CHAPTER: Polynomial

Grade: **10**
Board: **CBSE**
Book: **NCERT**

1.

Chapter Summary and Placement

<Read through the following to understand how this chapter is connected with the previous and next grades and also the important concepts covered in it.>

1. This chapter connects to polynomials chapter of grade 9
2. In this chapter the relation between zeros and coefficients of the polynomial are found.
3. It connects to chapters related to polynomial functions in future grades

2.

LEARNING OUTCOMES

<On completion of the chapter, students will be able to:>

1. Recall definitions degree of polynomials, types of polynomials, zeros of the polynomial
2. Acquire the knowledge of geometrical meaning of zero of the polynomial
3. Find zeros of the polynomial by reading the graph
4. Explore the relationship between zeros of the polynomial and coefficients of the polynomial
5. Restate division algorithm of polynomials

4. TEACHING PLAN

Follow this section for getting ideas on teaching the content covered in the chapter. The subtopic given in the first column with the matching learning outcome can be explained using instructions and resources listed subsequently

Subtopic and Learning Objective #1

Subtopic: Definitions
LO:
Recall definitions degree of polynomials, types of polynomials, zeros of the polynomial
1.To make the student recall definitions

Teacher Activity

1. Recap of definitions/ perform **Activity 1** for recap
2. **Resource 1 can be used for recap**

Check for understanding:

a) Degree of a quadratic polynomial is _____

b) A polynomial with degree 3 is called as_____

c) Maximum number of zeros of linear polynomial _____

Ask number of questions to get practice

Resources Suggested

Blackboard, pen and paper
Resource 1 can be used for recap

Activity 1

Name of the activity- Flash card identification
Type of activity – group/ individual
Use it for – recap
Materials – flashcards with definitions
Procedure –

1. Divide the class into a few groups and ask a student from each group to pick up a card and answer the question on the card.
2. The questions should be on the various terms related to polynomials and the categories of polynomials.
3. Algebraic expressions can also be written on the cards and ask the students to identify if the expressions are polynomials.
4. If a given expression is a polynomial, ask them to identify its degree and name it. Also, ask the student to find the zeroes of the polynomial.

Subtopic and Learning Objective #2

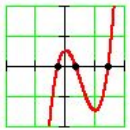
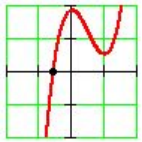
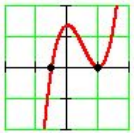
Sub topic:Zeros of the polynomials from graph
LO:
Acquire the knowledge of geometrical meaning of zero of the polynomial
1. To make student acquire the knowledge of reading number of zeros of polynomials from graph
LO:
Find zeros of the polynomial by reading the graph
2. To make student acquire the knowledge of reading number of zeros of polynomials from graph

Teacher Activity

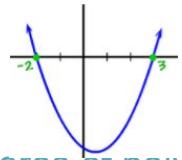
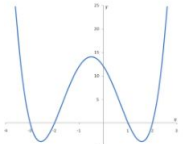
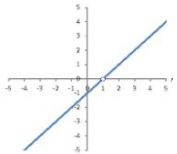
1. Introduce the method of reading graph and find out number of zeros of polynomial
2. Introduce the method of reading graph and find out zeros of polynomials
3. **Resource 2** can be used for explanation
4. Perform **Activity 2**

Check for understanding:

Find the number of zeros of polynomials from the following graphs

a)  b)  c)  d) 

Find the zeros of the given polynomials

a)  b)  c) 

Find zeros of polynomial x^2-1

Resources Suggested

Blackboard, pen and paper
Resource 2 can be used for recap

Activity 2

Name of the activity- Graph activity
Type of activity – group/individual
Use it for – practice
Materials – graph papers
Procedure –

1. Divide the class into a few groups
2. Write down a linear polynomial on the board and ask each student to plot the polynomial on the graph paper [As the students learnt plotting of linear graph in grade 8 they can do it easily. As an extension even quadratic polynomial were also given]
3. Once the students have plotted the graph, ask them to find the zeroes of the polynomials

