CHAPTER: Arithmetic Progression

Grade: **10**
Board: **CBSE**
Book: **NCERT**

1.

Chapter Summary and Placement

<Read through the following to understand how this chapter is connected with the previous and next grades and also the important concepts covered in it.>

1. This chapter is connected indirectly to the sequences and patterns learnt in lower grades
2. Here we learn to find unknown terms and sum of the series
3. This connects to chapters related to sequences and series of future grades

2.

LEARNING OUTCOMES

<On completion of the chapter, students will be able to:>

1. Recognize the patterns in numbers
2. Acquire the knowledge of Arithmetic progression and its components
3. Find general term and specific term of progression
4. Find sum of first n terms of progression
5. Apply the concepts to solve Higher order thinking problems[worksheet]

4. TEACHING PLAN

Follow this section for getting ideas on teaching the content covered in the chapter. The subtopic given in the first column with the matching learning outcome can be explained using instructions and resources listed subsequently

Subtopic and Learning Objective #1

Subtopic:Sequence and identification of pattern
LO:
Recognize the patterns in numbers
1.To make the student understand the meaning of sequence and its pattern

Teacher Activity

Introduce definition with examples and write the rule of pattern

Check for understanding:

a) Check whether following are in sequential order and write the rule behind it

1) 2, 1, 5, -6, 10…….

2) 2,4,8,16,32……..

3) 4,20,100,500,2500……..

4) 1000,998,996,994,992……..

5) If the sequence follows the rule n(n+1)/2 write the sequence

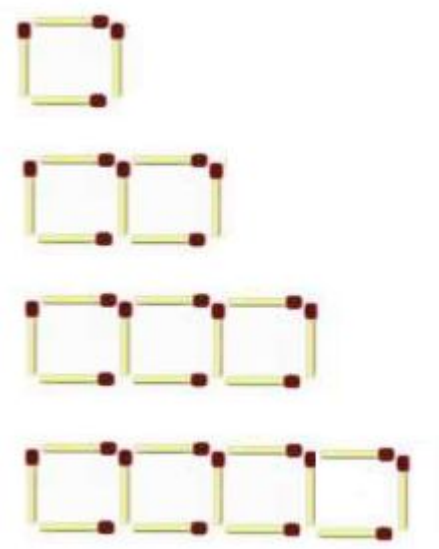
Give some more examples for practice

Resources Suggested

Blackboard, pen and paper
Resource 1 for reference

Resource 1

Sequence: An ordered arrangement of numbers according to the given rule is called sequence
Observe the following pattern of squares formed by match sticks



Resource 1

Number of squares	1	2	3	4
Number of match sticks used	4	7	10	13

Rule followed here is to find number of match sticks= 3 x number of squares + 1

If we have to find number of match sticks used for a pattern of 20 squares = 3 x 20 + 1 =61

The above sequence can be written as 4, 7, 10, 13, 16, 19, 22, ……
Here are some more sequences:
3, 7, 11, 15, 19……. Rule of the sequence: write 3 and add 4 to ge next number
4, 1, -2, -5, -8…….Rule of the sequence: write 4 and subtract 3 to get next number
10, -10, 10, -10, 10, -10……..Rule of the sequence: Write 10 and -10 alternatively
4, 12, 36, 108……Rule of the sequence: Write 4 and multiply by 3 to get next number
0.1, 0.01, 0.001, 0.0001……...Rule of the sequence: write 0.1 and multiply by 0.1 to get next number/ write 0.1 and divide by 10 to get next number

Resource 1

3, 9, 27, 81……. Rule of the sequence: Powers of 3 starting from 3^1
In the given sequences some are increasing sequences and some are decreasing sequences

If the general rule is given sequence can be written by using it
Sequence with the rule 2^n-1 starting with n=1 is: 1, 3, 7, 15, 31…….
Sequence with the rule 5n+1 starting with n=0 is: 1, 6, 11, 16, 21, 26……..

