rightarrow \cos \theta = \frac{-1}{\sqrt{2}} \text{ and } \sin \theta = \frac{1}{\sqrt{2}}$$

$$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \quad [\text{As } \theta \text{ lies in the II quadrant}]$$

Therefore, the modulus and argument of the given complex number are $\sqrt{2}$ and $\frac{3\pi}{4}$ respectively.

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Q14 Find the real number x and y if $(x - iy)(3 + 5i)$ is the conjugate of $-6 - 24i$

Answer.

$$\text{Let } z = (x - iy)(3 + 5i)$$

$$z = 3x + 5xi - 3yi - 5yi^2 = 3x + 5xi - 3yi + 5y = (3x + 5y) + i(5x - 3y)$$

$$\therefore \bar{z} = (3x + 5y) - i(5x - 3y)$$

$$\text{It is given that, } \bar{z} = -6 - 24i$$

$$\therefore (3x + 5y) - i(5x - 3y) = -6 - 24i$$

Equating real and imaginary parts, we obtain

$$3x + 5y = -6$$

$$5x - 3y = 24$$

Multiplying equation (i) by 3 and equation (ii) by 5 and then adding them, we obtain

Putting the value of x in equation (i), we obtain

$$3(3) + 5y = -6$$

$$\Rightarrow 5y = -6 - 15$$

$$\Rightarrow y = -3$$

Thus, the values of x and y are 3 and -3 respectively.

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Q15 Find the modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$

Answer.

$$\begin{aligned} \frac{1+i}{1-i} - \frac{1-i}{1+i} &= \frac{(1+i)^2 - (1-i)^2}{(1-i)(1+i)} \\ &= \frac{1+i^2+2i-1-i^2+2i}{1^2+1^2} \end{aligned}$$

$$\therefore = \frac{4i}{2} = 2i$$

$$\therefore \left| \frac{1+i}{1-i} - \frac{1-i}{1+i} \right| = |2i| = \sqrt{2^2} = 2$$

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Q16

$(x + iy)^3 = u + iv$, then show that $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$

Answer.

$$(x + iy)^3 = u + iv$$

$$\Rightarrow x^3 + (iy)^3 + 3 \cdot x \cdot iy(x + iy) = u + iv$$

$$\Rightarrow x^3 + i^3y^3 + 3x^2yi + 3xy^2i = u + iv$$

$$\Rightarrow x^3 - 4y^3 + 3x^2yi - 3xy^2i = u + iv$$

$$\Rightarrow (x^3 - 3xy^2) + i(3x^2y - y^3) = u + iv$$

$$\begin{aligned}
u &= x^3 - 3xy^2, v = 3x^2y - y^3 \\
\therefore \frac{u}{x} + \frac{v}{y} &= \frac{x^3 - 3xy^2}{x} + \frac{3x^2y - y^3}{y} \\
&= \frac{x(x^2 - 3y^2)}{x} + \frac{y(3x^2 - y^2)}{y} \\
&= x^2 - 3y^2 + 3x^2 - y^2 \\
&= 4(x^2 - 4y^2) \\
&= 4(x^2 - y^2) \\
\therefore \frac{u}{x} + \frac{v}{y} &= 4(x^2 - y^2)
\end{aligned}$$

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Q17 if α and β are different complex numbers with $|\beta| = 1$, then find $\left| \frac{\beta - \alpha}{1 - \bar{\alpha}\beta} \right|$

Answer.

Let $a = a + ib$ and $\beta = x + iy$

It is given that, $|\beta| = 1$

$$\therefore \sqrt{x^2 + y^2} = 1$$

$$\Rightarrow x^2 + y^2 = 1$$

$$\begin{aligned}
\left| \frac{\beta - \alpha}{1 - \bar{\alpha}\beta} \right| &= \left| \frac{(x + iy) - (a + ib)}{1 - (a - ib)(x + iy)} \right| \\
&= \left| \frac{(x - a) + i(y - b)}{1 - (ax + ai y - ibx + by)} \right| \\
&= \left| \frac{(x - a) + i(y - b)}{(1 - ax - by) + i(bx - ay)} \right| \\
&= \frac{|(x-a)+i(y-b)|}{|(1-ax-by)+i(bx-ay)|} \\
&= \frac{\sqrt{(x-a)^2 + (y-b)^2}}{\sqrt{(1-ax-by)^2 + (bx-ay)^2}} \\
&= \frac{\sqrt{x^2 + a^2 - 2ax + y^2 + b^2 - 2by}}{\sqrt{1 + a^2x^2 + b^2y^2 - 2ax + 2abxy - 2by + b^2x^2 + a^2y^2 - 2abxy}} \\
&= \frac{\sqrt{(x^2 + y^2) + a^2 + b^2 - 2ax - 2by}}{\sqrt{1 + a^2(x^2 + y^2) + b^2(y^2 + x^2) - 2ax - 2by}} \\
&= \frac{\sqrt{1 + a^2 + b^2 - 2ax - 2by}}{\sqrt{1 + a^2 + b^2 - 2ax - 2by}} [U \sin g(1)] \\
&= 1 \\
\therefore \left| \frac{\beta - \alpha}{1 - \bar{\alpha}\beta} \right| &= 1
\end{aligned}$$

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Q18 Find the number of non-zero integral solutions of the equation $|1 - i|^x = 2^x$

Answer.

$$|1 - i|^x = 2^x$$

$$\Rightarrow \left(\sqrt{1^2 + (-1)^2} \right)^x = 2^x$$

$$\Rightarrow (\sqrt{2})^x = 2^x$$

$$\Rightarrow 2^{\frac{x}{2}} = 2^x$$

$$\Rightarrow x = 2x$$

$$\Rightarrow 2x - x = 0$$

$$\Rightarrow x = 0$$

Thus, 0 is the only integral solution of the given equation. Therefore, the number of non-integral solutions of the given equation is 0.

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Q19

If $(a + ib)(c + id)(e + if)(g + ih) = A + iB$, then show that

$$(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$$

Answer.

$$(a + ib)(c + id)(e + if)(g + ih) = A + iB$$

$$\therefore |(a + ib)(c + id)(e + if)(g + ih)| = |A + iB|$$

$$\Rightarrow \sqrt{a^2 + b^2} \times \sqrt{c^2 + d^2} \times \sqrt{e^2 + f^2} \times \sqrt{g^2 + h^2} = \sqrt{A^2 + B^2}$$

On squaring both sides, we obtain

$$(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$$

Hence, proved.

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Q20

$$\left(\frac{1+i}{1-i} \right)^m = 1, \text{ then find the least positive integral value of } m$$

Answer.

$$\left(\frac{1+i}{1-i} \right)^m = 1$$

$$\Rightarrow \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i} \right)^m = 1$$

$$\Rightarrow \left(\frac{(1+i)^2}{1^2+1^2} \right)^m = 1$$

$$\Rightarrow \left(\frac{1^2+i^2+2i}{2} \right)^m = 1$$

$$\Rightarrow \left(\frac{1-1+2i}{2} \right)^m = 1$$

$$\Rightarrow \left(\frac{2i}{2} \right)^m = 1$$

$$\Rightarrow i^m = 1$$

$\therefore m = 4k$, where k is some integer.

Therefore, the least positive integer is 1.

Thus, the least positive integral value of m is $4 (= 4 \times 1)$.