NCERT SOLUTIONS

CLASS - 11th



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Class : 11th
Subject : Maths
Chapter : 3
Chapter Name : Trigonometric Functions

Exercise 3.1

Q1 Find the radian measures corresponding to the following degree measures:
(i) 25° (ii) $-47^{\circ}30'$ (iii) 240° (iv) 520°

Answer.
(i) 25°
We know that $180^{\circ} = \pi$ radian
$\therefore 25^{\circ} = \frac{\pi}{180} \times 25$ radian $= \frac{5\pi}{36}$ radian
(ii) $-47^{\circ}30'$
$-47^{\circ}30' = -47\frac{1}{2}$
$= \frac{-95}{2}$
Since $180^{\circ} = \pi$ radian
(iii) 240°
We know that $180^{\circ} = \pi$ radian
$\therefore 240^{\circ} = \frac{\pi}{180} \times 240$ radian $= \frac{4}{3}\pi$ radian
(iv) 520°
We know that $180^{\circ} = \pi$ radian
$\therefore 520^{\circ} = \frac{\pi}{180} \times 520$ radian $= \frac{26\pi}{9}$ radian

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Q2 Find the degree measures corresponding to the following radian measures (Use $\pi = \frac{22}{7}$).
(i) $\frac{11}{16}$ (ii) -4 (iii) $\frac{5\pi}{3}$ (iv) $\frac{7\pi}{6}$

Answer.
(i) $\frac{11}{16}$
We know that π radian $= 180^{\circ}$

$\therefore \frac{11}{16}\text{radain} = \frac{180}{\pi} \times \frac{11}{16} \text{ degree } = \frac{45 \times 11}{\pi \times 4} \text{ deg ree}$

$= \frac{45 \times 11 \times 7}{22 \times 4} \text{ deg ree } = \frac{315}{8} \text{ degree}$

$= 39\frac{3}{8} \text{ deg ree}$

$= 39^\circ + \frac{3 \times 60}{8} \text{ min utes} \quad [1^\circ = 60^\circ]$

$= 39^\circ + 22' + \frac{1}{2} \text{ min utes}$

$= 39^\circ 22' 30'' \quad [1' = 60'']$

$(ii) -4$

We know that π radian $= 180^\circ$

-4 radian $= \frac{180}{\pi} \times (-4)$ deg ree $= \frac{180 \times 7(-4)}{22}$ degree

$= \frac{-2520}{11}$ deg ree $= -229\frac{1}{11}$ degree

$= -229^\circ + \frac{1 \times 60}{11}$ min utes $\quad [1^\circ = 60^\circ]$

$= -229^\circ 5^{12} 27^n \quad [1' = 60'']$

(iii) $\frac{5\pi}{3}$

We know that π radlan $= 180^\circ$

$\therefore \frac{5\pi}{3}$ radian $= \frac{180}{\pi} \times \frac{5\pi}{3}$ degree $= 300^\circ$

(iv) $\frac{7\pi}{6}$

We know that π radian $= 180^\circ$

$\therefore \frac{7\pi}{6}$ radian $= \frac{180}{\pi} \times \frac{7\pi}{6} = 210^\circ$

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Q3 A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

Answer. Number of revolutions made by the wheel in 1 minute = 360
$\therefore$ Number of revolutions made by the wheel in 1 second = $\frac{360}{60} = 6$
Hence, In 6 complete revolutions, it will turn an angle of $6 \times 2\pi$ radian, i.e., 12 π radian
Thus, In one second, the wheel turns and angle of 12 π radian.

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Q4 Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm (Use $\pi = \frac{22}{7}$)

Answer. We know that in a circle of radius r unit, if an arc of length l unit subtends and angle θ radian at the centre, then
$\theta = \frac{1}{r}$
Therefore, for r = 100 cm, l = 22 cm, we have

$\theta = \frac{22}{100}$ radian $= \frac{180}{\pi} \times \frac{22}{100}$ deg ree $= \frac{180 \times 7 \times 22}{22 \times 100}$ deg ree

$= \frac{126}{10}$ deg ree $= 12\frac{3}{5}$ deg ree $= 12^\circ 36^\circ \quad \left[1^\circ = 60'\right]$

Thus, the required angle is $12^\circ 36'$.

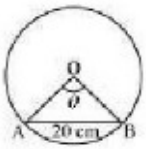
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Q5 In a circle of diameter 40 cm, the length of a circle is 20 cm. Find the length of minor arc of the chord.

Answer. Diameter of the circle = 40 cm

Radius of the circle = $\frac{40}{2}\text{cm} = 20\text{cm}$

Let AB be a chord (length = 20 cm) of the circle.



In $\triangle OAB, OA = OB =$ Radius of circle $= 20\text{cm}$

Also, $AB = 20\text{cm}$

Thus, $\triangle OAB$ is an equilateral triangle.

$\therefore \theta = 60^\circ = \frac{\pi}{3}$

We know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre, then $\theta = \frac{l}{r}$

$\frac{\pi}{3} = \frac{\widetilde{AB}}{20} \Rightarrow \widetilde{AB} = \frac{20\pi}{3}\text{cm}$

Thus, the length of the minor arc of the chord is $\frac{20\pi}{3}\text{cm}$.

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Q6 In the two circles , arcs of the same length subtend angles 60° and 75° at the centre, find the ratio of their radii.

Answer. Let the radii of the two circles be r_1 and r_2 . Let an arc of length l subtend an angle of 60° at the centre of the circle of radius r_1 , while let an arc of length l subtend an angle of 60° at the centre of the circle of radius r_2.

Now, $60^\circ = \frac{\pi}{3}$ radian and $75^\circ = \frac{5\pi}{12}$

We know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre, then $\theta = \frac{l}{r}$ or $l = r\theta$

$\therefore l = \frac{r_1\pi}{3}$ and $l = \frac{r_2 5\pi}{12}$

$\Rightarrow \frac{r_1\pi}{3} = \frac{r_2 5\pi}{12}$

$\Rightarrow r_1 = \frac{r_2 5}{4}$

$\Rightarrow \frac{r_1}{r_2} = \frac{5}{4}$

Thus, the ratio of the radii is 5 : 4.

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Q7 Find the angle in radian through which a pendulum swings if its length is 75 cm and the tip describes an arc of length.
(i) 10 cm
(ii) 15 cm
(iii) 21 cm

Answer. We know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre, then $\theta = \frac{l}{r}$.
It is given that r = 75 cm
(i) Here, l = 10 cm
$\theta = \frac{10}{75}$ radian $= \frac{2}{15}$ radian
(ii) Here, l = 15 cm
$\theta = \frac{15}{75}$ radian $= \frac{1}{5}$ radian
(iii) Here, l = 21 cm
$\theta = \frac{21}{75}$ radian $= \frac{7}{25}$ radian

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Exercise 3.2

Q1 Find the values of other five trigonometric functions in $\cos x = -\frac{1}{2}$, x lies in third quadrant.

Answer.
$\cos x = -\frac{1}{2}$
$\therefore \sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{1}{2}\right)} = -2$
$\sin^2 x + \cos^2 x = 1$
$\Rightarrow \sin^2 x = 1 - \cos^2 x$
$\Rightarrow \sin^2 x = 1 - \left(-\frac{1}{2}\right)^2$
$\Rightarrow \sin^2 x = 1 - \frac{1}{4} = \frac{3}{4}$
$\Rightarrow \sin x = \pm\frac{\sqrt{3}}{2}$
Since x lies in the 3rd quadrant, the value of sin x will be negative.

$\therefore \sin x = -\frac{\sqrt{3}}{2}$

$cosecx = \frac{1}{\sin x} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$

$\tan x = \frac{\sin x}{\cos x} = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\cos x} = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = \sqrt{3}$

$\cot x = \frac{1}{\tan x} = \frac{1}{\sqrt{3}}$

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Q2 Find the values of other five trigonometric functions in $\sin x = \frac{3}{5}$, x lies in second quadrant.