Terms of the sequence: The individual numbers that forms the sequence are called terms of the sequence. And are denoted by $a_1,a_2,a_3,.....a_n$

The sequence is finite of Infinite as the number of terms of the sequence is finite or infinite

Arithmetic Progression: A sequence is called **arithmetic progression** if the difference of any two consecutive terms is constant

In the sequence 4, 7, 10, 13, 16, 19, 22, ……; 7-4=3; 16-13=3; 22-19=3

Resource 1

This difference is constant and so called as **common difference** and is denoted by ‘d’.

First term of arithmetic sequence is generally denoted as ‘a’

Hence general form of arithmetic sequence is a, a+d, a+2d, a+3d, a+4d…….a+(n-1)d
Where nth term (or) general term a_n=a+(n-1)d

And also common difference can be obtained as d= a_n - a_{n-1}

In a finite sequence n^{th} term from the last can be found out by the formula **l-(n-1)d,** where l is last term
Eg.: 50, 100, 150, 200, 250, 300, 350, 400, 450, 500
In the above sequence 4^{th} term from the end can be found as 500-(4-1) x 50=500-150=350

Resource 1

Sum Of n numbers of an A.P

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] \qquad = \frac{n}{2}(l + a)$$

Where a and l are first and last terms of AP

Subtopic and Learning Objective #2

Sub topic:Arithmetic progression
LO:
Acquire the knowledge of Arithmetic progression and its components
1. To make student to understand arithmetic progression and its components

Teacher Activity

Introduce the definition and its components with good number of examples

Check for understanding:

a) Which of the following are in AP. Justify your answer

1) 3, 7, 11, 15, 19…….
2) 4, 1, -2, -5, -8……..
3) 10, -10, 10, -10, 10, -10……..
4) 4, 12, 36, 108……
5) 0.1, 0.01, 0.001, 0.0001……
6) 3, 9, 27, 81…….

Resources Suggested

Blackboard, pen and paper
Resource 1 for reference

Activity

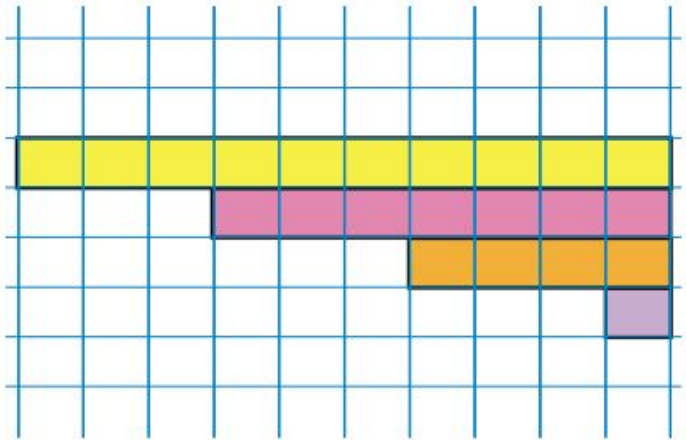
Name of the activity- Checking progression geometry
Type of activity – Individual
Use it for – understanding
Materials – graph papers-2, color papers, scissors, glue stick
Procedure –

1. Take two Sequences 1,4,7,10..... and 1,4,6,9..... To check whether in AP or not
2. Cut the lengths of the strips as the numbers in sequences
3. Paste the strips on different graph sheets
4. And check if the increase in lengths is uniform or not. If they are uniform then the sequence is in AP otherwise not

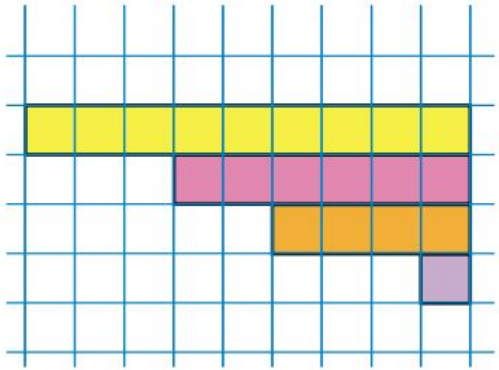
Here are the images for teachers reference

Activity

Sequence of an AP image.



