PLEASE ENSURE THAT THIS QUESTION BOOKLET CONTAINS 120 QUESTIONS SERIALLY NUMBERED FROM 1 TO 120 PRINTED PAGES 32.

1. $\int_0^1 \frac{x}{x^2-4}\,dx =$

(A) $-\frac{\pi^2}{6}$ (B) $-\frac{22}{7}$ (C) $\ln\left(\frac{\sqrt{3}}{2}\right)$ (D) $\ln\left(\frac{3}{2}\right)$ (E) $\ln\left(\frac{3}{\sqrt{2}}\right)$

2. If $(2,-6), (5,2)$ and $(-2,2)$ constitute the vertices of a triangle, then the line joining the origin and its orthocentre is

(A) $x+4y=0$ (B) $x-4y=0$ (C) $4x-y=0$

(D) $4x+y=0$ (E) $x-y=0$

3. If a straight line in XY plane passes through $(-a,-b), (a,b), (k,k), (a^2,a^3)$, for some real numbers a, b and k, where $a \neq 0$, then which of the following options is correct ?

(A) $k = 0$ when $a \neq b$

(B) k is necessarily a positive real number when $a = b$

(C) k is any positive real number when $a \neq b$

(D) $k = a$ or $k = b$ necessarily

(E) $k \neq 0$ when $a \neq b$

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4. The line perpendicular to $4x - 5y + 1 = 0$ and passing through the point of intersection of the straight lines $x + 2y - 10 = 0$ and $2x + y + 5 = 0$ is

(A) $5x + 4y = 0$ (B) $y + \frac{5}{4}x = \frac{50}{3}$ (C) $5x + 4y = 1$

(D) $y + \frac{5}{4}x = -\frac{50}{3}$ (E) $4x + 5y = 0$

5. A thin particle moves from $(0, 1)$ and gets reflected upon hitting the x-axis at $(\sqrt{3}, 0)$. Then the slope of the reflected line is

(A) $\frac{1}{\sqrt{3}}$ (B) $-\frac{1}{\sqrt{3}}$ (C) $\sqrt{3}$ (D) $-\sqrt{3}$ (E) 0

6. If the two sides AB and AC of a triangle are along $4x - 3y - 17 = 0$ and $3x + 4y - 19 = 0$, then the equation of the bisector of the angle between AB and AC is

(A) $x + 7y + 2 = 0$ (B) $7x - y - 36 = 0$ (C) $7x - y + 36 = 0$

(D) $x = y$ (E) $x - 7y + 2 = 0$

7. A point moves in such a way that it remains equidistant from each of the lines $3x \pm 2y = 5$. Then the path along which the point moves is

(A) $x = -\frac{5}{3}$ (B) $y = \frac{5}{3}$ (C) $x = \frac{5}{3}$ (D) $y = -\frac{5}{3}$ (E) $x = 0$

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8. Suppose the line $mx - y + 5m - 4 = 0$ meets the lines $x + 3y + 2 = 0$, $2x + 3y + 4 = 0$ and $x - y - 5 = 0$ at the points R, S and T, respectively. If R, S and T are at distances r_1, r_2 and r_3, respectively, from $(-5, -4)$ and $\left(\frac{15}{r_1}\right)^2 + \left(\frac{10}{r_2}\right)^2 = \left(\frac{6}{r_3}\right)^2$ then the value of m is

(A) $-\frac{2}{3}$ (B) $\frac{2}{3}$ (C) $\frac{3}{2}$ (D) $-\frac{3}{2}$ (E) 1

9. Suppose the point $P(1, 1)$ is translated to Q in the direction of $y = 2x$. If $PQ = 1$, then Q is

(A) (2, 0) (B) (0, 2) (C) $\left(\frac{\sqrt{2}+1}{\sqrt{2}}, \frac{\sqrt{2}+1}{\sqrt{2}}\right)$

(D) $\left(\frac{\sqrt{5}+1}{\sqrt{5}}, \frac{\sqrt{5}+2}{\sqrt{5}}\right)$ (E) $\left(\frac{2+\sqrt{3}}{2}, \frac{3}{2}\right)$