Answer.

$\sin x = \frac{3}{5}$

$cosecx = \frac{1}{\sin x} = \frac{1}{\left(\frac{3}{5}\right)} = \frac{5}{3}$

$\sin^2 x + \cos^2 x = 1$

$\Rightarrow \cos^2 x = 1 - \sin^2 x$

$\Rightarrow \cos^2 x = 1 - \left(\frac{3}{5}\right)^2$

$\Rightarrow \cos^2 x = 1 - \frac{9}{25}$

$\Rightarrow \cos^2 x = \frac{16}{25}$

$\Rightarrow \cos x = \pm\frac{4}{5}$

Since x lies in the 2nd quadrant, the value of cos x will be negative

$\therefore \cos x = -\frac{4}{5}$

$\sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{4}{5}\right)} = -\frac{5}{4}$

$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{3}{5}\right)}{\cos x} = \frac{\left(\frac{3}{5}\right)}{\cos x} = \frac{\left(-\frac{4}{5}\right)}{\left(-\frac{4}{5}\right)} = -\frac{3}{4}$.

$\cot x = \frac{1}{\tan x} = -\frac{4}{3}$

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Q3 Find the values of other five trigonometric functions in $\cot x = \frac{3}{4}$, x lies in third quadrant.

Answer.

$\cot x = \frac{3}{4}$

$\tan x = \frac{1}{\cot x} = \frac{1}{\left(\frac{3}{4}\right)} = \frac{4}{3}$

$1 + \tan^2 x = \sec^2 x$

$\Rightarrow 1+\left(\frac{4}{3}\right)^2=\sec^2 x$

$\Rightarrow 1+\frac{16}{9}=\sec^2 x$

$\Rightarrow \frac{25}{9}=\sec^2 x$

$\Rightarrow \sec x=\pm\frac{5}{3}$

Since, x lies in the 3rd quadrant, the value of sec x will be negative.

$\therefore \sec x=-\frac{5}{3}$

$\cos x=\frac{1}{\sec x}=\frac{1}{\left(-\frac{5}{3}\right)}=-\frac{3}{5}$

$\tan x=\frac{\sin x}{\cos x}$

$\Rightarrow \frac{4}{3}=\frac{\sin x}{\left(\frac{-3}{5}\right)}$

$\Rightarrow \sin x=\left(\frac{4}{3}\right)\times\left(\frac{-3}{5}\right)=-\frac{4}{5}$

$cosecx=\frac{1}{\sin x}=-\frac{5}{4}$

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Q4 Find the values of other five trigonometric functions in $\sec x=\frac{13}{5}$, lies in fourth quadrant.

Answer.

$\sec x=\frac{13}{5}$

$\cos x=\frac{1}{\sec x}=\frac{1}{\left(\frac{13}{5}\right)}=\frac{5}{13}$

$\sin^2 x+\cos^2 x=1$

$\Rightarrow \sin^2 x=1-\cos^2 x$

$\Rightarrow \sin^2 x=1-\left(\frac{5}{13}\right)^2$

$\Rightarrow \sin^2 x=1-\frac{25}{169}=\frac{144}{169}$

$\Rightarrow \sin x=\pm\frac{12}{13}$

Since x lies in the 4th quadrant, the value of sin x will be negative.

$\therefore \sin x=-\frac{12}{13}$

$cosecx=\frac{1}{\sin x}=\frac{1}{\left(-\frac{12}{13}\right)}=-\frac{13}{12}$

$\tan x=\frac{\sin x}{\cos x}=\frac{\left(\frac{-12}{13}\right)}{\cos x}=\frac{\left(\frac{-12}{13}\right)}{\left(\frac{5}{13}\right)}=-\frac{12}{5}$

$\cot x=\frac{1}{\tan x}=\frac{1}{\left(-\frac{12}{5}\right)}=-\frac{12}{5}$

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Q5 Find the values of other five trigonometric functions in $\tan x = -\frac{5}{12}$, x lies in second quadrant.

Answer.
$\tan x = -\frac{5}{12}$
$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{5}{12}\right)} = -\frac{12}{5}$
$1 + \tan^2 x = \sec^2 x$
$\Rightarrow 1 + \left(-\frac{5}{12}\right)^2 = \sec^2 x$
$\Rightarrow 1 + \frac{25}{144} = \sec^2 x$
$\Rightarrow \frac{169}{144} = \sec^2 x$
$\Rightarrow \sec x = \pm\frac{13}{12}$
Since x lies in the 2nd quadrant, the value of sec x will be negative.
$\therefore \sec x = -\frac{13}{12}$
$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{13}{12}\right)} = -\frac{12}{13}$
$\tan x = \frac{\sin x}{\cos x}$
$\Rightarrow -\frac{5}{12} = \frac{\sin x}{\left(-\frac{12}{13}\right)}$
$\Rightarrow \sin x = \left(-\frac{5}{12}\right) \times \left(-\frac{12}{13}\right) = \frac{5}{13}$
$cosecx = \frac{1}{\sin x} = \frac{1}{\left(\frac{5}{13}\right)} = \frac{13}{5}$.

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Q6 Find the values of the trigonometric functions in $\sin 765^\circ$.

Answer. It is known that the values of sin x repeat after an interval of 2π or 360^deg.
$\therefore \sin 765^\circ = \sin(2 \times 360^\circ + 45^\circ) = \sin 45^\circ = \frac{1}{\sqrt{2}}$

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Q7 Find the values of the trigonometric functions in $cosec\,(-1410^\circ)$.

Answer. It is known that the values of cosec x repeat after an interval of 2π or 360^$\circ$.
$\therefore$ cosec $(-1410^\circ) = cosec\,(-1410^\circ + 4 \times 360^\circ)$
$= cosec\,(-1410^\circ + 1440^\circ)$.
$= cosec 30^\circ = 2$

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Q8 Find the values of the trigonometric functions in $\tan \frac{19\pi}{3}$.

Answer. It is known that the values of tan x repeat after an interval of π or 180° .
$\therefore \tan \frac{19\pi}{3}=\tan 6\frac{1}{3}\pi=\tan\left(6\pi+\frac{\pi}{3}\right)=\tan \frac{\pi}{3}=\tan 60^{\circ}=\sqrt{3}.$

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Q9 Find the values of the trigonometric functions in $\sin\left(-\frac{11\pi}{3}\right)$.

Answer. It is known that the values of sin x repeat after an interval of π or 360° .
$\therefore \sin\left(-\frac{11\pi}{3}\right)=\sin\left(-\frac{11\pi}{3}+2\times 2\pi\right)=\sin\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$

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Q10 Find the values of the trigonometric functions in $\cot\left(-\frac{15\pi}{4}\right)$.

Answer. It is known that the values of cot x repeat after an interval of π or 180° .
$\therefore \cot\left(-\frac{15\pi}{4}\right)=\cot\left(-\frac{15\pi}{4}+4\pi\right)=\cot \frac{\pi}{4}=1.$

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Exercise 3.3

Q1 Prove that: $\sin^2 \frac{\pi}{6}+\cos^2 \frac{\pi}{3}-\tan^2 \frac{\pi}{4}=-\frac{1}{2}$

Answer.
$\text{L. H.S. }=\sin^2 \frac{\pi}{6}+\cos^2 \frac{\pi}{3}-\tan^2 \frac{\pi}{4}$
$=\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2-(1)^2$
$=\frac{1}{4}+\frac{1}{4}-1=-\frac{1}{2}$
$=\text{ R.H.S.}$

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Q2 Prove that: $2\sin^2 \frac{\pi}{6}+cosec^2 \frac{7\pi}{6}\cos^2 \frac{\pi}{3}=\frac{3}{2}$

Answer.