Subtopic and Learning Objective #3

Sub topic: Relation between zeros and coefficients of the polynomial
LO:
Explore the relationship between zeros of the polynomial and coefficients of the polynomial
1. To make student to understand the relation between zeros and coefficients of the polynomial

Teacher Activity

1. Introduce the relation between zeros and coefficients of the polynomial
2. **Resource 3 can be used for explanation**

Check for understanding:

a) Find the quadratic polynomial whose sum and product of the zeros are 0 and 4 respectively

b) Find the zeroes of the quadratic polynomial $4u^2 + 8u$ and verify the relationship between the zeroes and the coefficients.

c) If α and β are the zeros of the quadratic polynomial f(x0= $5x^2$-7x+1 then find the value of 1+1

d) If α , and β are the zeros of the polynomial f(x) =x^2+px+q form polynomial whose zeros are $(\alpha+\beta)^2$ and $(\alpha-\beta)^2$

Resources Suggested

Blackboard, pen and paper
Resource 3 can be used for recap

Subtopic and Learning Objective #4

Subtopic: Division algorithm of polynomials
LO:
Restate division algorithm of polynomials
1.To make the student state division algorithm and use it to find zeros of polynomials

Teacher Activity

1. Recap division algorithm and explain how it can be used to find zeros of polynomials
2. **Resource 4** can be used for explanation

Check for understanding:

a) Give an example of polynomials f(x), g(x), q(x) and r(x) satisfying that f(x)= g(x) q(x) + r(x) = 0

b) Check whether the polynomial t^2-3 is a factor of $2t^4+3t^3-2t^2-9t-12$ by division method

c) Find the value of 'k' for which the polynomial $x^4+10x^3+25x^2+15x+k$ is exactly divisible by (x+7)

d) Obtain all other zeros of $3x^4+6x^3-2x^2-10x-5$. If two of its zeros are 53and -53

Resources Suggested

Blackboard, pen and paper
Resource 3 can be used for recap

Resource 1

Only one example is given for each definition. Make sure that each definition must be given plenty of examples at the time of explanation

Variable: Alphabets used in place of numbers in algebra are called variables whose value can be changed

Eg: a, x, y…...

Constant : Numbers used in algebra are called constants whose value is fixed

Eg:1, 2, -3……...

Term: A combination of variable and constant with the operation multiplication is called a term

Eg: $2xy$, $3x^2$, $-4a^3b^3$,.....

Like terms: Terms with same variables and powers

Eg: $-3xy$, $4xy$; $--4a^3b^3$, a^3b^3……...

Unlike terms: terms with different variables

Eg: $3xy^2$, $3x^2y^2$ these two terms are unlike as the powers of x are not same

Algebraic expression: A combination of terms with operations + or - or both is called an algebraic expression

Eg: $-xy+8x^2-3x^2y^2+12$, x^3-x^2+4x+7

Monomial: An algebraic expression consisting of one term with positive integral variables is called monomial

Eg: 12, $-3xy$,$-4a^3b^3$ ……..

Resource 1

Binomial: An algebraic expression consisting of two terms with positive integral variables is called binomial
Eg: $-3x^2y^2+12$
Trinomial: An algebraic expression consisting of three terms with positive integral variables is called Trinomial
Eg: $8x^2-3x^2y^2+1$
Polynomial: An algebraic expression consisting of more than three terms with positive integral variables is called Polynomial
Eg: $-xy+ 8x^2-3x^2y^2+2$
Multinomial: An algebraic expression consisting of positive integral variables or negative integral variables or both is called Multinomial
Eg: xy, a, $a^{-4}b^2c^3+ab-3$……..
Degree of a term: The degree of a term is the sum of the exponents of the variables
Eg: -xy the degree of the term is 1+1=2
$2^2a^3b^3$,the degree of the term is 3+3=6[the power of constant should not be added]

Resource 1

Degree of polynomial: The degree of a polynomial is the highest degree of its monomials (individual terms) with non-zero coefficients.