10. Suppose the line joining distinct points P and Q on $(x-2)^2 + (y-1)^2 = r^2$ is the diameter of $(x-1)^2 + (y-3)^2 = 4$. Then the value of r is

(A) 2 (B) 3 (C) 1 (D) 9 (E) 4

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11. The equation of the circle that can be inscribed in the square formed by $x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$ is

(A) $x^2 - 8x - 14y + 61 = 0$
(B) $x^2 - 8x - 14y + 71 = 0$
(C) $x^2 - 4x - 7y + 61 = 0$
(D) $x^2 - 4x - 7y + 71 = 0$
(E) $x^2 + 8x + 14y - 61 = 0$

12. For the circle $C: x^2 + y^2 - 6x + 2y = 0$, which of the following is incorrect

(A) the radius of C is $\sqrt{10}$
(B) $(3, -1)$ lies inside of C
(C) $(7, 3)$ lies outside of C
(D) the line $x + 3y = 0$ intersects C
(E) one of diameters of C is not along $x + 3y = 0$

13. For $i = 1, 2, 3, 4$, suppose the points $(\cos\theta_i, \sec\theta_i)$ lie on the boundary of a circle, where $\theta_i \in [0, \frac{\pi}{6})$ are distinct. Then $\cos\theta_1 \cos\theta_2 \cos\theta_3 \cos\theta_4$ equals

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $\frac{1}{16}$ (E) 1

14. The set of points of the form $(t^2 + t + 1, t^2 - t + 1)$, where t is a real number, represents a/an

(A) circle
(B) parabola
(C) ellipse
(D) hyperbola
(E) pair of straight lines

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15. Suppose a and b are the lengths of major and minor axes of an ellipse that passes through the points $(4, 3)$ and $(-1, 4)$. If the major axis of the ellipse lies along the x-axis, then the value of $\frac{1}{a^2} + \frac{16}{b^2}$ is

(A) 4 (B) $\frac{1}{4}$ (C) 2 (D) $\frac{1}{2}$ (E) 1

16. For a real number t, the equation $(1+t)x^2 + (t-1)y^2 + t^2 - 1 = 0$ represents a hyperbola provided

(A) $|t| < 1$ (B) $|t| > 1$ (C) $|t| = 1$

(D) $t \in (1, \infty]$ (E) $t \in (-\infty, -1]$

17. Given the points $A(6, -7, 0), B(16, -19, -4), C(0, 3, -6)$ and $D(2, -5, 10)$, the point of intersection of the lines AB and CD is

(A) $(-1, 1, 2)$ (B) $(1, -1, 2)$ (C) $(1, -1, -2)$

(D) $(-1, 1, -2)$ (E) $(1, 1, 2)$

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18. If the $xz-$ plane divides the straight line joining the points $(2,4,7)$ and $(3,-5,8)$ in the ratio $\alpha:1$, then the value of α is

(A) $\frac{5}{4}$ (B) $\frac{1}{3}$ (C) $\frac{7}{8}$ (D) $\frac{4}{5}$ (E) $\frac{5}{2}$

19. If θ_1, θ_2 and θ_3 are the angles made by a line with the positive directions of the x, y, z axes, then the value of $\cos 2\theta_1 + \cos 2\theta_2 + \cos 2\theta_3$ is

(A) -1 (B) 1 (C) 2 (D) -2 (E) 0

20. The angle between the lines, whose direction cosines are proportional to $4, \sqrt{3}-1, -\sqrt{3}-1$ and $4, -\sqrt{3}-1, \sqrt{3}-1$, is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$ (E) π

30 45 60 90

21. Suppose P is the point on the line joining $(-9,4,5)$ and $(11,0,-1)$ that lies closest to the origin O. Then $|OP|^2$ equals to

(A) 3 (B) 4 (C) 2 (D) 9 (E) 1

22. The plane that is perpendicular to the planes $x-y+2z-4=0$ and $2x-2y+z=0$ and passes through $(1,-2,1)$ is

(A) $x+y+1=0$ (B) $2x+y+z-1=0$ (C) $x+y+z=0$

(D) $2x+y-z+1=0$ (E) $x+z-2=0$

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23. The line of intersection of the planes $3x - 6y - 2z - 15 = 0$ and $2x + y - 2z - 5 = 0$ is

