NCERT SOLUTIONS

CLASS - 11th

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Class : 11th
Subject : Maths
Chapter : 4
Chapter Name : Principle of Mathematical Induction

Exercise 4.1

Q1 Prove the following by using the principle of mathematical induction for all n ∈ N:
$1+3+3^2+\ldots+3^{a-1}=\frac{(3^n-1)}{2}$

Answer.
Let the given statement be $P(n),$ i.e.,
$P(n): 1+3+3^2+\ldots+3^{n-1}=\frac{(3^n-1)}{2}$, which is true
For $n=1,$ we have
$P(1): 1=\frac{(3'-1)}{2}=\frac{3-1}{2}=\frac{2}{2}=1$
Let $P(k)$ be true for some positive integer $k,$ i.e.,
$1+3+3^2+\ldots+3^{k-1}=\frac{\left(3^k-1\right)}{2}$
We shall now prove that $P(k+1)$ is true.
Consider
$1+3+3^2+\ldots+3^{k-1}+3^{(k+1)-1}$
$=\left(1+3+3^2+\ldots+3^{k-1}\right)+3^k$
$=\frac{(3^t-1)}{2}+3^k$
$=\frac{(3^t-1)+2.3^k}{2}$ [using (i)]
$=\frac{(1+2)3^k-1}{2}$
$=\frac{3.3^k-1}{2}$
$=\frac{3^{k+1}-1}{2}$
Thus, $P(k+1)$ is true whenever $P(k)$ is true.
Hence, by the principle of mathematical induction, statement $P(n)$ is true for all natural numbers l.e.,

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Q2 Prove the following by using the principle of mathematical induction for all n ∈ N:

$1^3+2^3+3^3+\ldots+n^3=\left(\frac{n(n+1)}{2}\right)^2$

Answer.

Let the given statement be $P(n),$ i.e.,

$1^3+2^3+3^3+\ldots+n^3=\left(\frac{n(n+1)}{2}\right)^2$

For $n=1,$ we have

For $n=1,$ we have

$P(1): 1^3=1=\underline{\left(1(1+1)\right.}^2=\left(\frac{1.2}{2}\right)^2=1^2=1$

, which is true.

Let $P(k)$ be true for some positive integer $k,$ l.e.,

$1^3+2^3+3^3+\ldots\ldots+k^3=\left(\frac{k(k+1)}{2}\right)^2$

We shall now prove that $P(k+1)$ is true.

Consider

$1^3+2^3+3^3+\ldots+k^3+(k+1)^3$

$=\left(\frac{k(k+1)}{2}\right)^2+(k+1)^3 \quad [\text{ Using } (i)]$

$=\frac{k^2(k+1)^2}{4}+(k+1)^3$

$=\frac{k^2(k+1)^2+4(k+1)^3}{4}$

$=\frac{(k+1)^2\left\{k^2+4(k+1)\right\}}{4}$

$=\frac{(k+1)^2\{k^2+4k+4\}}{4}$

$=\frac{(k+1)^2(k+2)^2}{4}$

$=\frac{(k+1)^2(k+1+1)^2}{4}$

$=\left(1^3+2^3+3^3+\ldots.+k^3\right)+(k+1)^3=\left(\frac{(k+1)(k+1+1)}{2}\right)^2$

Thus, $P(k+1)$ is true whenever $P(k)$ is true.

Hence, by the principle of mathematical induction, statement $P(n)$ is true for all natural numbers l.e.,

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Q3 Prove the following by using the principle of mathematical induction for all n ∈ N:

$1+\frac{1}{(1+2)}+\frac{1}{(1+2+3)}+\ldots+\frac{1}{(1+2+3+\ldots n)}=\frac{2n}{(n+1)}$

Answer.

Let the given statement be $P(n),$ i.e.,

$\mathrm{P(n)}: 1+\frac{1}{1+2}+\frac{1}{1+2+3}+\ldots+\frac{1}{1+2+3+\ldots n}=\frac{2n}{n+1}$

For $n=1,$ we have

$P(1): 1=\frac{2.1}{1+1}=\frac{2}{2}=1$

Let $P(k)$ be true for some positive integer $k,$ i.e.,

$1+\frac{1}{1+2}+\ldots+\frac{1}{1+2+3}+\ldots+\frac{1}{1+2+3+\ldots+k}=\frac{2k}{k+1}$.....(i)

We shall now prove that $\mathrm{P}(k+1)$ is true.

Consider

$1+\frac{1}{1+2}+\frac{1}{1+2+3}+\ldots+\frac{1}{1+2+3+\ldots+k}+\frac{1}{1+2+2+k+(k+1)}$

$=\left(1+\frac{1}{1+2}+\frac{1}{1+2+3}+\ldots+\frac{1}{1+2+3+.k}\right)+\frac{1}{1+2+3+\ldots+k+(k+1)}$

$=\frac{2k}{k+1}+\frac{1}{1+2+3+\ldots+k+(k+1)} \quad [\text{ Using (i) }]$

$=\frac{2k}{k+1}+\frac{1}{\left(\frac{(k+1)(k+1+1)}{2}\right)} \quad \left[1+2+3+\ldots+n=\frac{n(n+1)}{2}\right]$

$=\frac{2k}{(k+1)}+\frac{2}{(k+1)(k+2)}$

$=\frac{2}{(k+1)}\left(k+\frac{1}{k+2}\right)$

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Q4 Prove the following by using the principle of mathematical induction for all n ∈ N:

$1.2.3+2.3.4+\ldots+n(n+1)(n+2)=\frac{n(n+1)(n+2)(n+3)}{4}$

Answer.