Eg: $-xy+ 8x^2-3x^2y^2+2$

The degree of respective terms is 2, 2, 4, 0 so the degree of the polynomial is 4

Linear polynomial: If the degree of a polynomial is 1 then it is called linear polynomial

Eg: 2x-y, 4-3a, ……...

Quadratic polynomial: If the degree of the polynomial is 2 then it is called as quadratic polynomial

Eg: $3x^2-4y-12$

Cubic polynomial: If the degree of polynomial is 3 then it is called cubic polynomial

Eg: $4x^3-2x^2+x$

Coefficient : Each term of a polynomial has a coefficient

Eg:$3x^3-x^2+4x+7$, coefficient of x^3 is 3, coefficient of x^2 is -1, coefficient of x is 4, coefficient of x^0 is 7[as $x^0=1$ last term can be also written as x^07]

$-xy+ 8x^2-3x^2y^2+2$; coefficient of x is -y, coefficient of y is -x, coefficient of x^2 is $(8-3y^2)$ [The polynomial can be re written as $-xy+(8-3y^2)x^2 +2$,hence coefficient of x^2 is $(8-3y^2)$] , coefficient of y^2 is $-3x^2$, coefficient of x^0 is 2

Resource 1

Zero polynomial: A polynomial whose coefficient is zero is called zero polynomial

Eg: $0\ x^2 - 0\ x + 0$ which results to 0

Or

2 is called as constant polynomial. A constant polynomial 0 is called Zero polynomial

Zero of the polynomial: A value of the variable for the polynomial becomes zero is called zero of the polynomial (or) A value of the variable for the polynomial becomes zero polynomial is called zero of the polynomial

Eg: in 2x-6, x=3 makes the polynomial 0 [2 x 3 -6 =0] hence 3 is called as zero of the polynomial

Note: At this level we consider zero of the polynomial for polynomials with one variable.

Note: Number of zeros of the polynomial depends on it's degree

A linear polynomial has i zero of the polynomial, a quadratic polynomial has 2 and a cubic polynomial has 3

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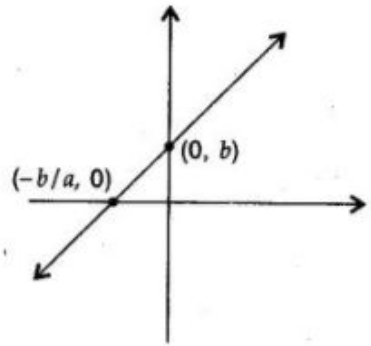
Resource 2

Geometrical meaning of zeros of a polynomial:

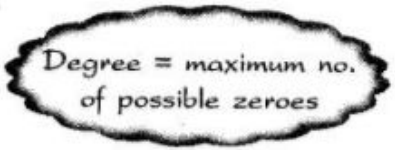
The zero of a polynomial is abscissa of the graph that intersects the x-axis (or) The zeroes of a polynomial p(x) are precisely the x-coordinates of the points, where the graph of y = p(x) intersects the x -axis.

Case 1: Polynomial is a linear polynomial of the form f(x)=ax+b, a≠0

Zero: x= - ba , graph is a straight line



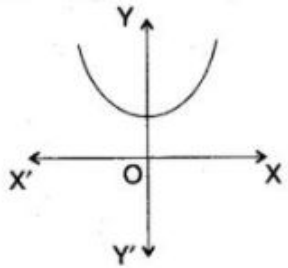
Degree=1 so at most 1 zero for a Linear polynomial

