CHAPTER: REAL NUMBERS

Grade: **10**
Board: **CBSE**
Book: **NCERT**

1.

Chapter Summary and Placement

<Read through the following to understand how this chapter is connected with the previous and next grades and also the important concepts covered in it.>

1. This chapter connects directly to the number system chapter in grade 9
2. This chapter establishes the theoretical concept of prime factorisation[Fundamental theorem of arithmetic] and computing HCF of two numbers[Euclid’s division lemma]
3. It connects to the chapters related to number theory in future grades

2.

LEARNING OUTCOMES

<On completion of the chapter, students will be able to:>

1. Prove Euclid's division lemma
2. Compute HCF using Euclid’s division algorithm
3. State fundamental theorem of arithmetic
4. Find HCF and LCM using prime factorisation
5. Recollect the product rule of HCF and LCM of two numbers
6. Use fundamental theorem of arithmetic to prove the given number as irrational
7. Check whether the given rational number is terminating or non terminating decimal without actual division

3 . Keywords

<On completion of the chapter, students will be able to:>

1. Lemma: A subsidiary or intermediate theorem in an argument or proof. In precise a "helping theorem"
2. Algorithm: A process or set of rules to be followed in calculations or other problem-solving operations

4. TEACHING PLAN

Follow this section for getting ideas on teaching the content covered in the chapter. The subtopic given in the first column with the matching learning outcome can be explained using instructions and resources listed subsequently

Subtopic and Learning Objective #1

Subtopic:Euclid's division lemma
LO:
Prove Euclid's division lemma
1.To make students understand Euclid’s division lemma

Teacher Activity

1. Introduce Definition of Lemma and algorithm as given in Key words above.
2. Introduce Euclid's division lemma with an example as in NCERT textbook
3. **Resource 1** can be used for statement explanation

Check for understanding:

a) State Euclid's division lemma

Resources Suggested

Blackboard, pen and paper
Resource 1

Subtopic and Learning Objective #2

Sub topic:Finding HCF by using division algorithm
LO:
Compute HCF using Euclid’s division algorithm
1. To help students to find HCF of two numbers using division algorithm

Teacher Activity

1. Introduce method of finding HCF by using Euclid's division algorithm
2. **Resource 1** can be used for explanation through example

Check for understanding:

a) Find HCF of 17017, 274170 by using Euclid's division algorithm

Repeat more number of examples as above

Resources Suggested

Blackboard, pen and paper
Resource 1

Resource 1

Euclid's Division Lemma

Euclidean Algorithm :

Let a and b be positive integers with a≥ b.

If b divides a, then (a, b) = b.

If b divides a, then apply the division algorithm repeatedly as follows:

$a = b\,q_0 + r_0$ with $0 \le r_0 < b$

$b = r_0\,q_1 + r_1$ with $0 \le r_1 < r_0$

$r_0 = r_1\,q_2 + r_2$ with $0 \le r_2 < r_1$

$r_1 = r_2\,q_3 + r_3$ with $0 \le r_3 < r_2$

$r_2 = r_3\,q_4 + r_4$ with $0 \le r_4 < r_3$ .....

This process ends when the remainder 0 is obtained. This must occur after a finite number of steps; that is, for some k

$r_{k-2} = r_{k-1}\,q_k + r_k$ with $0 \le r_k < r_{k-1}$

$r_{k-1} = r_k\,q_{k+1} + 0$

Then r_k the last non zero remainder, is the greatest common divisor of a and b.