Let the given statement be $P(n),$ i.e.,

$P(n): 1.2.3+2.3.4+\ldots+n(n+1)(n+2)=\frac{n(n+1)(n+2)(n+3)}{4}$

For $n=1,$ we have

$P(1): 1.2\cdot 3=6=\frac{1(1+1)(1+2)(1+3)}{4}=\frac{1.23.4}{4}=6$, which is true

Let P(k) be true for some positive integer k, i.e.,

$1.2.3+2.3.4+\ldots+k(k+1)(k+2)=\frac{k(k+1)(k+2)(k+3)}{4}$ .....(i)

We shall now prove that $P(k+1)$ is true.

Consider

$1.2.3+2.3.4+\ldots+k(k+1)(k+2)+(k+1)(k+2)(k+3)$

$=\{1.2.3+2.3.4+\ldots+k(k+1)(k+2)\}+(k+1)(k+2)(k+3)$

$= \frac{k(k+1)(k+2)(k+3)}{4} + (k+1)(k+2)(k+3) \quad [\text{ Using } (i)]$

$= (k+1)(k+2)(k+3)\left(\frac{k}{4}+1\right)$

$= \frac{(k+1)(k+2)(k+3)(k+4)}{4}$

$= \frac{(k+1)(k+1+1)(k+1+2)(k+1+3)}{4}$

Thus, $P(k+1)$ is true whenever $P(k)$ is true.

Hence, by the principle of mathematical induction, statement $P(n)$ is true for all natural numbers l.e., n .

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Q5 Prove the following by using the principle of mathematical induction for all n ∈ N:

$1.3 + 2.3^2 + 3.3^3 + \ldots + n3^n = \frac{(2n-1)3^{n+1}+3}{4}$

Answer.

Let the given statement be $P(n),$ i.e.,

$1.3 + 2.3^2 + 3.3^\circ + \ldots + n3^n = \frac{(2n-1)3^{n+1}+3}{4}$

$P(n)$:

For $n = 1,$ we have

$P(1) : 1.3 = 3 \quad = \frac{(2.1-1)3^{1+1}+3}{4} = \frac{3^2+3}{4} = \frac{12}{4} = 3$

Let $P(k)$ be true for some positive integer $k,$ i.e.,

$1.3 + 2.3^2 + 3.3^3 + \ldots + k3^4 = \frac{(2k-1)3^{k+1}+3}{4}$ .....(i)

We shall now prove that $P(k+1)$ is true.

Consider

$1.3 + 2.3^2 + 3.3^3 + \ldots + k3^k + (k+1)3^{k+1}$

$= \left(1.3 + 2.3^2 + 3.3^3 + \ldots + k.3^k\right) + (k+1)3^{k+1}$

$= \frac{(2k-1)3^{k+1}+3}{4} + (k+1)3^{k+1} \quad [\text{ Using } (\mathbf{i})]$

$= \frac{(2k-1)3^{3+1}+3+4(k+1)3^{k+1}}{4}$

$= \frac{3^{t+1}\{2k-1+4(k+1)\}+3}{4}$

$= \frac{3^{k+1}\{6k+3\}+3}{4}$

$= \frac{3^{k+1}\cdot 3\{2k+1\}+3}{4}$

$= \frac{3^{(k+1)+1}\{2k+1\}+3}{4}$

$= \frac{\{2(k+1)-1\}3^{(k+1)+1}+3}{4}$

Thus, P(k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers

i.e., n.

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Q6 Prove the following by using the principle of mathematical induction for all n ∈ N:
$1.2 + 2.3 + 3.4 + \ldots + n.\,(n+1) = \left[\frac{n(n+1)(n+2)}{3}\right]$

Answer. Let the given statement be P(n), i.e.,
P(n) : $1.2 + 2.3 + 3.4 + \ldots + n \cdot (n+1) = \left[\frac{n(n+1)(n+2)}{3}\right]$
For n = 1, we have
P(1): $1.2 = 2 = \frac{1(1+1)(1+2)}{3} = \frac{1.2.3}{3} = 2$which is true.
Let P(k) be true for some positive integer k, i.e.,
$1.2 + 2.3 + 3.4 + \ldots. + k.\,(k+1) = \left[\frac{k(k+1)(k+2)}{3}\right]$ .....(i)
We shall now prove that P(k + 1) is true.
Consider
1.2 + 2.3 + 3.4 + ... + k.(k + 1) + (k + 1).(k + 2)
= [1.2 + 2.3 + 3.4 + ... + k.(k + 1)] + (k + 1).(k + 2)
$= \frac{k(k+1)(k+2)}{3} + (k+1)(k+2) \quad [\text{ Using (i)}]$
$= (k+1)(k+2)\left(\frac{k}{3}+1\right)$
$= \frac{(k+1)(k+2)(k+3)}{3}$
$= \frac{(k+1)(k+1+1)(k+1+2)}{3}$
Thus, P(k + 1) is true whenever P(k) is true.
Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q7 Prove the following by using the principle of mathematical induction for all n ∈ N:
$1.3 + 3.5 + 5.7 + \ldots + (2n-1)(2n+1) = \frac{n\left(4n^2+6n-1\right)}{3}$