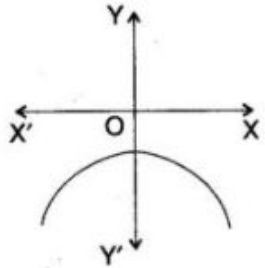


Resource 2

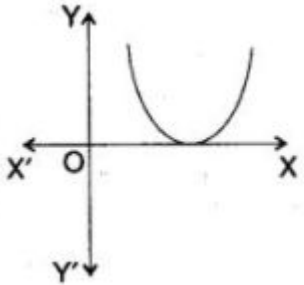
Case: 2

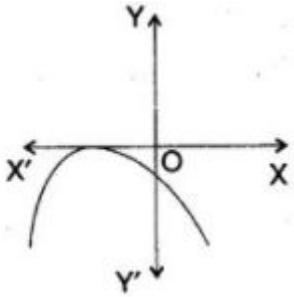
Quadratic polynomial of the general form $ax^2+bx+c=0$. At most zeros[that is no zero/ one zero/ two zeros]





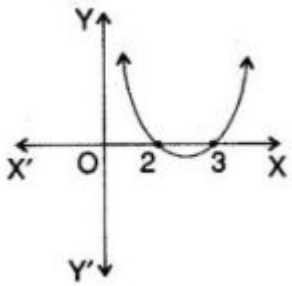
Case i: No zero. Parabola showing such type of polynomials is above or below the x-axis



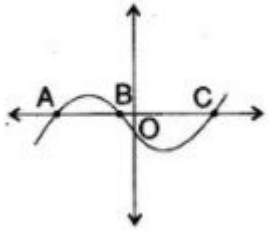


Resource 2

Case ii: One zero. Parabola showing such type of polynomials cuts the x-axis exactly at one point
Case iii: Two zeros. Parabola showing such type of polynomials cuts the x-axis at two points

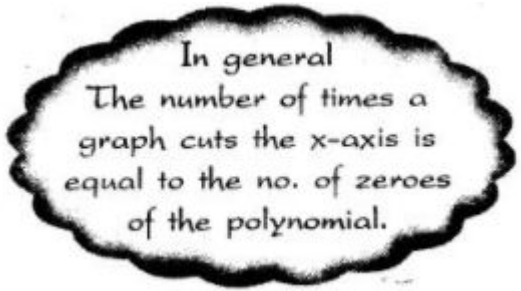


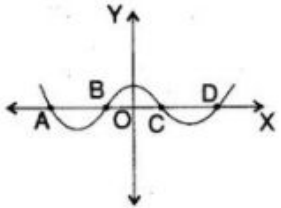
A graph representing a polynomial with three zeros