$\text{L.H.S. } = 2\sin^2\frac{\pi}{6} + cosec^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$

$= 2\left(\frac{1}{2}\right)^2 + \operatorname{cosec}^2\left(\pi + \frac{\pi}{6}\right)\left(\frac{1}{2}\right)^2$

$= 2 \times \frac{1}{4} + \left(-cosec\frac{\pi}{6}\right)^2\left(\frac{1}{4}\right)$

$= \frac{1}{2} + \frac{4}{4} = \frac{1}{2} + 1 = \frac{3}{2}$

$= \text{ R.H.S.}$

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Q3 Prove that: $\cot^2\frac{\pi}{6} + cosec\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6} = 6$

Answer.

$\text{L.H.S. } = \cot^2\frac{\pi}{6} + cosec\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6}$

$= (\sqrt{3})^2 + \operatorname{cosec}\left(\pi - \frac{\pi}{6}\right) + 3\left(\frac{1}{\sqrt{3}}\right)^2$

$= 3 + \operatorname{cosec}\frac{\pi}{6} + 3 \times \frac{1}{3}$

$= 3 + 2 + 1 = 6$

$= RHS$

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Q4 Prove that: $2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3} = 10$

Answer. $L.\text{H. S} = 2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3}$

$= 2\left\{\sin\left(\pi - \frac{\pi}{4}\right)\right\}^2 + 2\left(\frac{1}{\sqrt{2}}\right)^2 + 2(2)^2$

$= 2\left\{\sin\frac{\pi}{4}\right\}^2 + 2 \times \frac{1}{2} + 8$

$= 2\left(\frac{1}{\sqrt{2}}\right)^2 + 1 + 8$

$= 1 + 1 + 8$

$= 10$

$= \text{ R.H.S}$

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Q5 Find the value of :
(i) $\sin 75^\circ$
(ii) $\tan 15^\circ$

Answer.

$(i)\sin 75^\circ = \sin(45^\circ + 30^\circ)$

$= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$

$[\sin(x+y) = \sin x \cos y + \cos x \sin y]$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$

(ii) $\tan 15^\circ = \tan(45^\circ - 30^\circ)$

$= \frac{\tan 45^\circ - \tan 30^\circ}{1+\tan 45^\circ \tan 30^\circ} \quad \left[\tan(x-y) = \frac{\tan x - \tan y}{1+\tan x \tan y}\right]$

$= \frac{1-\frac{1}{\sqrt{3}}}{1+1\left(\frac{1}{\sqrt{3}}\right)} = \frac{\sqrt{3}-1}{\frac{\sqrt{3}}{\sqrt{3}}}$

$= \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{3+1-2\sqrt{3}}{(\sqrt{3})^2-(1)^2}$

$= \frac{4-2\sqrt{3}}{3-1} = 2-\sqrt{3}$

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Q6 Prove the following: $\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right) - \sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right) = \sin(x+y)$

Answer.

$\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right) - \sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)$

$= \frac{1}{2}\left[2\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)\right] + \frac{1}{2}\left[-2\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)\right]$

$= \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\} + \cos\left\{\left(\frac{\pi}{4}-x\right)-\left(\frac{\pi}{4}-y\right)\right\}\right]$

$\quad + \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\} - \cos\left\{\left(\frac{\pi}{4}-x\right)-\left(\frac{\pi}{4}-y\right)\right\}\right]$

$\begin{bmatrix}\because 2\cos A\cos B = \cos(A+B)+\cos(A-B)\\ -2\sin A\sin B = \cos(A+B)-\cos(A-B)\end{bmatrix}$

$= 2 \times \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-\mathrm{x}\right)+\left(\frac{\pi}{4}-\mathrm{y}\right)\right\}\right]$

$= \cos\left[\frac{\pi}{2}-(\mathrm{x}+\mathrm{y})\right]$

$= \sin(\mathrm{x}+y)$

$= \mathrm{R.H.S}$

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Q7 Prove the following: $\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)} = \left(\frac{1+\tan x}{1-\tan x}\right)^2$

Answer. It is known that

$\tan(A+B) = \frac{\tan A+\tan B}{1-\tan A\tan B}$ and $\tan(A-B) = \frac{\tan A-\tan B}{1+\tan A\tan B}$

$$\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\frac{\left(\frac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}\right)}{\left(\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right)}=\frac{\left(\frac{1+\tan x}{1-\tan x}\right)}{\left(\frac{1-\tan x}{1+\tan x}\right)}=\left(\frac{1+\tan x}{1-\tan x}\right)^2=\text{R. H. S.}$$

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Q8 Prove the following: $\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}=\cot^2 x$

Answer.

$$\begin{aligned}\text{L.H.S.}&=\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}\\&=\frac{[-\cos x][\cos x]}{(\sin x)(-\sin x]}\\&=\frac{-\cos^2 x}{-\cot^2 x}\\&=\cot^2 x\\&=R.H.S\end{aligned}$$

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Q9 Prove the following: $\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]=1$

Answer. L.H.S $=\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$

$=\sin x\cos x[\tan x+\cot x]$

$=\sin x\cos x\left(\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right)$

$=(\sin x\cos x)\left[\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}\right]$

$=1=\text{R. H. S.}$

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Q10 Prove the following: $\sin(n+1)x\sin(n+2)x+\cos(n+1)x\cos(n+2)x=\cos x$

Answer. L.H.S = $\sin(n+1)x\sin(n+2)x+\cos(n+1)x\cos(n+2)x$

$=\frac{1}{2}[2\sin(n+1)x\sin(n+2)x+2\cos(n+1)x\cos(n+2)x]$

$=\frac{1}{2}\begin{bmatrix}\cos\{(n+1)x-(n+2)x\}-\cos\{(n+1)x+(n+2)x\}\\+\cos\{(n+1)x+(n+2)x\}+\cos\{(n+1)x-(n+2)x\}\end{bmatrix}$

$\begin{bmatrix}\because -2\sin A\sin B=\cos(A+B)-\cos(A-B)\\2\cos A\cos B=\cos(A+B)+\cos(A-B)\end{bmatrix}$

$= \frac{1}{2} \times 2\cos\{(n+1)x-(n+2)x\}$
$= \cos(-x) = \cos x = R.HS$

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Q11 Prove the following: $\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right) = -\sqrt{2}\sin x$

Answer. It is known that
$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\cdot\sin\left(\frac{A-B}{2}\right)$
$\therefore$ = $\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)$
$= -2\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)+\left(\frac{3\pi}{4}-x\right)}{2}\right\}\cdot\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)-\left(\frac{3\pi}{4}-x\right)}{2}\right\}$
$= -2\sin\left(\frac{3\pi}{4}\right)\sin x$
$= -2\sin\left(\pi-\frac{\pi}{4}\right)\sin x$
$= -2\sin\frac{\pi}{4}\sin x$
$= -2\times\frac{1}{\sqrt{2}}\times\sin x$
$= -\sqrt{2}\sin x$
$= R.H.S$

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Q12 Prove the following: $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

Answer. It is known that
$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right), \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$
$\therefore$ L. H. S. $= \sin^2 6x - \sin^2 4x$
$= (\sin 6x + \sin 4x)(\sin 6x - \sin 4x)$
$= \left[2\sin\left(\frac{6x+4x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)\right]\left[2\cos\left(\frac{6x+4x}{2}\right)\cdot\sin\left(\frac{6x-4x}{2}\right)\right]$
$= (2\sin 5x\cos x)(2\cos 5x\sin x)$
$= (2\sin 5x\cos 5x)(2\sin x\cos x)$
$= \sin 10x \sin 2x$
= R. H . S.