Answer. Let the given statement be P(n), i.e.,
P(n) : $1.3 + 3.5 + 5.7 + \ldots + (2n-1)(2n+1) = \frac{n\left(4n^2+6n-1\right)}{3}$
$\text{For } n = 1, \text{ we have}$
$P(1) : 1.3 = 3 = \frac{1\left(4.1^2+6.1-1\right)}{3} = \frac{4+6-1}{3} = \frac{9}{3} = 3$, which is true
Let P(k) be true for sorne positive integer k, i.e.,
$1.3 + 3.5 + 5.7 + \ldots. + (2k-1)(2k+1) = \frac{k\left(4k^2+6k-1\right)}{3}$ .....(i)
$\text{We shall now prove that } P(k+1) \text{ is true.}$
Consider
$(1.3 + 3.5 + 5.7 + \ldots + (2k-1)(2k+1) + \{2(k+1)-1\}\{2(k+1)+1\}$

$= \frac{k(4k^2+6k-1)}{3} + (2k+2-1)(2k+2+1) \quad [\text{ Using (i) }]$
$= \frac{k(4k^2+6k-1)}{3} + (2k+1)(2k+3)$
$= \frac{k(4k^2+6k-1)}{3} + \left(4k^2+8k+3\right)$
$= \frac{k(4k^2+6k-1)+3(4k^2+8k+3)}{3}$
$= \frac{4k^3+6k^2-k+12k^2+24k+9}{3}$
$= \frac{4k^3+18k^2+93k+9}{3}$
$= \frac{4k^3+14k^2+9k+4k^2+14k+9}{3}$
$= \frac{k(4k^2+14k+9)+1(4k^2+14k+9)}{3}$
$= \frac{(k+1)(4k^2+14k+9)}{3}$
$= \frac{(k+1)\{4k^2+8k+4+6k+6-1\}}{3}$
$= \frac{(k+1)\{4(k^2+2k+1)+6(k+1)-1\}}{3}$
$= \frac{(k+1)\{4(k+1)^2+6(k+1)-1\}}{3}$

Thus, P(k + 1) is true whenever P(k) is true.
Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q8 Prove the following by using the principle of mathematical induction for all n ∈ N:
$1.2 + 2.2^2 + 3.2^3 + \ldots + n.2^{\circ} = (n-1)2^{n+1} + 2$

Answer. Let the given statement be P(n), i.e.,
$P(n) : 1.2 + 2.2^2 + 3.2^2 + \ldots + n.2^n = (n-1)2^{n+1} + 2$
For n = 1, we have
$P(1) : 1.2 = 2 = (1-1)2^{1+1} + 2 = 0 + 2 = 2,$
Let P(k) be true for some positive integer k, i.e.,
$1.2 + 2.2^2 + 3.2^2 + \ldots + \mathrm{k}.2^{\mathrm{k}} = (\mathrm{k}-1)2^{\mathrm{k}+1} + 2 \ldots..(\mathrm{i})$
We shall now prove that P(k + 1) is true.
Consider
$\left\{1.2 + 2.2^2 + 3.2^3 + \ldots + k.2^k\right\} + (k+1) \cdot 2^{k+1}$
$= (k-1)2^{k+1} + 2 + (k+1)2^{k+1}$
$= 2^{k+1}\{(k-1) + (k+1)\} + 2$
$= 2^{k+1} \cdot 2k + 2$
$= k.2^{(k+1)+1} + 2$
$= \{(k+1) - 1\}2^{(k+1)+1} + 2$

Thus, P(k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q9 Prove the following by using the principle of mathematical induction for all n ∈ N:
$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots+\frac{1}{2^n}=1-\frac{1}{2^n}$

Answer. Let the given statement be P(n), i.e.,
P(n): $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots+\frac{1}{2^n}=1-\frac{1}{2^n}$
For n = 1, we have
P(1): $\frac{1}{2}=1-\frac{1}{2^1}=\frac{1}{2}$, which is true
Let P(k) be true for some positive integer k, i.e.,
$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots+\frac{1}{2^k}=1-\frac{1}{2^k}$
We shall now prove that P(k + 1) is true.
Consider
$\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\ldots+\frac{1}{2^k}\right)+\frac{1}{2^{k+1}}$ [using(i))]
$=\left(1-\frac{1}{2^k}\right)+\frac{1}{2^{k+1}}$
$=1-\frac{1}{2^k}+\frac{1}{2\cdot 2^k}$
$=1-\frac{1}{2^k}\left(1-\frac{1}{2}\right)$
$=1-\frac{1}{2^k}\left(\frac{1}{2}\right)$
$=1-\frac{1}{2^{k+1}}$
Thus, P(k + 1) is true whenever P(k) is true.
Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q10 Prove the following by using the principle of mathematical induction for all n ∈ N:
$\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+\ldots+\frac{1}{(3n-1)(3n+2)}=\frac{n}{(6n+4)}$