Resource 1

Euclid’s division algorithm is “ If a and b are two positive integers, then there exists unique integers q and r satisfying the condition a=bq+r where0r<band and r rational number
Based on above algorithm HCF of 180670 and 2937 can be found as
180670=2937x61+1513
2937=1513x1+1424
1513=1424x1+89
1424=89x16+0
HCF of 180670 and 2937 is 89

Subtopic and Learning Objective #3

Subtopic: Fundamental theorem of arithmetic
LO:
State fundamental theorem of arithmetic
1. To help students understand fundamental theorem of arithmetic

Teacher Activity

1. Introduce fundamental theorem of arithmetic and also explain how it is theoretical concept of writing prime factors of a number

Check for understanding:

a) state fundamental theorem of arithmetic

Resources Suggested

Blackboard, pen and paper

Subtopic and Learning Objective #4

Sub topic:LCM and HCF of two numbers
LO:
Find HCF and LCM using prime factorisation
1. To make students find LCM and HCF of two numbers by Prime factorisation method
LO:Recollect the product rule of HCF and LCM of two numbers
2. To make the student to recollect the product rule of LCM and HCF

Teacher Activity

1. Introduce method of finding LCM and HCF of two numbers by prime factorisation method
2. **Resource 2** can be used for explanation
3. Introduce product rule of LCM and HCF. [Product of LCM and HCF of two numbers is equal to the product of numbers] and also product rule for three numbers as below

HCF (p, q, r) = [p. q. r. LCM (p, q, r)] / [LCM(p ,q). LCM(q , r). LCM(p ,r)]

LCM (p, q, r) = [p. q. r. HCF (p, q, r)] / [HCF(p ,q). HCF(q , r). HCF(p ,r)]

Check for understanding:

a) Find the HCF and LCM of given numbers[Numbers can be of teachers choice]

b) The product of two numbers is 36 and their HCF is 3 Find their LCM

Can repeat the questions by changing the numbers

Resources Suggested

Blackboard, pen and paper
Resource 3

Resource 2

Finding LCM and HCF of 12,16 and 72

$12=2^2 \times 3$

$16= 2^4$

$72= 2^3 \times 3^2$

To find LCM search for greatest factors it is 2^4 and 3^2. So LCM = $2^4 \times 3^2 = 144$

To find HCF search for common factor it is 2^2 so HCF= 4

Note:

LCM= Product of greatest power of each prime factor

HCF= Product of smallest power of each common prime factor

Subtopic and Learning Objective #5

Sub topic: To show the given number is irrational
LO: Use fundamental theorem of arithmetic to prove the given number as irrational
1. To make student to show the given number as irrational

Teacher Activity

1. Introduce the method of showing a number as irrational as given in NCERT text book

Check for understanding:

a) Prove that $\sqrt{7}$ as irrational

b) Prove that $\sqrt{33}$ as irrational

c) Prove that $\sqrt{5+2}$ as irrational

d) Prove that $\sqrt{4-5}$as irrational

Resources Suggested

Blackboard, pen and paper

Subtopic and Learning Objective #6

Sub topic:Revisiting rationals as terminating or non terminating decimals

LO:

Check whether the given rational number is terminating or non terminating decimal without actual division

1. To make student find terminating or non terminating decimals without actual division

Teacher Activity

1. Introduce the method of writing a number as Product of prime factors and check whether the given rational number is terminating or non terminating without actual division
2. Terminating if it's product of prime factors are $2^m x 5^n$ otherwise Non terminating

Check for understanding:

a) Check whether the given rational number is terminating or non terminating decimal[Numbers can be of teachers choice]

Resources Suggested

Blackboard, pen and paper

Projects

Students can be asked to select a subtopic as their project work and prepare notes and submit
Some examples are below
To find HCF and LCM of numbers using set theory-
https://drive.google.com/file/d/0B48LXwSwCo6BT2hraGNRY2tPeTg/view
To find HCF using euclid's division lemma-
https://drive.google.com/file/d/0B48LXwSwCo6BTmJ4cXFWRkJWQlk/view
Terminating and non terminating decimals
https://drive.google.com/file/d/0B48LXwSwCo6BdHc1LWlaZ3hwT1U/view

6. WORKSHEETS

Print or copy the worksheet questions on the blackboard and ask the students to attempt them. They can be given as homework or a class test as well.