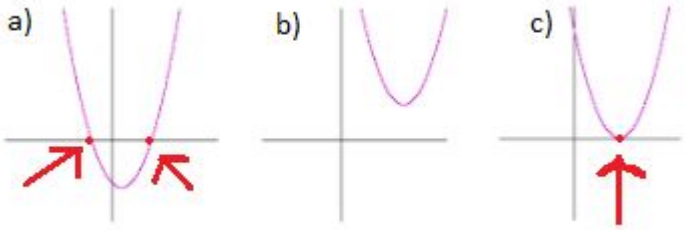
Resource 2

A graph representing a polynomial with four zeros



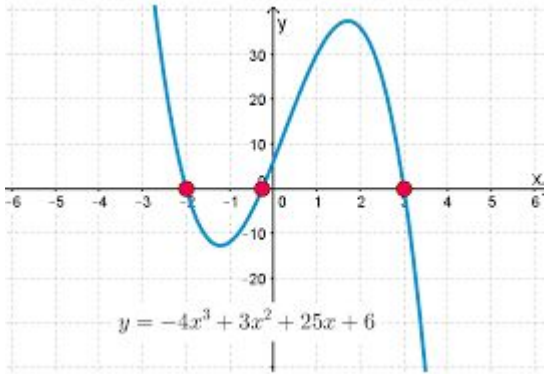


A graph showing zeros with arrow marks

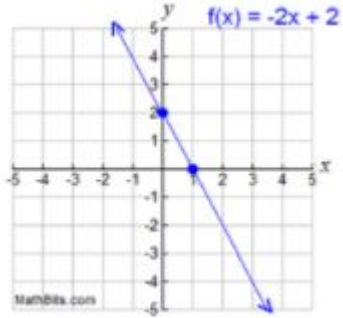


Resource 2

The zeros of the polynomial from the below graph are x= -2, -0.25, 3

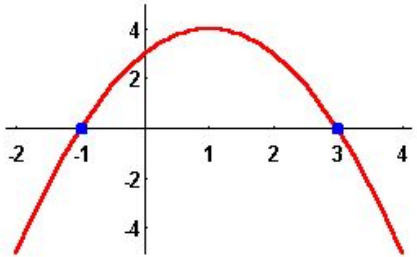


The zeros of the polynomial from the below graph are x= 1



Resource 2

The zeros of the polynomial from the below graph are x= -1,3



Resource 3

RELATIONSHIP BETWEEN ZEROES AND COEFFICIENT OF A POLYNOMIAL.

(*i*) If 'α' (alpha) and 'β' (beta) are the roots of a quadratic polynomial

$$ax^2 + bx + c = 0, \quad a \neq 0.$$

Then

$$\alpha + \beta = \frac{-b}{a}$$

$$\alpha\beta = \frac{c}{a}$$

(*ii*) If 'α' (alpha), β (beta) and γ (gamma) are the roots of the cubic polynomial

$$ax^3 + bx^2 + cx + d = 0$$

Then

$$\alpha + \beta + \gamma = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{-d}{a}$$

Hint : α, β, γ follow the rules of algebraic identities *i.e.*,

$$(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$$

$$\therefore \quad \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

Resource 3

1. Let the polynomial be $x^2 - 12x + 35$

   Comparing with ax^2+bx+c, $a=1$ $b=-12$ and $c=35$

   And zeros are $\alpha = -7$ and $\beta = -5$

   Sum of the zeros $\alpha + \beta = -7 + (-5) = -12 = -\frac{b}{a}$

   Product of the zeros $\alpha \times \beta = -7 \times (-5) = 35 = \frac{c}{a}$

2. Let the polynomial be $-4x^3+3x^2+25x+6$

   Comparing with ax^3+bx^2+cx+d, $a=-4$ $b=3$, $c=25$, $d=6$

   And zeros are $\alpha = -2$, $\beta = -1/4$, $\gamma = 3$

   Sum of the zeros $\alpha + \beta + \gamma = -2+(-1/4)+3 = 3/4 = -\frac{b}{a}$

   Product of zeros= $\alpha \times \beta \times \gamma = -2 \times -1/4 \times 3 = 3/2 = \frac{d}{a}$

   Sum of two taken at a time= $\alpha\beta + \beta\gamma + \gamma\alpha = -2 \times (-1/4) + (-1/4) \times 3 + 3 \times (-2) = -25/4 = \frac{c}{a}$

Resource 4

Division Algorithm For Polynomials

If $p(x)$ and $g(x)$ are any two polynomials with
$g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that
$p(x) = q(x) \times g(x) + r(x)$
where $r(x) = 0$ or degree of $r(x)$ < degree of $g(x)$.
The result is called Division Algorithm for polynomials.

Dividend = Quotient × Divisor + Remainder

Resource 4

Obtain all the zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Sol. Since two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

$x = \sqrt{\frac{5}{3}}, x = -\sqrt{\frac{5}{3}}$

$\Rightarrow \left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = x^2 - \frac{5}{3}$ Or $3x^2 - 5$ is a factor of the given polynomial.

Now, we apply the division algorithm to the given polynomial and $3x^2 - 5$.

$$
\begin{array}{r|l}
 & x^2 + 2x + 1 \\
\hline
3x^2 - 5 & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\
 & 3x^4 \quad\quad - 5x^2 \\
 & - \quad\quad + \\
\hline
 & 6x^3 + 3x^2 - 10x - 5 \\
 & 6x^3 \quad\quad - 10x \\
 & - \quad\quad + \\
\hline
 & 3x^2 \quad\quad - 5 \\
 & 3x^2 \quad\quad - 5 \\
 & - \quad\quad + \\
\hline
 & 0
\end{array}
$$

So, $3x^4 + 6x^3 - 2x^2 - 10x - 5$

$= (3x^2 - 5)(x^2 + 2x + 1) + 0$

Quotient $= x^2 + 2x + 1 = (x + 1)^2$

Zeroes of $(x + 1)^2$ are $-1, -1$.