Answer. Let the given statement be P(n), i.e.,
P(n): $\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+\ldots+\frac{1}{(3n-1)(3n+2)}=\frac{n}{(6n+4)}$
For n = 1, we have
$P(1)=\frac{1}{2.5}=\frac{1}{10}=\frac{1}{6.1+4}=\frac{1}{10}$, which is true.
Let P(k) be true for some positive integer k, i.e.,
$\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+\ldots+\frac{1}{(3k-1)(3k+2)}=\frac{k}{6k+4}$ ....(i)
We shall now prove that P(k + 1) is true.
Consider

$\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+\ldots\ldots+\frac{1}{(3k-1)(3k+2)}+\frac{1}{\{3(k+1)-1\}\{3(k+1)+2\}}$

$=\frac{k}{6k+4}+\frac{1}{(3k+3-1)(3k+3+2)}$ [Using (i)]

$=\frac{k}{6k+4}+\frac{1}{(3k+2)(3k+5)}$

$=\frac{k}{2(3k+2)}+\frac{1}{(3k+2)(3k+5)}$

$=\frac{1}{(3k+2)}\left(\frac{k}{2}+\frac{1}{3k+5}\right)$

$=\frac{1}{(3k+2)}\left(\frac{k(3k+5)+2}{2(3k+5)}\right)$

$=\frac{1}{(3k+2)}\left(\frac{3k^2+5k+2}{2(3k+5)}\right)$

$=\frac{1}{(3k+2)}\left(\frac{(3k+2)(k+1)}{2(3k+5)}\right)$

$=\frac{(k+1)}{6k+10}$

$=\frac{(k+1)}{6(k+1)+4}$

Thus, P(k + 1) is true whenever P(k) is true.
Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q11 Prove the following by using the principle of mathematical induction for all n ∈ N:
$\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+\ldots+\frac{1}{n(n+1)(n+2)}=\frac{n(n+3)}{4(n+1)(n+2)}$

Answer. Let the given statement be P(n),i.e., P(n):
$\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+\ldots+\frac{1}{n(n+1)(n+2)}=\frac{n(n+3)}{4(n+1)(n+2)}$
For n = 1, we have
$P(1):\frac{1}{1\cdot2\cdot3}=\frac{1\cdot(1+3)}{4(1+1)(1+2)}=\frac{1\cdot4}{4\cdot2\cdot3}=\frac{1}{1\cdot2\cdot3}$, which is true
Let P(k) be true for some positive integer k, i.e.,
$\frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+\frac{1}{3\cdot4\cdot5}+\ldots+\frac{1}{k(k+1)(k+2)}=\frac{k(k+3)}{4(k+1)(k+2)}$ .....(i)
We shall now prove that P(k + 1) is true.
Consider
$\left[\frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+\frac{1}{3\cdot4\cdot5}+\ldots+\frac{1}{k(k+1)(k+2)}\right]+\frac{1}{(k+1)(k+2)(k+3)}$

$=\frac{k(k+3)}{4(k+1)(k+2)}+\frac{1}{(k+1)(k+2)(k+3)}$ [Using (i)]

$=\frac{1}{(k+1)(k+2)}\left\{\frac{k(k+3)}{4}+\frac{1}{k+3}\right\}$

$=\frac{1}{(k+1)(k+2)}\left\{\frac{k(k+3)^2+4}{4(k+3)}\right\}$

$=\frac{1}{(k+1)(k+2)}\left\{\frac{k\left(k^2+6k+9\right)+4}{4(k+3)}\right\}$

$= \frac{1}{(k+1)(k+2)}\left\{\frac{k^3+6k^2+9k+4}{4(k+3)}\right\}$

$= \frac{1}{(k+1)(k+2)}\left\{\frac{k^3+2k^2+k+4k^2+8k+4}{4(k+3)}\right\}$

$= \frac{1}{(k+1)(k+2)}\left\{\frac{k(k^2+2k+1)+4(k^2+2k+1)}{4(k+3)}\right\}$

$= \frac{1}{(k+1)(k+2)}\left\{\frac{k(k+1)^2+4(k+1)^2}{4(k+3)}\right\}$

$= \frac{(k+1)^2(k+4)}{4(k+1)(k+2)(k+3)}$

$= \frac{(k+1)\{(k+1)+3\}}{4\{(k+1)+1\}\{(k+1)+2\}}$

Thus, P(k + 1) is true whenever P(k) is true.
Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q12 Prove the following by using the principle of mathematical induction for all n ∈ N:
$a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(r^n-1)}{r-1}$