Worksheet 1

Use it like: test after the chapter is complete, homework

Answer the following:

1. Prove that if x and y are both odd positive integers, then $x^2 + y^2$ is even but not divisible by 4
2. Prove that $n^2 - n$ is even for every positive integer n.
3. Prove that $n^2 – 1$, is divisible by 8, if n is an odd integer
4. Prove that one of every three consecutive positive integers is divisible by 3
5. Show that the square of any odd integer is of the form (4q + 1) for some integer q.

Worksheet 2

Use it like: test after the chapter is complete, homework

Answer the following:

1. Using Euclid's Division Algorithm, find the HCF of 6496 and 376
2. Write the HCF and LCM of the smallest odd composite number and the smallest odd prime number. If an odd number p divides q^2 then will it divide q^3 also. Explain
3. Using Euclid's Division Algorithm, find the HCF of 480 and 210
4. If the HCF of 65 and 117 is expressible in the form 65m – 117, then find the value of m?
5. Using Euclid’s division algorithm, find the largest number that divides 1251, 9377 and 1562 8 leaving remainders, 1, 2, and 3 respectively.

Worksheet 3

Use it like: test after the chapter is complete, homework

Answer the following:

1. The numbers 525 and 3000 are both divisible only by 3, 5, 15, 25 and 75, what is HCF of (3000, 525)? Justify your answer.
2. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$; x, y are prime numbers then HCF (a, b)
3. If two positive integers p and q can expressed as $p = ab^2$ and $q = a^3b$ where a and b being prime numbers, then LCM (p, q) is
4. Can two numbers have 18 as their HCF and 380 as their LCM? Give reasons.
5. If LCM (p,q,r) = 420, HCF (p,q,r) = 1, HCF (p,q) = 3 , HCF (q,r) = 5 and HCF (p,r) = 1, then find the the product of p,q and r

Worksheet 4

Use it like: test after the chapter is complete, homework

Answer the following:

1. The numbers 525 and 3000 are both divisible only by 3, 5, 15, 25 and 75, what is HCF of (3000, 525)? Justify your answer.
2. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$; x, y are prime numbers then HCF (a, b)
3. If two positive integers p and q can expressed as $p = ab^2$ and $q = a^3b$ where a and b being prime numbers, then LCM (p, q) is
4. Can two numbers have 18 as their HCF and 380 as their LCM? Give reasons.
5. If LCM (p,q,r) = 420, HCF (p,q,r) = 1, HCF (p,q) = 3 , HCF (q,r) = 5 and HCF (p,r) = 1, then find the the product of p,q and r

6. SOLUTIONS

This section gives you sample answers for activities and questions listed above.

Worksheet 1

1. Let $x=2n+1$ and $y=2m+1$ (Where n and m are positive integers)

   Therefore, $x^2+y^2 = (2n+1)^2+ (2m+1)^2 =4n^2+4n+1+4m^2+4m+1 =4(n^2+m^2+n+m) +2$

   Thus, $x^2 + y^2$ is even but not divisible by 4

2. To prove that $n^2 - n$ is even for every positive integer n.

   We know that Even × Even = Even

   Even - Even = Even

   Odd - Odd = Even

   Odd × Odd = Odd

   Suppose n is odd: Now, $n^2 - n = (odd)^2 - (odd) = odd \times odd - odd = odd - odd = Even$

   Suppose n is even: Now, $n^2 - n = (even)^2 - (even) = even \times even - even = even - even = even$

   Therefore, $n^2 - n$ is even for every positive integer n.

Worksheet 1

. 3.Let $p = n^2 - 1$, where n is any integer.

Case I: Let n is even, then $n = 2k$.

$p = (2k)^2 - 1$

$p = 4k^2 - 1$

Let $k = 0$, then $p = 4(0)^2 - 1 = -1$, which is not divisible by 8

$k = 2$, then $p = 4(2)^2 - 1 = 15$, which is not divisible by 8

$k = 4$, then $p = 4(4)^2 = 64 - 1 = 6$, which is not divisible by 8

So, n cannot be even integer.