(A) $\frac{x+3}{14} = \frac{y+1}{-2} = \frac{z}{15}$ (B) $\frac{x+3}{-14} = \frac{y+1}{2} = \frac{z}{15}$ (C) $\frac{x-3}{14} = \frac{y+1}{2} = \frac{z}{-15}$

(D) $\frac{x+3}{14} = \frac{y-1}{2} = \frac{z+1}{15}$ (E) $\frac{x-3}{14} = \frac{y+1}{2} = \frac{z}{15}$

24. The plane passing through the points $(2, 1, 0), (5, 0, 1)$ and $(4, 1, 1)$ intersects the x-axis at

(A) $(3, 0, 0)$ (B) $(-3, 0, 0)$ (C) $(0, 0, 0)$ (D) $(1, 0, 0)$ (E) $(-1, 0, 0)$

25. Suppose a line parallel to $ax + by = 0$ (where $b \neq 0$) intersects $5x - y + 4 = 0$ and $3x + 4y - 4 = 0$, respectively, at P and Q. If the midpoint of PQ is $(1, 5)$, then the value of $\frac{a}{b}$ is

(A) $\frac{107}{3}$ (B) $-\frac{107}{3}$ (C) $\frac{3}{107}$ (D) $-\frac{3}{107}$ (E) 1

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26. Let $f: \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = x^2 + 9$. The range of f is

(A) $\mathbb{R}$ (B) $(-\infty, -9] \cup [9, \infty)$ (C) $[9, \infty)$

(D) $[3, \infty)$ (E) $[3, \infty) \cup (-\infty, -3]$

27. Let $f(x) = \frac{x-1}{x+1}$. Let $S = \{x \in \mathbb{R} \mid fof^{-1}(x) = x \text{ does not hold}\}$.
The cardinality of S is

(A) a finite number, but not equal to 1, 2, 3 (B) 3

(C) 2 (D) 1

(E) infinite

28. The domain of the real valued function $f(x) = \sqrt{x^2 - 4} + \frac{1}{\sqrt{x^2 - 7x + 6}}$ is

(A) $\mathbb{R} - [-6, -2)$ (B) $\mathbb{R} - [-6, 2)$ (C) $\mathbb{R} - [-2, 6)$

(D) $\mathbb{R} - (2, 6]$ (E) $\mathbb{R} - (-2, 6]$

29. The number of solutions of the equation $\frac{1}{2}(x^3 + 1) = \sqrt[3]{2x - 1}$ is

(A) 0 (B) 6 (C) 9

(D) Infinite (E) 3

30. Let a, b, c, d be an increasing sequence of real numbers, which are in geometric progression. If $a + d = 112$ and $b + c = 48$, then the value of $\frac{a+c+8}{b}$ is

(A) 1 (B) 5 (C) 4 (D) 3 (E) 2

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31. Let a, b be two real numbers between 3 and 81 such that the resulting sequence $3, a, b, 81$ is in a geometric progression. The value of $a + b$ is

(A) 29 (B) 90 (C) 27 (D) 81 (E) 36

32. Let $a_1, a_2, a_3, \ldots$ be an increasing sequence of natural numbers, which are in an arithmetic progression with common difference d. Suppose $a_1 + a_2 + a_3 = 27$ and $a_1^2 + a_2^2 + a_3^2 = 275$. Then the values of a_1, d are

(A) $a_1 = 3;\ d = 2$ (B) $a_1 = -5;\ d = 4$

(C) $a_1 = 4;\ d = 5$ (D) $a_1 = -4;\ d = 5$

(E) $a_1 = 5;\ d = 4$

33. The sides of a right-angled triangle are in an arithmetic progression. If the area of the triangle is 54, then the length of the longest side is

(A) 6 (B) 12 (C) 15 (D) 9 (E) 18

34. Let A be $(2n + 1) \times (2n + 1)$ matrix with integer entries and positive determinant, where $n \in \mathbb{N}$. If $AA^T = I = A^T A$, then which of the following statements always holds ?