Hence, all its zeroes are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, -1, -1$.

5. WORKSHEETS

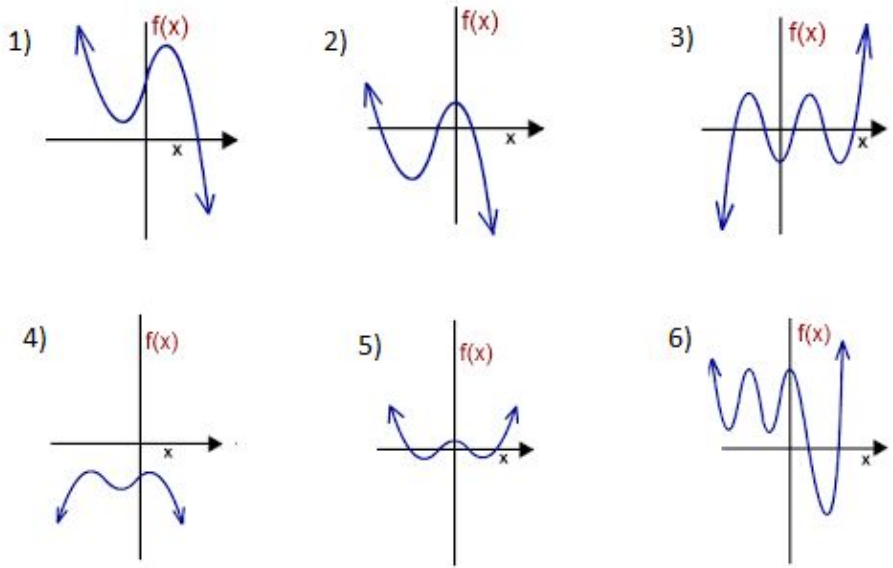
Print or copy the worksheet questions on the blackboard and ask the students to attempt them. They can be given as homework or a class test as well.

Worksheet 1

Use it like: test after the chapter is complete, homework

Answer the following:

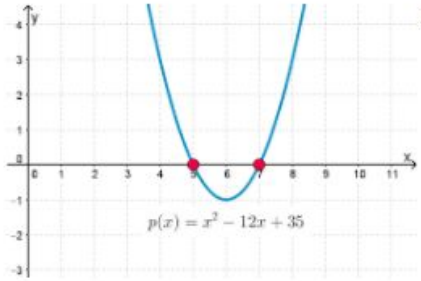
a) **Find the number of zeros of the polynomials represented as graphs below:**



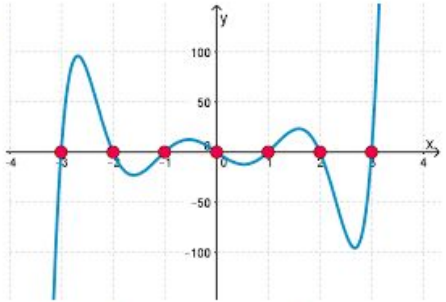
Worksheet 1

Find the zeros of the polynomials represented as graphs

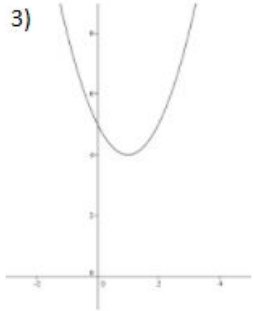
1)



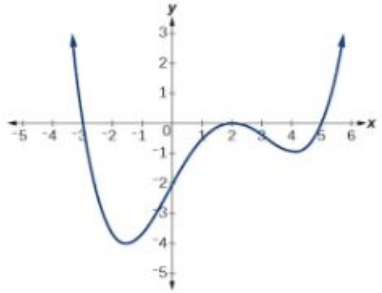
2)