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Q13 Prove the following: $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

Answer. It is known that

$\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right), \cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$\therefore \mathrm{L.H\cdot S}, =\cos^2 2x-\cos^2 6\mathrm{x}$

$=(\cos 2x+\cos 6x)(\cos 2x-6x)$

$=\left[2\cos\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right]\left[-2\sin\left(\frac{2x+6x}{2}\right)\sin\frac{(2x-6x)}{2}\right]$

$=[2\cos 4x\cos(-2x)][-2\sin 4x\sin(-2x)]$

$=[2\cos 4x\cos 2x][-2\sin 4x(-\sin 2x)]$

$=(2\sin 4x\cos 4x)(2\sin 2x\cos 2x)$

$=\sin 8x\sin 4x$

$=\mathrm{R.H.S.}$

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Q14 Prove the following: $\sin 2x+2\sin 4x+\sin 6x=4\cos^2 x\sin 4x$

Answer.

$\mathrm{LH.S.}=\sin 2x+2\sin 4x+\sin 6x$

$=[\sin 2x+\sin 6x]+2\sin 4x$

$=\left[2\sin\left(\frac{2\mathrm{x}+6\mathrm{x}}{2}\right)\left(\frac{2\mathrm{x}-6\mathrm{x}}{2}\right)\right]+2\sin 4\mathrm{x}$

$\left[\because \sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$

$=2\sin 4x\cos(-2x)+2\sin 4x$

$=2\sin 4x\cos 2x+2\sin 4x$

$=2\sin 4x(\cos 2x+1)$

$=2\sin 4x\left(2\cos^2 x-1+1\right)$

$=2\sin 4x\left(2\cos^2 x\right)$

$=4\cos^2 x\sin 4x$

$=R.H.S$

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Q15 Prove the following: cot 4x (sin 5x + sin 3x) = cot x (sin 5x − sin 3x)

Answer.

$\mathrm{L.H.S}\ =\cot 4x(\sin 5x+\sin 3x)$

$=\frac{\cot 4x}{\sin 4x}\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$

$\left[\because \sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$

$= \left(\frac{\cos 4x}{\sin 4x}\right)[2 \sin 4x \cos x]$

$= 2 \cos 4x \cos x$

$R.4.5. = \cot x(\sin 5x - \sin 3x)$

$= \frac{\cos x}{\sin x}\left[2 \cos\left(\frac{5x+3x}{2}\right) \sin\left(\frac{5x-3x}{2}\right)\right]$

$\left[\because \sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)\right]$

$= \frac{\cos x}{\sin x}[2 \cos 4x \sin x]$

$= 2 \cos 4x \cdot \cos x$

$L \cdot H \cdot S. = R.H.S$

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Q16 Prove the following: $\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\frac{\sin 2x}{\cos 10x}$

Answer. It is known that

$\cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right), \sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$

$\therefore L.H.S = \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x}$

$= \frac{-2 \sin\left(\frac{9x+5x}{2}\right) \cdot \sin\left(\frac{9x-5x}{2}\right)}{2 \cos\left(\frac{17x+3x}{2}\right) \cdot \sin\left(\frac{17x-3x}{2}\right)}$

$= \frac{-2 \sin 7x \cdot \sin 2x}{2 \cos 10x \cdot \sin 7x}$

$= -\frac{\sin 2x}{\cos 10x}$

$= R.H.S$

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Q17 Prove the following: $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$

Answer. It is known that

$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right), \cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$

$\therefore L \cdot H \cdot S. = \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$

$2 \sin\left(\frac{5x+3x}{2}\right) \cdot \cos\left(\frac{5x-3x}{2}\right)$

$2 \cos\left(\frac{5x+3x}{2}\right) \cdot \cos\left(\frac{5x-3x}{2}\right)$

$= \frac{2 \sin 4x \cdot \cos x}{2 \cos 4x \cdot \cos x}$

$= \frac{\sin 4x}{\cos 4x}$

= tan 4x = R.H.S

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Q18 Prove the following: $\frac{\sin x-\sin y}{\cos x+\cos y}=\tan \frac{x-y}{2}$

Answer. It is known that
$\sin A-\sin B=2 \cos \left(\frac{A+B}{2}\right) \sin \left(\frac{A-B}{2}\right), \cos A+\cos B=2 \cos \left(\frac{A+B}{2}\right) \cos \left(\frac{A-B}{2}\right)$
$\therefore =\frac{\sin x-\sin y}{\cos x+\cos y}$
$=\frac{2 \cos \left(\frac{x+y}{2}\right) \cdot \sin \left(\frac{x-y}{2}\right)}{2 \cos \left(\frac{x+y}{2}\right) \cdot \cos \left(\frac{x-y}{2}\right)}$

$$\sin \left(\frac{x-y}{2}\right)$$
$$=\frac{\sin \left(\frac{x-y}{2}\right)}{\cos \left(\frac{x-y}{2}\right)}$$
$$=\tan \left(\frac{x-y}{2}\right)=R.H.S$$

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Q19 Prove the following: $\frac{\sin x+\sin 3x}{\cos x+\cos 3x}=\tan 2x$

Answer. It is known that
$\sin A+\sin B=2 \sin \left(\frac{A+B}{2}\right) \cos \left(\frac{A-B}{2}\right), \cos A+\cos B=2 \cos \left(\frac{A+B}{2}\right) \cos \left(\frac{A-B}{2}\right)$
$\therefore L.H.S.=\frac{\sin x+\sin 3x}{\cos x+\cos 3x}$
$$=\frac{2 \sin \left(\frac{x+3x}{2}\right) \cos \left(\frac{x-3x}{2}\right)}{2 \cos \left(\frac{x+3x}{2}\right) \cos \left(\frac{x-3x}{2}\right)}$$
$$=\frac{\sin 2x}{\cos 2x}$$
$$=\tan 2x$$
$$=R.HS$$

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Q20 Prove the following: $\frac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}=2 \sin x$

Answer. It is known that
$\sin A-\sin B=2 \cos \left(\frac{A+B}{2}\right) \sin \left(\frac{A-B}{2}\right), \cos^2 A-\sin^2 A=\cos 2A$
$\therefore L \cdot H \cdot S=\frac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}$

$$2\cos\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)$$
$$=\frac{2\cos 2x\sin(-x)}{-\cos 2x}$$
$$=-2\times(-\sin x)$$
$$=2\sin x=R.H.S$$

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Q21 Prove the following: $\frac{\cos 4x+\cos 3x+\cos 2x}{\sin 4x+\sin 3x+\sin 2x}=\cot 3x$

Answer. L.H.S $=\frac{\cos 4x+\cos 3x+\cos 2x}{\sin 4x+\sin 3x+\sin 2x}$
$$=\frac{(\cos 4x+\cos 2x)+\cos 3x}{(\sin 4x+\sin 2x)+\sin 3x}$$
$$2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right)+\cos 3x$$
$$=\frac{2\sin(4x+2x}{2})\cos\left(\frac{4x-2x}{2}\right)+\sin 3x$$
$$\left[\because\cos A+\cos B=2\cos\left(\tfrac{A+B}{2}\right)\cos\left(\tfrac{A-B}{2}\right),\sin A+\sin B=2\sin\left(\tfrac{A+B}{2}\right)\cos\left(\tfrac{A-B}{2}\right)\right]$$
$$=\frac{2\cos 3x\cos x+\cos 3x}{2\sin 3x\cos x+\sin 3x}$$
$$=\frac{\cos 3x(2\cos x+1)}{\sin 3x(2\cos x+1)}$$
$$=\cot 3x=R.H.S$$

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Q22 Prove the following: $\cot x\cot 2x-\cot 2x\cot 3x-\cot 3x\cot x=1$