Case II: Let n is odd then $n = 2k + 1$

$p = (2k + 1)^2 - 1 = 4k^2 + 1 + 4k - 1$

$p = 4k (k + 1)$

Let $k = 1$, $p = 4(1) [1+1] = 8$ is divisible by 8

$k = 3$ $p = 4 \times 3 (3+1) = 48 = 8 \times 6$, is divisible by 8

$k = 5$ $p = 4(5) (5 + 1) = 120, = 8 \times 15$ is divisible by 8

So, $n^2 - 1$ is divisible by 8 if n is odd number.

Worksheet 1

4 Let three consecutive positive integers are n , n+1 and n+2 for some integer n

Now, Number divisible 3 are of form 3q , 3(q +1) and 3(q + 2)

For n = 3q

n = 3q is divisible by 3.

n + 1 = 3q + 1 not divisible by 3

n + 2 = 3q + 2 not divisible by 3

For n = 3q+1

n = 3q+1 not divisible by 3

n + 1 = (3q +1) + 1 = 3q+2 not divisible by 3

n + 2 = (3q + 1) + 2 = 3(q+1), which is divisible by 3.

For n = 3q+2

n = 3q+2 not divisible by 3

n + 1 = (3q +2) + 1 = 3(q+1) which is divisible by 3

n + 2 = (3q + 2) + 2 = (3q+3) +1 = 3(q+1) + 1, which is not divisible by 3.

So, we can say that one of the numbers among n, n + 1 and n + 2 is always divisible by 3

Worksheet 1

5.By Euclid's division algorithm, a = bq + r where a, b, q, r are non- negative integers and $0 \leq r < 4$.

On putting b = 4 we get $a=4q+r$…(i)

When r = 0, a = 4q which is even (as it is divisible by 2)

When r = 1, a = 4q + 1 which is odd (it is not divisible by 2)

Squaring the odd number (4q + 1), we get

$(4q + 1)^2 = (4q)^2 + (1)^2 + 2(4q)$

$= 4[4q^2 + 2q] + 1$

$= 4m + 1$ is perfect square for $m = 4q^2 + 2q$

When r = 2, a = 4q + 2 [From (i)]

⇒ a = 2(2q + 1) is divisible by 2 so it is even.

When r = 3, a = 4q + 3 = 4q + 2 +1

= 2[2q + 1] + 1 is not divisible by 2 so it is odd.

Squaring the odd number (4g + 3), we get

$(4q+3)^2 = (4q)^2 + (3)^2 + 2(4q)\ (3)$

$= 16q^2 + 9 + 24q$

$= 16q^2 + 24q + 8 + 1$

$= 4[4q^2 + 6q + 2] + 1$

Worksheet 2

1. 6496 > 376. Therefore, a = 6496 and b = 376,
   By applying the Euclid's Division Algorithm(a = bq + r)
   6496 = 376 × 17 + 104
   376 = 104 × 3 + 64
   104 = 64 × 1 + 40
   64 = 40 × 1 + 24
   40 = 24 × 1 + 16
   24 = 16 × 1 + 8
   16 = 8 × 2 + 0
   Since, r = 0, the process stops. The divisor at this stage is 8. Thus, the HCF of the pair 6496 and 376 is 8.

Worksheet 2

(i) Smallest odd composite number is 9, smallest odd prime number is 3

LCM of 9 and 3 is 9, HCF of 9 and 3 is 3

(ii) An odd number p divides q^2

=> p is one of the factor of q^2 ----------(1)

=>$q^3 = q^2 \times q$

=>q^2 is one of the factor of q^3 ----------(ii)

from (i) and (ii), q^2 and q^3 has common factor p

Thus p divide q^3 also.

3. 480 > 210. Therefore, a = 480 and b = 210,

By applying the Euclid's Division Algorithm(a = bq + r) we get, 480 = 210 × 2 + 60

210 = 60 × 3 + 30

60 = 30 × 2 + 0

Since, r = 0, the process stops. The divisor at this stage is 30. Thus, the HCF of the pair 480 and 210 is 30.