Answer. Let the given statement be P(n), i.e.,
P(n): $a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(r^n-1)}{r-1}$
For n = 1, we have
$\mathrm{P}(1) : a = \frac{a(r^1-1)}{(r-1)} = a$, which is true.
Let P(k) be true for some positive integer k, i.e.,
$a + ar + ar^2 + \ldots\ldots\ldots + ar^{k-1} = \frac{a(r^k-1)}{r-1}$ .....(i)
We shall now prove that P(k + 1) is true.
Consider
$\left\{a + ar + ar^2 + \ldots\ldots + ar^{k-1}\right\} + ar^{(k+1)-1}$
$= \frac{a(r^k-1)}{r-1} + ar^k$ [Using(i)]

$= \frac{a\left(r^k-1\right)+ar^k(r-1)}{r-1}$

$= \frac{a\left(r^k-1\right)+ar^{k+1}-ar^k}{r-1}$

$= \frac{ar^k-a+ar^{k+1}-ar^k}{r-1}$

$= \frac{ar^{k+1}-a}{r-1}$

$= \frac{a(r^{k+1}-1)}{r-1}$

Thus, P(k + 1) is true whenever P(k) is true.
Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q13 Prove the following by using the principle of mathematical induction for all n ∈ N:
$\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\ldots\left(1+\frac{(2n+1)}{n^2}\right)=(n+1)^2$

Answer.Let the given statement be P(n), i.e.,
$\mathrm{P}(n):\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\ldots\left(1+\frac{(2n+1)}{n^2}\right)=(n+1)^2$
For n = 1, we have
$\mathrm{P}(1):\left(1+\frac{3}{1}\right)=4=(1+1)^2=2^2=4,$
Let P(k) be true for some positive integer k, i.e.,
$\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\ldots\left(1+\frac{(2k+1)}{k^2}\right)=(k+1)^2$ .....(1)
We shall now prove that P(k + 1) is true.
Consider
$\left[\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\ldots\left(1+\frac{(2k+1)}{k^2}\right)\right]\left\{1+\frac{\{2(k+1)+1\}}{(k+1)^2}\right\}$
$=(k+1)^2\left(1+\frac{2(k+1)+1}{(k+1)^2}\right)$ [(Using (1)]
$=(k+1)^2\left[\frac{(k+1)^2+2(k+1)+1}{(k+1)^2}\right]$
$=(k+1)^2+2(k+1)+1$
$=\{(k+1)+1\}^2$
Thus, P(k + 1) is true whenever P(k) is true.
Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q14 Prove the following by using the principle of mathematical induction for all n ∈ N:
$\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\ldots\left(1+\frac{1}{n}\right)=(n+1)$

Answer. Let the given statement be P(n), i.e.,
P(n): $\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\ldots\left(1+\frac{1}{n}\right)=(n+1)$
For n = 1, we have
$\mathrm{P}(1):\left(1+\frac{1}{1}\right)=2=(1+1)$, which is true
Let P(k) be true for some positive integer k, i.e.,
$\mathrm{P}(k):\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\ldots\left(1+\frac{1}{k}\right)=(k+1)$ .....(1)
We shall now prove that P(k + 1) is true.
Consider
$\left[\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\ldots\left(1+\frac{1}{k}\right)\right]\left(1+\frac{1}{k+1}\right)$

$= (k+1)\left(1+\frac{1}{k+1}\right)$ [Using(1)]

$= (k+1)\left(\frac{(k+1)+1}{(k+1)}\right)$

$= (k+1)+1$

Thus, P(k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q15 Prove the following by using the principle of mathematical induction for all n ∈ N:

$1^2+3^2+5^2+\ldots+(2n-1)^2=\frac{n(2n-1)(2n+1)}{3}$

Answer. Let the given statement be P(n), i.e.,

$P(n)=1^2+3^2+5^2+\ldots+(2n-1)^2=\frac{n(2n-1)(2n+1)}{3}$

$\text{For } n=1, \text{ we have}$

$P(1)=1^2=1=\frac{1(2.1-1)(2.1+1)}{3}=\frac{1.1.3}{3}=1, \text{ which is true.}$

Let P(k) be true for some positive integer k, i.e.,

$P(k)=1^2+3^2+5^2+\ldots+(2k-1)^2=\frac{k(2k-1)(2k+1)}{3}$ .....(1)

We shall now prove that P(k + 1) is true.