Answer.
$$\text{L. H.S. }=\cot x\cot 2x-\cot 2x\cot 3x-\cot 3x\cot x$$
$$=\cot x\cot 2x-\cot 3x(\cot 2x+\cot x)$$
$$=\cot x\cot 2x-\cot(2x+x)(\cot 2x+\cot x)$$
$$=\cot x\cot 2x-\left[\frac{\cot 2x\cot x-1}{\cot x+\cot 2x}\right](\cot 2x+\cot x)$$
$$\left[\because\cot(A+B)=\frac{\cot A\cot B-1}{\cot A+\cot B}\right]$$
$$=\cot x\cot 2x-(\cot 2x\cot x-1)$$
$$=1=R.H.S$$

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Q23 Prove the following: $\tan 4x=\frac{4\tan x\left(1-\tan^2 x\right)}{1-6\tan^2 x+\tan^4 x}$

Answer. It is known that

$\tan 2\mathrm{A}=\frac{2 \tan \mathrm{A}}{1-\tan ^2 \mathrm{A}}$

$\therefore \mathrm{L} \mathrm{H.S.}=\tan 4 \mathrm{x}=\tan 2(2 \mathrm{x})$

$=\frac{2 \tan 2 \mathrm{x}}{1-\tan ^2(2 \mathrm{x})}$

$=\frac{2\left(\frac{2 \tan \mathrm{x}}{1-\tan ^2 \mathrm{x}}\right)}{1-\left(\frac{2 \tan \mathrm{x}}{1-\tan ^2 \mathrm{x}}\right)^2}$

$=\frac{\left(\frac{4 \tan x}{1-\tan ^2 x}\right)}{1-\frac{4 \tan ^2 x}{\left(1-\tan ^2 x\right)^2}]}$

$=\frac{\left(\frac{4 \tan x}{1-\tan ^2 x}\right)}{\left[\frac{\left(1-\tan ^2 x\right)^2-4 \tan ^2 x}{\left(1-\tan ^2 x\right)^2}\right]}$

$=\frac{4 \tan x\left(1-\tan ^2 x\right)}{1+\tan ^4 x-2 \tan ^2 x-4 \tan ^2 x}$

$=\frac{4 \tan x\left(1-\tan ^2 x\right)}{1-6 \tan ^2 x+\tan ^4 x}=RHS$

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Q24 Prove the following: $\cos 4x=1-8 \sin ^2 x \cos ^2 x$

Answer.

$\mathrm{L.H.S.}=\cos 4x$

$=\cos 2(2x)$

$=1-2 \sin ^2 2x\left[\cos 2A=1-2 \sin ^2 A\right]$

$=1-2\left(2 \sin x \cos ^2 x\right)^2[\sin 2A=2 \sin A \cos A]$

$=1-8 \sin ^2 x \cos ^2 x$

$=\mathrm{R.H.S.}$

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Q25 Prove the following: $\cos 6x=32 \cos ^6 x-48 \cos ^4 x+18 \cos ^2 x-1$

Answer.

$\mathrm{L.H.S.}=\cos 6x$

$=\cos 3(2x)$

$=4 \cos ^3 2x-3 \cos 2x\left[\cos 3A=4 \cos ^3 A-3 \cos A\right]$

$=4\left[\left(2 \cos ^2 x-1\right)^3-3\left(2 \cos ^2 x-1\right)\left[\cos 2x=2 \cos ^2 x-1\right]\right.$

$=4\left[\left(2 \cos ^2 x\right)^3-(1)^3-3\left(2 \cos ^2 x\right)^2+3\left(2 \cos ^2 x\right)\right]-6 \cos ^2 x+3$

$= 4\left[8\cos^6 x - 1 - 12\cos^4 x + 6\cos^2 x\right] - 6\cos^2 x + 3$
$= 32\cos^6 x - 4 - 48\cos^4 x + 24\cos^2 x - 6\cos^2 x + 3$
$= 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$
$= \text{R. H. S}$

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Exercise 3.4

Q1 Find the principal and general solutions of the following equations:
$\tan x = \sqrt{3}$

Answer. $\tan x = \sqrt{3}$
It is known that $\tan \frac{\pi}{3} = \sqrt{3}$ and $\tan\left(\frac{4\pi}{3}\right) = \tan\left(\pi + \frac{\pi}{3}\right) = \tan \frac{\pi}{3} = \sqrt{3}$
Therefore, the principal solutions are x = $\frac{\pi}{3}$ and $\frac{4\pi}{3}$
Now, tan x = $\tan \frac{\pi}{3}$
$\Rightarrow x = n\pi + \frac{\pi}{3},$ where $n \in Z$
Therefore, the general solution is $x = n\pi + \frac{\pi}{3},$ where $n \in Z.$

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Q2 Find the principal and general solutions of the following equations: sec x = 2

Answer. Sec x = 2
It is known that $\sec \frac{\pi}{3} = 2$ and $\sec \frac{5\pi}{3} = \sec\left(2\pi - \frac{\pi}{3}\right) = \sec \frac{\pi}{3} = 2$
Therefore, the principal solutions are x = $\frac{\pi}{3}$ and $\frac{5\pi}{3}$.
Now $_7 \sec x = \sec \frac{\pi}{3}$
$\Rightarrow \cos x = \cos \frac{\pi}{3}$
$\Rightarrow x = 2n\pi \pm \frac{\pi}{3},$ where $n \in Z$
Therefore, the general solution is $x = 2n\pi \pm \frac{\pi}{3},$ where $n \in Z$

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Q3 Find the principal and general solutions of the following equations: cot x = $-\sqrt{3}$

Answer. Cot x = $-\sqrt{3}$
It is known that $\cot \frac{\pi}{6} = \sqrt{3}$

$\therefore \cot\left(\pi - \frac{\pi}{6}\right) = -\cot\frac{\pi}{6} = -\sqrt{3}$ and $\cot\left(2\pi - \frac{\pi}{6}\right) = -\cot\frac{\pi}{6} = -\sqrt{3}$

i.e., $\cot\frac{5\pi}{6} = -\sqrt{3}$ and $\cot\frac{11\pi}{6} = -\sqrt{3}$

Therefore, the principal solutions are x = $\frac{5\pi}{6}$ and $\frac{11\pi}{6}$

Now, $\cot x = \cot\frac{5\pi}{6}$

$\Rightarrow \tan x = \tan\frac{5\pi}{6} \quad \left[\cot x = \frac{1}{\tan x}\right]$

$\Rightarrow x = n\pi + \frac{5\pi}{6}$, where $n \in Z$

Therefore, the general solution is $x = n\pi + \frac{5\pi}{6}$, where $n \in Z$.

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Q4 Find the principal and general solutions of the following equations: cosec x = – 2

Answer. Cosec x = -2

It is known that

$cosec\frac{\pi}{6} = 2$

$\therefore \operatorname{cosec}\left(\pi + \frac{\pi}{6}\right) = -\operatorname{cosec}\frac{\pi}{6} = -2$ and $\operatorname{cosec}\left(2\pi - \frac{\pi}{6}\right) = -\operatorname{cosec}\frac{\pi}{6} = -2$

$cosec\frac{7\pi}{6} = -2$ and $cosec\frac{11\pi}{6} = -2$

Therefore, the principal solutions are x = $\frac{7\pi}{6}$ and $\frac{11\pi}{6}$

Now, cosec x = $\operatorname{cosec}\frac{7\pi}{6}$

$\Rightarrow \sin x = \sin\frac{7\pi}{6} \quad \left[cosecx = \frac{1}{\sin x}\right]$

$\Rightarrow x = \mathbf{n}\pi + (-1)^n\frac{7\pi}{6}$, where $n \in Z$

Therefore, the general solution is $x = n\pi + (-1)^n\frac{7\pi}{6}$, where $n \in Z$.