Worksheet 2

HCF of 65, 117 by any method let by factorisation

117= 65 x1 + 52

65= 52 x 1+ 13

52=13 x 4 + 0

So, HCF of 65 and 117 = 13

But given 65m – 117 = 13

⇒ 65m = 130

⇒ m = 2

5. As 1, 2, and 3 are the remainders when required largest number (HCF) divides 1251, 9377 and 15628 respectively.

We have the numbers for HCF (1251 – 1), (9377 – 2) and (15628 – 3),

i.e., 1250, 9375, 15625

For HCF of 1250, 9375, 15625 let a = 15625, b = 9375

By Euclid's division algorithm, a = bq + r

15625 = 9375 × 1 + 6250

9375 = 6250 × 1 + 3125

6250 = 3125 × 2 + 0

HCF (15625, 9375) = 3125

Now, let d = 1250 and c = 3125

By Euclid's division algorithm, c = dq + r

3125 = 1250 × 2 + 625

1250 = 625 × 2 + 0

Hence, required HCF (15625, 1250 and 9375) is 625.

Worksheet 3

1. The numbers 525 and 3000 are both divisible only 3, 5, 15, 25 and 75, So, highest common factors out of 3, 5, 15, 25 and 75 is 75 or HCF of (525, 3000) is 75.
   Verification: $525 = 5 \times 5 \times 3 \times 7 = 3 \times 5^2 \times 7^1$
   $3000 = 2^3 \times 5^3 \times 3^1 = 2^3 \times 3^1 \times 5^3$
   $HCF = 3^1 \times 5^2 = 75$
   Hence, verified.
2. $a = x^3y^2$
   $b = xy^3$
   HCF= Product of smallest power of each common prime factor
   So, HCF of a and b = xy^2

Worksheet 3

$p = ab^2$

$q = a^3b$

LCM = product of the highest powers of each factor.

So, LCM = a^3b^2.

4. As we know that

HCF (a, b) × LCM (a, b) = (a × b)

18 must be factor of 380.

So, 38018 should be a natural number.

But 38018 is not a natural number or 380 is not divisible by 18.

So, 380 and 18 are not the LCM and HCF of any two numbers

5. We know that : HCF (p, q, r) = [p. q. r. LCM (p, q, r)] / [LCM(p ,q). LCM(q , r). LCM(p ,r)]

LCM (p, q, r) = [p. q. r. HCF (p, q, r)] / [HCF(p ,q). HCF(q , r). HCF(p ,r)]

420 = [pqr x1]/[3x5x1]

pqr = 420 /(3x5x1) = 28

Worksheet 4

1. Terminates after 4 decimals

   $145871250=14587542$

   By multiplying numerator and denominator by 2^3

   $14587542=145872354223=11669610000=11.6696$

2. Denominator of the rational number 2575000 is 5000.

   $5000 = 2^3 \times 5^4$ which is of the form $2^m \times 5^n$

   where m = 3 and n = 4

   $2575000=257235422=5142454=51410000= 0.0514$

3. 327.7081 is terminating decimal so in the form of

   $pq=327708110000$

   $q= 2^4 \times 5^4$

   So, q has only factors of 2 and 5 so it is terminating decimal.

Worksheet 4

14^n can end with digit 0 or 5 for any natural number n, if $14^n = 2^x \times 5^y$

But $14^n = (2^n \times 7^n)$

Thus ,14^n cannot end with digit 0 or 5 for any natural number n.

5. $2520 = 2^3 \times 3^2 \times 5 \times 7 = 2^3 \times 3^p \times q \times 7$

So, p = 2 , q = 5

7. EXTRA RESOURCES

Carry out the following suggested activities/ tasks if you have the time and resources to do them

Teacher can use these videos if time and facilities permit:

1. https://www.youtube.com/watch?v=qra1hHzCkpo - Preview of euclid's division lemma