Consider

$\left\{1^2+3^2+5^2+\ldots+(2k-1)^2\right\}+\{2(k+1)-1\}^2$

$=\frac{k(2k-1)(2k+1)}{3}+(2k+2-1)^2$ [Using (1)]

$=\frac{k(2k-1)(2k+1)}{3}+(2k+1)^2$

$=\frac{k(2k-1)(2k+1)+3(2k+1)^2}{3}$

$=\frac{(2k+1)\{k(2k-1)+3(2k+1)\}}{3}$

$=\frac{(2k+1)\left\{2k^2-k+6k+3\right\}}{3}$

$=\frac{(2k+1)\left\{2k^2+5k+3\right\}}{3}$

$=\frac{(2k+1)\left\{2k^2+2k+3k+3\right\}}{3}$

$=\frac{(2k+1)\{2k(k+1)+3(k+1)\}}{3}$

$=\frac{(2k+1)(k+1)(2k+3)}{3}$

$=\frac{(k+1)\{2(k+1)-1\}\{2(k+1)+1\}}{3}$

Thus, P(k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q16 Prove the following by using the principle of mathematical induction for all n ∈ N:
$\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\ldots+\frac{1}{(3n-2)(3n+1)}=\frac{n}{(3n+1)}$

Answer. Let the given statement be P(n), i.e.,
$P(n):\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\ldots+\frac{1}{(3n-2)(3n+1)}=\frac{n}{(3n+1)}$
$\text{For } n=1, \text{ we have}$
$P(1)=\frac{1}{1.4}=\frac{1}{3.1+1}=\frac{1}{4}=\frac{1}{1.4}, \text{ which is true.}$
Let P(k) be true for some positive integer k, i.e.,
$P(k)=\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\ldots+\frac{1}{(3k-2)(3k+1)}=\frac{k}{3k+1}$ .....(1)
We shall now prove that P(k + 1) is true.
Consider
$\left\{\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\ldots+\frac{1}{(3k-2)(3k+1)}\right\}+\frac{1}{\{3(k+1)-2\}\{3(k+1)+1\}}$
$=\frac{k}{3k+1}+\frac{1}{(3k+1)(3k+4)}$ [Using (1)]
$=\frac{1}{(3k+1)}\left\{k+\frac{1}{(3k+4)}\right\}$
$=\frac{1}{(3k+1)}\left\{\frac{k(3k+4)+1}{(3k+4)}\right\}$
$=\frac{1}{(3k+1)}\left\{\frac{3k^2+4k+1}{(3k+4)}\right\}$
$=\frac{1}{(3k+1)}\left\{\frac{3k^2+3k+k+1}{(3k+4)}\right\}$
$=\frac{(3k+1)(k+1)}{(3k+1)(3k+4)}$
$=\frac{(k+1)}{3(k+1)+1}$
Thus, P(k + 1) is true whenever P(k) is true.
Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q17 Prove the following by using the principle of mathematical induction for all n ∈ N:
$\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+\ldots+\frac{1}{(2n+1)(2n+3)}=\frac{n}{3(2n+3)}$

Answer. Let the given statement be P(n), i.e.,
$P(n):\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+\ldots+\frac{1}{(2n+1)(2n+3)}=\frac{n}{3(2n+3)}$
For n = 1, we have
$P(1):\frac{1}{3.5}=\frac{1}{3(2.1+3)}=\frac{1}{3.5}$, which is true.
Let P(k) be true for some positive integer k, i.e.,
$P(k):\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+\ldots+\frac{1}{(2k+1)(2k+3)}=\frac{k}{3(2k+3)}$ .....(1)
We shall now prove that P(k + 1) is true.
Consider

$\left[\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+\ldots+\frac{1}{(2k+1)(2k+3)}\right]+\frac{1}{\{2(k+1)+1\}\{2(k+1)+3\}}$

$=\frac{k}{3(2k+3)}+\frac{1}{(2k+3)(2k+5)}\ [\text{U sing}(1)]$

$=\frac{1}{(2k+3)}\left[\frac{k}{3}+\frac{1}{(2k+5)}\right]$

$=\frac{1}{(2k+3)}\left[\frac{k(2k+5)+3}{3(2k+5)}\right]$

$=\frac{1}{(2k+3)}\left[\frac{2k^2+5k+3}{3(2k+5)}\right]$

$=\frac{1}{(2k+3)}\left[\frac{2k^2+2k+3k+3}{3(2k+5)}\right]$

$=\frac{1}{(2k+3)}\left[\frac{2k(k+1)+3(k+1)}{3(2k+5)}\right]$

$=\frac{(k+1)(2k+3)}{3(2k+3)(2k+5)}$

$=\frac{(k+1)}{3\{2(k+1)+3\}}$

Thus, P(k + 1) is true whenever P(k) is true.
Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q18 Prove the following by using the principle of mathematical induction for all n ∈ N:
$1+2+3+\ldots+n<\frac{1}{8}(2n+1)^2$

Answer. Let the given statement be P(n), i.e.,
$\mathrm{P}(n): 1+2+3+\ldots+n<\frac{1}{8}(2n+1)^2$
It can be noted that P(n) is true for n = 1 since $1<\frac{1}{8}(2.1+1)^2=\frac{9}{8}$
Let P(k) be true for some positive integer k, i.e.,
$1+2+\ldots+k<\frac{1}{8}(2k+1)^2$ .....(1)
We shall now prove that P(k + 1) is true whenever P(k) is true.
Consider
$(1+2+\ldots+k)+(k+1)<\frac{1}{g}(2k+1)^2+(k+1)\ [\text{ Using }(1)]$