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Q5 Find the general solution for each of the following equations: cos 4 x = cos 2 x

Answer.

$\cos 4x = \cos 2x$

$\Rightarrow \cos 4x - \cos 2x = 0$

$\Rightarrow -2\sin\left(\frac{4x+2x}{2}\right)\sin\left(\frac{4x-2x}{2}\right) = 0$

$\left[\because \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$

$\Rightarrow \sin 3x \sin x = 0$

$\Rightarrow \sin 3x = 0 \quad$ or $\quad \sin x = 0$

$\therefore 3x = n\pi$

$\Rightarrow x = \frac{n\pi}{3} \quad$ or $\quad x = n\pi$, where $n \in Z$

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Q6 Find the general solution for each of the following equations: cos 3x + cos x – cos 2x = 0

Answer.

$\cos 3x + \cos x - \cos 2x = 0$

$\Rightarrow 2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) - \cos 2x = 0 \quad \left[\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$

$\Rightarrow 2\cos 2x \cos x - \cos 2x = 0$

$\Rightarrow \cos 2x(2\cos x - 1) = 0$

$\Rightarrow \cos 2x = 0 \quad \text{or} \quad 2\cos x - 1 = 0$

$\Rightarrow \cos 2x = 0 \quad \text{or} \quad \cos x = \frac{1}{2}$

$\therefore 2\text{x} = (2\text{n}+1)\frac{\pi}{2} \quad \text{or} \quad \cos \text{x} = \cos\frac{\pi}{3},\ \text{where n} \in Z$

$\Rightarrow \text{x} = (2\text{n}+1)\frac{\pi}{4} \quad \text{or} \quad \text{x} = 2\text{n}\pi \pm \frac{\pi}{3},\ \text{where n} \in Z$

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Q7 Find the general solution for each of the following equations: sin 2x + cos x = 0

Answer.

$\sin 2x + \cos x = 0$

$\Rightarrow 2\sin x \cos x + \cos x = 0$

$\Rightarrow \cos x(2\sin x + 1) = 0$

$\Rightarrow \cos x = 0 \quad \text{or} \quad 2\sin x + 1 = 0$

$\text{Now, } \cos x = 0 \Rightarrow \cos x = (2n+1)\frac{\pi}{2},\ \text{where } n \in Z$

$2\sin x + 1 = 0$

$\Rightarrow \sin x = \frac{-1}{2} = -\sin\frac{\pi}{6} = \sin\left(\pi + \frac{\pi}{6}\right) = \sin\left(\pi + \frac{\pi}{6}\right) = \sin\frac{7\pi}{6}$

$\Rightarrow \text{x} = \text{n}\pi + (-1)^{\text{n}}\frac{7\pi}{6},\ \text{where n} \in Z$

Therefore, the general solution is $(2n+1)\frac{\pi}{2}$ or $n\pi + (-1)^n\frac{7\pi}{6}, n \in Z.$

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Q8 Find the general solution for each of the following equations: $\sec^2 2x = 1 - \tan 2x$

Answer.

$\sec^2 2x = 1 - \tan 2x$

$\Rightarrow 1 + \tan^2 2x = 1 - \tan 2x$

$\Rightarrow \tan^2 2x + \tan 2x = 0$

$\Rightarrow \tan 2x(\tan 2x + 1) = 0$

$\Rightarrow \tan 2x = 0 \quad \text{or} \quad \tan 2x + 1 = 0$

$\text{Now, } \tan 2x = 0$
$\Rightarrow \tan 2x = \tan 0$
$\Rightarrow 2x = n\pi + 0, \text{ where } n \in Z$
$\Rightarrow x = \frac{n\pi}{2}, \text{ where } n \in Z$
$\tan 2x + 1 = 0$
$\Rightarrow \tan 2x = -1 = -\tan \frac{\pi}{4} = \tan\left(\pi - \frac{\pi}{4}\right) = \tan \frac{3\pi}{4}$
$\Rightarrow 2x = n\pi + \frac{3\pi}{4}, \text{ where } n \in Z$
$\Rightarrow x = \frac{n\pi}{2} + \frac{3\pi}{8}, \text{ where } n \in Z$
Therefore, the general solution is $\frac{\mathrm{n}\pi}{2}$ or $\frac{\mathrm{n}\pi}{2} + \frac{3\pi}{8}, \mathrm{n} \in Z.$

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Q9 Find the general solution for each of the following equations: sin x + sin 3x + sin 5x = 0

Answer. sin x + sin 3x + sin 5x = 0
$(\sin x + \sin 5x) + \sin 3x = 0$
$\Rightarrow \left[2 \sin\left(\frac{x+5x}{2}\right) \cos\left(\frac{x-5x}{2}\right)\right] + \sin 3x = 0 \quad \left[\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)\right]$
$\Rightarrow 2 \sin 3x \cos(-2x) + \sin 3x = 0$
$\Rightarrow 2 \sin 3x \cos 2x + \sin 3x = 0$
$\Rightarrow \sin 3x(2 \cos 2x + 1) = 0$
$\Rightarrow \sin 3x = 0 \quad \text{or} \quad 2 \cos 2x + 1 = 0$
$\text{Now, } \sin 3x = 0 \Rightarrow 3x = n\pi, \text{ where } n \in Z$
$\text{i.e., } x = \frac{n\pi}{3}, \text{ where } n \in Z$
$2 \cos 2x + 1 = 0$
$\Rightarrow \cos 2x = \frac{-1}{2} = -\cos \frac{\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right)$
$\Rightarrow \cos 2x = \cos \frac{2\pi}{3}$
$\Rightarrow 2x = 2n\pi \pm \frac{2\pi}{3}, \text{ where } n \in Z$
$\Rightarrow x = n\pi \pm \frac{\pi}{3}, \text{ where } n \in Z$
Therefore, the general solution is $\frac{\mathrm{n}\pi}{3}$ or $\mathrm{n}\pi \pm \frac{\pi}{3}, \mathrm{n} \in Z$

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Miscellaneous Exercise

Q1 Prove that: $2 \cos \frac{\pi}{13} \cos \frac{9\pi}{13} + \cos \frac{3\pi}{13} + \cos \frac{5\pi}{13} = 0$

Answer.

$$
\begin{aligned}
&\text{L.H.S.}\\
&=2\cos\tfrac{\pi}{13}\cos\tfrac{9\pi}{13}+\cos\tfrac{5\pi}{13}+\cos\tfrac{5\pi}{13}\\
&=2\cos\tfrac{\pi}{13}\cos\tfrac{9\pi}{13}+2\cos\left(\frac{\frac{3\pi}{13}+\frac{5\pi}{13}}{2}\right)\cos\left(\frac{\frac{3\pi}{13}-\frac{5\pi}{13}}{2}\right)\left[\cos x+\cos y=2\cos\left(\tfrac{x+y}{2}\right)\cos\left(\tfrac{x-y}{2}\right)\right]\\
&=2\cos\tfrac{\pi}{13}\cos\tfrac{9\pi}{13}+2\cos\tfrac{4\pi}{13}\cos\left(\tfrac{-\pi}{13}\right)\\
&=2\cos\tfrac{\pi}{13}\cos\tfrac{9\pi}{13}+2\cos\tfrac{4\pi}{13}\cos\tfrac{\pi}{13}\\
&=2\cos\tfrac{\pi}{13}\left[\cos\tfrac{9\pi}{13}+\cos\tfrac{4\pi}{13}\right]\\
&=2\cos\tfrac{\pi}{13}\left[2\cos\left(\tfrac{9\pi}{13}+\tfrac{4\pi}{13}\right)\cos\left(\frac{\frac{9\pi}{13}-\frac{4\pi}{13}}{2}\right)\right]\\
&=2\cos\tfrac{\pi}{13}\left[2\cos\tfrac{\pi}{2}\cos\tfrac{5\pi}{26}\right]\\
&=2\cos\tfrac{\pi}{13}\times2\times0\times\cos\tfrac{5\pi}{26}\\
&=0=\text{R. H. S}
\end{aligned}
$$