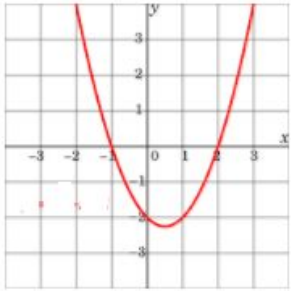
3)



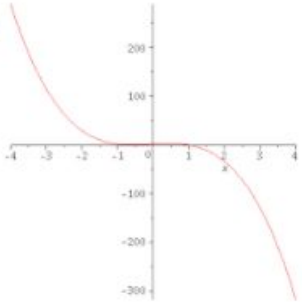
4)



5)



6)



Worksheet 2

Use it like: test after the chapter is complete, homework

Answer the following:

1. If $P(x)= 2x^2-3x+5$ then find $P(-1)$.
2. Find the zeros of the polynomial $3x-5$
3. Find the zeros of $P(x) = x^2-2x-3$
4. Write the quadratic polynomial whose zeros are 3 and -5.
5. Obtain all zeros of the polynomial $x^2+7x+10$ and verify the relation between the zeros and coefficients

Worksheet 3

Use it like: test after the chapter is complete, homework

Answer the following:

1. The sum and product of the zeros of a quadratic polynomial are 2 and -15 respectively. Find the quadratic polynomial?
2. Find the quadratic polynomial whose sum and product of the zeros are a and 1a
3. If one zero of the quadratic polynomial $f(x) = 4x^2-8kx-9$ is negative of the other,find the value of 'k'.
4. If α and β are the zeroes of the quadratic polynomial $f(x)= x^2-x-4$ then find the value of 1+1-
5. If $\alpha \ and \ \beta$ are the zeroes of the polynomial $p\ (x) = 2x^2 + 5x + k$ satisfying the relation $\alpha^2 + \beta^2 + \alpha\beta$ = 214 , then find the value of k.

Worksheet 4

Use it like: test after the chapter is complete, homework

Answer the following:

1. Divide $3x^2-x^3-3x-5$ by $x-1-x^2$ and verify the division algorithm.
2. What must be subtracted from the polynomial $f(x)= x^4+2x^3-13x^2-12x+21$ so that the resulting polynomial is exactly divisible by $g(x)= x^2-4x+3$
3. Find the value of a and b so that $8x^4 + 14x^3 - 2x^2 + ax + b$ is exactly divisible by $4x^2 + 3x - 2$.
4. On dividing a polynomial $3x^3 + 4x^2 + 5x - 13$ by a polynomial g(x), the quotient and the remainder were (3x + 10) and (16x – 43) respectively. Find g(x).
5. Find all the zeroes of the polynomial $f(x) = 2x^4-3x^3-3x^2+6x-2$, if being given that two of its zeroes are 2 and -2

6. SOLUTIONS

This section gives you sample answers for activities and questions listed above.

Worksheet 1

a) 1) one 2) three 3) five 4) No zeros 5) four 6) two

b) 1) x= 5 and 7 2) x= -3, -2, -1, 0, 1, 2, 3 3) No zeros 4) x=-3, 2, 5

5) x= -1, 2 6) x= -1, 0 , 1

Worksheet 2

1. 10
2. 5/3
3. 3 and -1
4. $x^2+2x-15$
5. Zeros are -5 and -2. Verification need to be checked

Worksheet 3

1. x^2-2x-15
2. x2-ax+1a
3. k=0
4. 15/4
5. k=2

Worksheet 4

1. Quotient= -x-4, Remainder=-1. Verification need to be checked
2. 2x-3[remainder when f(x) is divided by g(x)] must be subtracted
3. a= -7, b=2
4. $x^2 - 2x + 3$
5. x=1, 12

7. EXTRA RESOURCES

Carry out the following suggested activities/ tasks if you have the time and resources to do them

Teacher can use these videos if time and facilities permit:

1. https://www.youtube.com/watch?v=rP-__zFngio