Sequence of that is not in AP image



Subtopic and Learning Objective #3

Sub topic:Specific term and general term of the sequence
LO:
Find general term and specific term of progression
1. To make student
To find general term and specific term by using formulae

Teacher Activity

Introduce formulae of general term of the AP and method of finding specific term as given in NCERT text book
Introduce the method of finding number of terms, n^{th} term from the back of an AP

Check for understanding:

a) Find the general term or n^{th} term of an whose first term is 20 and common difference is -4
b) Find the common difference if $a_{18}-a_{13}=5$
c) 19 th term of the arithmetic progression 9, 4, -1, -6 is ____
d) Find the first term, common difference and the next term of the sequence 44,56,68,80,92,104,116,128
e) Check whether the following terms form n^{th} term of an AP or not? Justify your answer
1) $2n-3$
2) $3n^2+5$
3) $1+n+n^2$

Resources Suggested

Blackboard, pen and paper
Resource 1 for reference

Subtopic and Learning Objective #4

Sub topic:Sum of first n terms of an AP
LO: **Find sum of first n terms of progression**
1. To make student to find sum of first n terms

Teacher Activity

Introduce method of finding sum of first n terms of an AP.

Check for understanding:

a) What is the value of the following series: 6 + 13 + 20 + 27 + ... + 111?

b) The first and the last term of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum

c) The first term of an AP is 5 and the last term is 45 and the sum is 400. Find the number of terms and common difference of AP

d) If the nth term of an arithmetic progression is given by -7 - 6n, then what is the sum of the first 20 terms of the AP?

Resources Suggested

Blackboard, pen and paper
Resource 1 for reference

5. WORKSHEETS

Print or copy the worksheet questions on the blackboard and ask the students to attempt them. They can be given as homework or a class test as well.

Worksheet 1

Use it like: test after the chapter is complete, homework

Answer the following:

Find out the rule behind the following sequences where n is number of shapes:

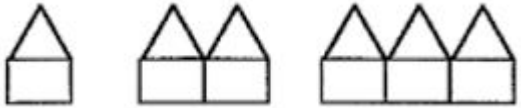
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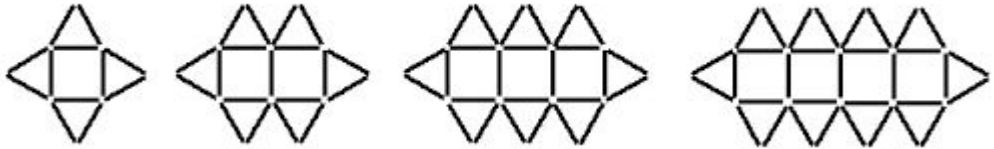
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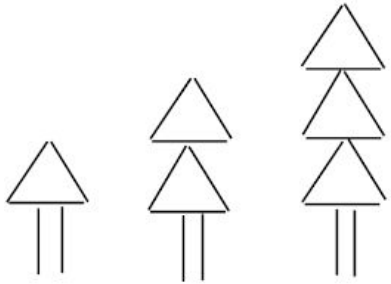
3.



4.



5.



Worksheet 2

Use it like: test after the chapter is complete, homework

a) Check which of the following situations form a sequence in AP write 5 terms of sequence

1) Fee charged by a school from class I to XII monthly, when class I monthly fee is ₹ 400 and increases by ₹ 50 for every class
2) Number of people left in an auditorium if the people leave as a batch of 25 members when the initial number is 1000
3) A sum of ₹ 1000 is to be used to give cash prizes to students of a school for their overall academic performance. If each prize is Rs 50 less than its preceding prize. If the highest prize is ₹300
4) A bacteria doubles for every second

b) Answer the following:

1) Write the first five terms of the sequence whose first term is -5 and common difference is 5/2
2) Find the first term, common difference and the missing terms of the sequence 34,46,58,70,82,94,.....142
3) A decreasing sequence with seven terms of the sequence starting with 5 and differs by 1/3

Worksheet 3

Use it like: test after the chapter is complete, homework

Answer the following:

1. Find the 19 th term of the arithmetic progression 9, 4, -1, -6……..
2. What is the difference between the 28^{th} term and 20^{th} term in the arithmetic progression 11, 25, 39, 53?
3. Which term of the A.P 3, 8, 13, 18, … is 78
4. Find the number of terms of the sequence 5, 4.75, 4.5……0
5. Which term will be the first negative term in the arithmetic progression 98, 92, 86,...?
6. Find whether 55 is a term of the sequence 7, 10, 13…….. If yes find which term it is
7. Find the number of integers between 100 and 200 which are divisible by 9?
8. Find the 21^{st} term from the end of an AP -2,-4,-6, ……..-100
9. If the 3^{rd} and the 9^{th} terms of an AP are 4 and -8 respectively. Which term of this A.P is zero
10. The sum of 5th and 7th terms of an AP is 52 and 10th term is 46. Find the AP?

Worksheet 4

Answer the following:

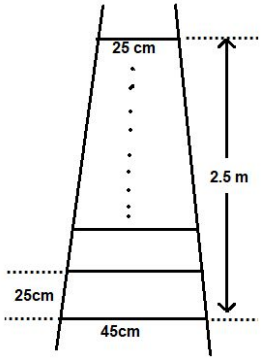
1. If the nth term of an arithmetic progression is given by -7 - 6n, then what is the sum of the first 20 terms of the AP?
2. The sum of first 10 terms of an arithmetic progression is 230. The sum of the first 20 terms is 860. What is the sum of the first 30 terms of the AP?
3. Find the sum of first 22 terms of an AP is which d= 7 and 22^{nd} term is 149
4. Find the sum of first 50 positive integers divisible by 6.
5. In an AP $S_n=n(4n+1)$, Find AP

Worksheet 5

Use it like:Practice

Answer the following:

1. The sum of 3rd and 7th terms of an AP is 6 and their product is 8. Find the common difference,first term and sum of first sixteen terms of AP?



2. A ladder has rungs 25 cm apart as in figure. The rungs decrease uniformly in length from 45cm at the bottom to 25 cm at the top. If top and bottom rungs are 2.5m apart,What is the length of the wood required for rungs?

3. The houses of row are numbered consecutively from 1 to 49. Show that there is a value of ***x*** such that sum of numbers of the houses preceding the ***x*** is equal to the sum of the numbers of the following it. Find ***x***?

[Hint: Sum of numbers of the houses before x= Sum of numbers of the houses after x

S_{x-1}=Sum of numbers of the houses x+1 to 49

But, Sum of numbers of the houses from x+1 to 49= $S_{49} - S_x$

$S_{x-1} = S_{49} - S_x$]

6. SOLUTIONS

This section gives you sample answers for activities and questions listed above.

Worksheet 1

1. $3n+1$
2. $2n+1$
3. $5n+1$
4. $7n+5$
5. $3n+2$

Worksheet 2

a)

1) In AP; 400, 450, 500, 550, 600…….

2) In AP; 1000, 975, 950, 900, 875, ……...

3) In AP; 300, 250, 200, 150, 100.

4) Not in AP; 1, 2, 4, 8, 16, ...

b)

1) -5, -5/2, 0, 5/2, 5

2) a=34, d=12, missing terms= 106, 118, 130

c) 5, 14/3, 13/3, 4, 11/3, 10/3, 3.

Worksheet 3

1. -81
2. 112
3. 16^{th} term
4. 21 terms
5. 18^{th} term
6. Yes, 17^{th} term
7. 11 numbers
8. -60
9. 5^{th} term
10. 1, 6, 11, 16, 21……..

Worksheet 4

1. -1700
2. S_{30}=1890
3. S_{22}=1661
4. S_{50}= 7650
5. 5, 13, 21, 29, 37……...

Worksheet 5

1. d= ±1/2,
   a=1 when d= 1/2
   a=5 when d= -½
   S_{16}=20
2. Number of rungs=number of parts the ladder has been divided[250/25=10]+1 = 11
   Wood required= S_{11}= 350 cm
3. x=35

7. EXTRA RESOURCES

Carry out the following suggested activities/ tasks if you have the time and resources to do them

Teacher can use these videos if time and facilities permit:

1. https://www.youtube.com/watch?v=IWX2MobGsnE - Arithmetic progression formulae