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Q2 Prove that: (sin 3x + sin x) sin x + (cos 3x − cos x) cos x = 0

Answer.

$$
\begin{aligned}
&\text{L.H.S.}\\
&=(\sin3x+\sin x)\sin x+(\cos3x-\cos x)\cos x\\
&=\sin3x\sin x+\sin^2x+\cos3x\cos x-\cos^2x\\
&=\cos3x\cos x+\sin3x\sin x-\left(\cos^2x-\sin^2x\right)\\
&=\cos(3x-x)-\cos2x\quad[\cos(A-B)=\cos A\cos B+\sin A\sin B]\\
&=\cos2x-\cos2x\\
&=0\\
&=R.H.S
\end{aligned}
$$

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Q3 Prove that: $(\cos x+\cos y)^2+(\sin x-\sin y)^2=4\cos^2\frac{x+y}{2}$

Answer.

$$
\begin{aligned}
&\text{L.H.S.}\ =(\cos x+\cos y)^2+(\sin x-\sin y)^2\\
&=\cos^2x+\cos^2y+2\cos x\cos y+\sin^2x+\sin^2y-2\sin x\sin y\\
&=\left(\cos^2x+\sin^2x\right)+\left(\cos^2y+\sin^2y\right)+2(\cos x\cos y-\sin x\sin y)\\
&=1+1+2\cos(x+y)\\
&=2+2\cos(x+y)
\end{aligned}
\qquad[\cos(A+B)=(\cos A\cos B-\sin A\sin B)]
$$

$= 2[1 + \cos(\mathrm{x} + \mathrm{y})]$

$= 2\left[1 + 2\cos^2\left(\dfrac{\mathrm{x} + \mathrm{y}}{2}\right) - 1\right] \quad \left[\cos 2\mathrm{A} = 2\cos^2 \mathrm{A} - 1\right]$

$= 4\cos^2\left(\dfrac{\mathrm{x} + \mathrm{y}}{2}\right) = \mathrm{R.\,H.\,S}$

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Q4 Prove that: $(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2 \frac{x-y}{2}$

Answer. L.H.S $= (\cos x - \cos y)^2 + (\sin x - \sin y)^2$

$= \cos^2 x + \cos^2 y - 2\cos x \cos y + \sin^2 x + \sin^2 y - 2\sin x \sin y$

$= \left(\cos^2 x + \sin^2 x\right) + \left(\cos^2 y + \sin^2 y\right) - 2[\cos x \cos y + \sin x \sin y]$

$= 1 + 1 - 2[\cos(x - y)] \quad [\cos(A - B) = \cos A \cos B + \sin A \sin B]$

$= 2[1 - \cos(x - y)]$

$= 2\left[1 - \left\{1 - 2\sin^2\left(\frac{x-y}{2}\right)\right\}\right] \quad \left[\cos 2A = 1 - 2\sin^2 A\right]$

$= 4\sin^2\left(\frac{x-y}{2}\right) = R.\,H.\,S$

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Q5 Prove that: sin x + sin 3x + sin 5x + sin 7x = 4 cos x cos 2x sin 4x

Answer. It is known that $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)$

$\mathrm{L.H.S.} = \sin x + \sin 3x + \sin 5x + \sin 7x$

$= (\sin x + \sin 5x) + (\sin 3x + \sin 7x)$

$= 2\sin\left(\frac{x+5x}{2}\right) \cdot \cos\left(\frac{x-5x}{2}\right) + 2\sin\left(\frac{3x+7x}{2}\right)\cos\left(\frac{3x-7x}{2}\right)$

$= 2\sin 3x \cos(-2x) + 2\sin 5x \cos(-2x)$

$= 2\sin 3x \cos 2x + 2\sin 5x \cos 2x$

$= 2\cos 2x[\sin 3x + \sin 5x]$

$= 2\cos 2x\left[2\sin\left(\frac{3x+5x}{2}\right) \cdot \cos\left(\frac{3x-5x}{2}\right)\right]$

$= 2\cos 2x[2\sin 4x \cdot \cos(-x)]$

$= 4\cos 2x \sin 4x \cos x = \mathrm{R.H.S.}$

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Q6 Prove that: $\frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} = \tan 6x$

Answer. It is known that

$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right), \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)$

$$\text{L.H.S} = \frac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)}$$
$$= \frac{\left[2\sin\left(\frac{7x+5x}{2}\right)\cdot\cos\left(\frac{7x-5x}{2}\right)\right]+\left[2\sin\left(\frac{9x+3x}{2}\right)\cdot\cos\left(\frac{9x-3x}{2}\right)\right]}{\left[2\cos\left(\frac{7x+5x}{2}\right)-\cos\left(\frac{7x-5x}{2}\right)\right]+\left[2\cos\left(\frac{9x+3x}{2}\right)\cdot\cos\left(\frac{9x-3x}{2}\right)\right]}$$
$$= [2\sin 6x\cdot\cos x]+[2\sin 6x\cdot\cos 3x]$$
$$= [2\cos 6x\cdot\cos x]+[2\cos 6x\cdot\cos 3x]$$
$$= \frac{2\sin 6x[\cos x+\cos 3x]}{2\cos 6x[\cos x+\cos 3x]}$$
$$= \tan 6x$$
$$= \text{ R.H.S.}$$

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Q7 Prove that: $\sin 3x+\sin 2x-\sin x = 4\sin x\cos\frac{x}{2}\cos\frac{3x}{2}$

Answer. $\text{L.H.S} = \sin 3x+\sin 2x-\sin x$
$$= \sin 3x+(\sin 2x-\sin x)$$
$$= \sin 3x+\left[2\cos\left(\frac{2x+x}{2}\right)\sin\left(\frac{2x-x}{2}\right)\right] \quad \left[\sin A-\sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$
$$= \sin 3x+\left[2\cos\left(\frac{3x}{2}\right)\sin\left(\frac{x}{2}\right)\right]$$
$$= \sin 3x+2\cos\frac{3x}{2}\sin\frac{x}{2}$$
$$= 2\sin\frac{3x}{2}\cdot\cos\frac{3x}{2}+2\cos\frac{3x}{2}\sin\frac{x}{2} \quad [\sin 2A = 2\sin A\cdot\cos B]$$
$$= 2\cos\left(\frac{3x}{2}\right)\left[\sin\left(\frac{3x}{2}\right)+\sin\left(\frac{x}{2}\right)\right]$$
$$= 2\cos\left(\frac{3x}{2}\right)\left[2\sin\left\{\frac{\left(\frac{3x}{2}\right)+\left(\frac{x}{2}\right)}{2}\right\}\cos\left\{\frac{\left(\frac{3x}{2}\right)-\left(\frac{x}{2}\right)}{2}\right\}\right] \left[\left[\sin A+\sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]\right.$$
$$= 2\cos\left(\frac{3\text{x}}{2}\right)\cdot 2\sin\text{x}\cos\left(\frac{\text{x}}{2}\right)$$
$$= 4\sin\text{x}\cos\left(\frac{\text{x}}{2}\right)\cos\left(\frac{3\text{x}}{2}\right) = R.H.S$$

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Q8 Find $\sin\frac{x}{2}, \cos\frac{x}{2}$ and $\tan\frac{x}{2}$ in $\tan' x = -\frac{4}{3}$, x in quadrant II

Answer. Here, x is in quadrant II.
$$\frac{\pi}{2} < x < \pi$$
$$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$
Therefore, $\sin\frac{x}{2}, \cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are all positive.
It is given that $\tan x = -\frac{4}{3}$

$\sec^2 x = 1 + \tan^2 x = 1 + \left(\frac{-4}{3}\right)^2 = 1 + \frac{16}{9} = \frac{25}{9}$

$\therefore \cos^2 x = \frac{9}{25}$

$\Rightarrow \cos x = \pm\frac{3}{5}$

As x is in quadrant II, cos x is negative.