$<\frac{1}{8}\left\{(2k+1)^2+8(k+1)\right\}$

$<\frac{1}{8}\left\{4k^2+4k+1+8k+8\right\}$

$<\frac{1}{8}\left\{4k^2+12k+9\right\}$

$<\frac{1}{8}(2k+3)^2$

$<\frac{1}{8}\{2(k+1)+1\}^2$

Hence, $(1+2+3+\ldots+k)+(k+1)<\frac{1}{8}(2k+1)^2+(k+1)$
Thus, P(k + 1) is true whenever P(k) is true.
Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q19 Prove the following by using the principle of mathematical induction for all n ∈ N:
$n(n+1)(n+5)$ is a multiple of 3

Answer. Let the given statement be P(n), i.e.,
P(n): n (n + 1) (n + 5), which is a multiple of 3.
It can be noted that P(n) is true for n = 1 since 1 (1 + 1) (1 + 5) = 12, which is a multiple of 3.
Let P(k) be true for some positive integer k, i.e.,
k (k + 1) (k + 5) is a multiple of 3.
∴k (k + 1) (k + 5) = 3m, where m ∈ N ... (1)
We shall now prove that P(k + 1) is true whenever P(k) is true.
Consider

$$
\begin{aligned}
&(k+1)\{(k+1)+1\}\{(k+1)+5\} \\
&=(k+1)(k+2)\{(k+5)+1\} \\
&=(k+1)(k+2)(k+5)+(k+1)(k+2) \\
&=\{k(k+1)(k+5)+2(k+1)(k+5)\} \\
&=3m+(k+1)\{2(k+5)+(k+2)\} \\
&=3m+(k+1)\{2k+10+k+2\} \\
&=3m+(k+1)\{2k+12) \\
&=3m+3(k+1)(k+4) \\
&=3\{m+(k+1)(k+4)\}=3\times q, \text{ where } q=\{m+(k+1)(k+4)\} \text{ is some natural number} \\
&\text{Therefore, } (k+1)\{(k+1)+1\}\{(k+1)+5\} \text{ is a multiple of 3.}
\end{aligned}
$$

Thus, P(k + 1) is true whenever P(k) is true.
Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q20 Prove the following by using the principle of mathematical induction for all n ∈ N:
$10^{1n-1}+1$ is divisible by 11

Answer. Let the given statement be P(n), i.e.,

$$
\begin{aligned}
&\mathrm{P(n)}: 10^{2\mathrm{n}-1}+1 \text{ is divisible by } 11. \\
&\text{It can be observed that } \mathrm{P(n)} \text{ is true for } \mathrm{n}=1 \text{ since } \mathrm{P}(1)=10^{2.1-1}+1= \\
&11, \text{ which is divisible by } 11.
\end{aligned}
$$

Let P(k) be true for some positive integer k, i.e.,

$$
\begin{aligned}
&10^{2\mathrm{k}-1}+1 \text{ is divisible by } 11. \\
&\therefore 10^{2\mathrm{k}-1}+1=11\mathrm{m}, \text{ where } \mathrm{m} \in \mathrm{N} \ldots(1)
\end{aligned}
$$

We shall now prove that P(k + 1) is true whenever P(k) is true.
Consider

$10^{2(k+1)-1}+1$
$=10^{2k+2-1}+1$
$=10^{2k+1}+1$
$=10^2\left(10^{2k-1}+1-1\right)+1$
$=10^2\left(10^{2k-1}+1\right)-10^2+1$
$=10^2 \cdot 11m-100+1\ [\text{ Using } (1)]$
Thus, P(k + 1) is true whenever P(k) is true.
Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q21 Prove the following by using the principle of mathematical induction for all n ∈ N:
$x^{2x}-y^{3n}$ is divisible by $x+y$

Answer. Let the given statement be P(n), i.e.,
$P(n): x^{2n}-y^{2n}$ is divisible by $x+y$.
It can be observed that $P(n)$ is true for $n=1$.
This is so because $x^2 \times 1-y^2 \times 1=x^2-y^2=(x+y)(x-y)$ is divisible by $(x+y)$.
Let P(k) be true for some positive integer k, i.e.,
$x^{2k}-y^{2k}$ is divisible by $x+y$.
$\therefore x^{2k}-y^{2k}=m(x+y),$ where $m \in N \ldots (1)$
We shall now prove that P(k + 1) is true whenever P(k) is true.
Consider
$x^{2(k+1)}-y^{2(k+1)}$
$=x^{2k} \cdot x^2-y^{2k} \cdot y^2$
$=x^2\left(x^{2k}-y^{2k}+y^{2k}\right)-y^{2k} \cdot y^2$
$=x^2\left\{m(x+y)+y^{2k}\right\}-y^{2k} \cdot y^2 \quad [\text{ Using } (1)]$
$=m(x+y)x^2+y^{2k} \cdot x^2-y^{2k} \cdot y^2$
$=m(x+y)x^2+y^{2k}\left(x^2-y^2\right)$
$=m(x+y)x^2+y^{2k}(x+y)(x-y)$
$=(x+y)\left\{mx^2+y^{2k}(x-y)\right\},$ which is a factor of $(x+y)$
Thus, P(k + 1) is true whenever P(k) is true.
Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q22 Prove the following by using the principle of mathematical induction for all n ∈ N:
$3^{2n+2}-8n-9$ is divisible by 8