$\cos x = \frac{-3}{5}$

$\text{Now, } \cos x = 2\cos^2 \frac{x}{2} - 1$

$\Rightarrow \frac{-3}{5} = 2\cos^2 \frac{x}{2} - 1$

$\Rightarrow 2\cos^2 \frac{x}{2} = 1 - \frac{3}{5}$

$\Rightarrow 2\cos^2 \frac{x}{2} = \frac{2}{5}$

$\Rightarrow \cos^2 \frac{x}{2} = \frac{1}{5}$

$\Rightarrow \cos \frac{x}{2} = \frac{1}{\sqrt{5}} \qquad \because \cos \frac{x}{2} \text{ is positive }]$

$\therefore \cos \frac{x}{2} = \frac{\sqrt{5}}{5}$

$\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} = 1$

$\Rightarrow \sin^2 \frac{x}{2} + \left(\frac{1}{\sqrt{5}}\right)^2 = 1$

$\Rightarrow \sin^2 \frac{x}{2} = 1 - \frac{1}{5} = \frac{4}{5}$

$\Rightarrow \sin \frac{x}{2} = \frac{2}{\sqrt{5}} \quad \left[\because \sin \frac{x}{2} \text{ is positive }\right]$

$\therefore \sin \frac{x}{2} = \frac{2\sqrt{5}}{5}$

$\tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \frac{\left(\frac{2}{\sqrt{5}}\right)}{\left(\frac{1}{\sqrt{5}}\right)} = 2$

Thus, the respective values of $\sin \frac{x}{2}, \cos \frac{x}{2} \text{ and } \tan \frac{x}{2} are \frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5}, \text{ and } 2$

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Q9 Find $\sin \frac{x}{2}, \cos \frac{x}{2} \text{ and } \tan \frac{x}{2}$ in $\cos x = -\frac{1}{3}, x$ in quadrant III.

Answer. Here, x is in quadrant III.

$\text{i.e., } \pi < \text{x} < \frac{3\pi}{2}$

$\Rightarrow \frac{\pi}{2} < \frac{\text{x}}{2} < \frac{3\pi}{4}$

Therefore,

$\cos \frac{x}{2} and \tan \frac{x}{2}$ are negative, whereas $\sin \frac{x}{2}$ is positive.

It is given that $\cos x = -\frac{1}{3}$.

$\cos x = 1 - 2\sin^2 \frac{x}{2}$

$\Rightarrow \sin^2 \frac{x}{2} = \frac{1 - \cos x}{2}$

$\Rightarrow \sin^2 \frac{x}{2} = \frac{1 - \left(-\frac{1}{3}\right)}{2} = \frac{\left(1 + \frac{1}{3}\right)}{2} = \frac{3}{2} = \frac{2}{3}$

$\Rightarrow \sin \frac{x}{2}=\frac{\sqrt{2}}{\sqrt{3}} \quad\left[\because \sin \frac{x}{2} \text{ is positive }\right]$

$\therefore \sin \frac{x}{2}=\frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{6}}{3}$

$\cos x=2 \cos ^{2} \frac{x}{2}-1$

$\Rightarrow \cos ^{2} \frac{x}{2}=\frac{1+\cos x}{2}=\frac{1+\left(-\frac{1}{3}\right)}{2}=\frac{\left(\frac{3-1}{3}\right)}{2}=\frac{\left(\frac{2}{3}\right)}{2}=\frac{1}{3}$

$\Rightarrow \cos \frac{x}{2}=-\frac{1}{\sqrt{3}}$

$\therefore \cos \frac{x}{2}=-\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}=\frac{-\sqrt{3}}{3}$

$\tan \frac{x}{2}=\frac{\sin \frac{x}{2}}{\cos \frac{x}{2}}=\frac{\left(\frac{\sqrt{2}}{\sqrt{3}}\right)}{\left(\frac{-1}{\sqrt{3}}\right)}=-\sqrt{2}$

Thus, the respective values of $\sin \frac{x}{2}, \cos \frac{x}{2}$ and $\tan \frac{x}{2}$ are $\frac{\sqrt{6}}{3}, \frac{-\sqrt{3}}{3}$, and $-\sqrt{2}$.

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Q10 Find $\sin \frac{x}{2}, \cos \frac{x}{2}$ and $\tan \frac{x}{2}$ in $\sin x=\frac{1}{4}$, x is in quadrant II.

Answer. Here, x is in quadrant II.

i.e., $\frac{\pi}{2}<x<\pi$

$\Rightarrow \frac{\pi}{4}<\frac{x}{2}<\frac{\pi}{2}$

Therefore,

$\sin \frac{x}{2}, \cos \frac{x}{2}$ and $\tan \frac{x}{2}$ are all positive.

It is given that $\sin x=\frac{1}{4}$

$\cos ^{2} x=1-\sin ^{2} x=1-\left(\frac{1}{4}\right)^{2}=1-\frac{1}{16}=\frac{15}{16}$

$\Rightarrow \cos x=-\frac{\sqrt{15}}{4}[\cos x \text{ is negative in quadrant II }]$

$\sin ^{2} \frac{x}{2}=\frac{1-\cos x}{2}=\frac{1-\left(-\frac{\sqrt{15}}{4}\right)}{2}=\frac{4+\sqrt{15}}{8}$

$\Rightarrow \sin \frac{x}{2}=\sqrt{\frac{4+\sqrt{15}}{8}} \quad\left[\because \sin \frac{x}{2} \text{ is positive }\right]$

$=\sqrt{\frac{4+\sqrt{15}}{8} \times \frac{2}{2}}$

$=\sqrt{\frac{8+2 \sqrt{15}}{16}}$

$=\frac{\sqrt{8+2 \sqrt{15}}}{4}$

$\cos ^{2} \frac{x}{2}=\frac{1+\cos x}{2}=\frac{1+\left(-\frac{\sqrt{15}}{4}\right)}{2}=\frac{4-\sqrt{15}}{8}$

$\Rightarrow \cos \frac{x}{2}=\sqrt{\frac{4-\sqrt{15}}{8}} \quad\left[\because \cos \frac{x}{2} \text{ is positive }\right]$

$= \sqrt{\frac{4-\sqrt{15}}{8} \times \frac{2}{2}}$

$= \sqrt{\frac{8-2\sqrt{15}}{16}}$

$= \frac{\sqrt{8-2\sqrt{15}}}{4}$

$\tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \frac{\left(\frac{\sqrt{8+2\sqrt{15}}}{4}\right)}{\frac{\sqrt{8-2\sqrt{15}}}{4}}) = \frac{\sqrt{8+2\sqrt{15}}}{\sqrt{8-2\sqrt{15}}}$

$= \sqrt{\frac{8+2\sqrt{15}}{8-2\sqrt{15}} \times \frac{8+2\sqrt{15}}{8+2\sqrt{15}}}$

$= \sqrt{\frac{(8+2\sqrt{15})^2}{64-60}} = \frac{8+2\sqrt{15}}{2} = 4+\sqrt{15}$

Thus, the respective values of $\sin \frac{x}{2}, \cos \frac{x}{2}$ and $\tan \frac{x}{2}$ are $\frac{\sqrt{8+2\sqrt{15}}}{4}, \frac{\sqrt{8-2\sqrt{15}}}{4}$, and $4+\sqrt{15}$.

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