Answer. Let the given statement be P(n), i.e.,
$\mathrm{P(n)}: 3^{2\mathrm{n}+2}-8\mathrm{n}-9$ is divisible by 8
$\text{It can be observed that } P(n) \text{ is true for } n=1 \text{ since } 3^{2\times 1+2}-8\times 1-9=64, \text{ which is divisible by } 8 .$
Let P(k) be true for some positive integer k, i.e.,
$3^{2k+2}-8k-9$ is divisible by 8 .
$\therefore 3^{2k+2}-8k-9=8m; \text{ where } m \in N \ldots (1)$
We shall now prove that P(k + 1) is true whenever P(k) is true.
Consider
$3^{2(k+1)+2}-8(k+1)-9$
$=3^{2k+2}\cdot 3^2-8k-8-9$
$=3^2\left(3^{2k+2}-8k-9+8k+9\right)-8k-17$
$=3^2\left(3^{2k+2}-8k-9\right)+3^2(8k+9)-8k-17$
$=9.8m+9(8k+9)-8k-17$
$=9.8m+9(8k+9)-8k-17$
$=9.8m+72k+81-8k-17$
$=9.8m+64k+64$
$=8(9m+8k+8)$
$=8r, \text{ where } r=(9m+8k+8) \text{ is a natural number}$
$\text{Therefore, } 3^{2(k+1)+2}-8(k+1)-9 \text{ is divisible by } 8 .$
Thus, P(k + 1) is true whenever P(k) is true.
Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q23 Prove the following by using the principle of mathematical induction for all n ∈ N:
$41^n-14^n \text{ is a multiple of } 27$

Answer. Let the given statement be P(n), i.e.,
$\mathrm{P(n)}: 41^{\mathrm{n}}-14^{\mathrm{n}} \text{ is a multiple of } 27$
It can be observed that P(n) is true for n = 1 since $41^1-14^1=27$ which is a multiple of 27.
Let P(k) be true for some positive integer k, i.e.,
$41^{\mathrm{k}}-14^{\mathrm{k}} \text{ is a multiple of } 27$
$\therefore 41^{\mathrm{k}}-14^{\mathrm{k}}=27\mathrm{m}, \text{ where } \mathrm{m} \in \mathrm{N} \ldots (1)$
We shall now prove that P(k + 1) is true whenever P(k) is true.
Consider

$41^{k+1} - 14^{k+1}$

$= 41^k \cdot 41 - 14^k \cdot 14$

$= 41\left(41^k - 14^k + 14^k\right) - 14^k \cdot 14$

$= 41\left(41^k - 14^k\right) + 41.14^k - 14^k \cdot 14$

$= 41.27m + 14^k(41 - 14)$

$= 41.27m + 27.14^k$

$= 27\left(41m - 14^k\right)$

$= 27 \times r, \text{ where } r = \left(41m - 14^k\right) \text{ is a natural number}$

$\text{Therefore, } 41^{k+1} - 14^{k+1} \text{ is a multiple of } 27$

Thus, P(k + 1) is true whenever P(k) is true.
Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q24 Prove the following by using the principle of mathematical induction for all n ∈ N:
$(2n + 7) < (n + 3)^2$

Answer. Let the given statement be P(n), i.e.,
$\mathrm{P(n)} : (2\mathrm{n} + 7) < (\mathrm{n} + 3)^2$

$\text{It can be observed that } \mathrm{P(n)} \text{ is true for } \mathrm{n} = 1 \text{ since } 2.1 + 7 = 9 < (1 + 3)^2$

$= 16, \text{ which is true.}$

Let P(k) be true for some positive integer k, i.e.,
$(2k + 7) < (k + 3)^2 \ldots (1)$
We shall now prove that P(k + 1) is true whenever P(k) is true.
Consider
$\{2(k + 1) + 7\} = (2k + 7) + 2$

$\therefore \{2(\mathrm{k} + 1) + 7\} = (2\mathrm{k} + 7) + 2 < (\mathrm{k} + 3)^2 + 2 \quad [\text{ using } (1)]$

$2(\mathrm{k} + 1) + 7 < \mathrm{k}^2 + 6\mathrm{k} + 9 + 2$

$2(\mathrm{k} + 1) + 7 < \mathrm{k}^2 + 6\mathrm{k} + 11$

$\text{Now, } \mathrm{k}^2 + 6\mathrm{k} + 11 < \mathrm{k}^2 + 8\mathrm{k} + 16$

$\therefore 2(\mathrm{k} + 1) + 7 < (\mathrm{k} + 4)^2$

$2(\mathrm{k} + 1) + 7 < \{(\mathrm{k} + 1) + 3\}^2$

Thus, P(k + 1) is true whenever P(k) is true.
Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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