(A) $\det(A) = 0$ (B) $\det(A + I) \neq 0$

(C) $\det(A + I) = 0$ (D) $\det(A - I) = 0$

(E) $\det(A - I) \neq 0$

35. The inequality $\frac{2x-1}{3} \geq \frac{3x-2}{4} - \frac{(2-x)}{5}$ holds for x belonging to

(A) $\mathbb{R}$ (B) $(-\infty, 3]$

(C) $(-\infty, -3] \cup [3, \infty)$ (D) $(-\infty, 2]$

(E) $(-\infty, 2] \cup [4, \infty)$

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36. The contrapositive of the statement "If the number is not divisible by 3, then it is not divisible by 15" is

(A) If the number is not divisible by 3, then it is not divisible by 15
(B) If the number is not divisible by 15, then it is not divisible by 3
(C) If the number is not divisible by 15, then it is divisible by 3
(D) If the number is divisible by 15, then it is divisible by 3
(E) If the number is divisible by 15, then it is not divisible by 3

37. Let A be an invertible matrix of size 4×4 with complex entries. If the determinant of $adj(A)$ is 5, then the number of possible value of determinant of A is

(A) 1 (B) 4 (C) 6 (D) 3 (E) 2

38. The determinant of the matrix

$$\begin{bmatrix} 1 & 4 & 8 \\ 1 & 9 & 27 \\ 1 & 16 & 64 \end{bmatrix}$$

(A) 13 (B) 208 (C) 104
(D) 26 (E) 52

39. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A.adjA = AA^T$, then which of the following statements is true

(A) $5a - b = -5$ (B) $5a + b = 10$ (C) $\det(A) < 0$
(D) A is symmetric (E) $\det(A) \geq 0$

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40. Suppose $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ is an adjoint of the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$.

The value of $\frac{a_1+b_2+c_3}{b_1 a_2}$ is

(A) 0 (B) 3 (C) 1 (D) 2 (E) 4

41. If $x + iy = \frac{1}{(1+\cos\theta)+i\sin\theta}$, then the value of $x^2 + 1$ is

(A) $\frac{7}{4}$ (B) $\frac{13}{4}$ (C) $\frac{1}{4}$

(D) $\frac{9}{4}$ (E) $\frac{5}{4}$

42. If α, β, γ are the cube roots of -2, then the value of $\frac{x\alpha+y\beta+z\gamma}{x\beta+y\gamma+z\alpha}$ is (x, y, z are variables)

(A) $e^{i\pi/3}$ (B) $e^{2\pi i/3}$ (C) 1

(D) -1 (E) $e^{4\pi i/3}$

43. Let $x + \frac{1}{x} = 2\cos\alpha$. For any $n \in \mathbb{N}$, the value of $x^n - \frac{1}{x^n}$ is

(A) $\cos(n\alpha)$ (B) $2\cos(n\alpha)$ (C) $2i\sin(n\alpha)$

(D) $i\sin(n\alpha)$ (E) $4\cos(n\alpha)$

44. If $f(z) = z^n + a_{n-1}z^{n-1} + \cdots + a_1 z^n + a_0 \in \mathbb{R}[z]$ is a polynomial in z with no root over $\mathbb{R}$, then $\deg(f)$ is

(A) 9 (B) always ≤ 4

(C) an odd number (D) always ≥ 4

(E) an even number

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45. Let $S = \{n \in \mathbb{N} | n^3 + 3n^2 + 5n + 3$ is not divisible by $3\}$. Then, which of the following statements is true about S

(A) $S = \phi$

(B) $|S| \geq 2$ and $|S|$ is a multiple of 5

(C) S is non-empty but $|S|$ is finite

(D) $|S|$ is infinite

(E) S is non-empty and $|S|$ is a multiple of 3

46. If the coefficients of $(5r + 4)^{th}$ term and $(r - 1)^{th}$ term in the expansion of $(1 + x)^{25}$ are equal, then r is

(A) 6 (B) 3 (C) 5 (D) 2 (E) 4

47. For any $n \geq 0$, the value of $\frac{\sum_{r=0}^{n} (4r+3) \cdot ({}^nC_r)^2}{(2n+3)}$ is

(A) ${}^{2n}C_{n-1}$ (B) ${}^{8n}C_n$ (C) ${}^{2n}C_{n+1}$ (D) ${}^nC_{n-2}$ (E) ${}^{2n}C_n$

48. The number of ways in which we can distribute n identical balls in k boxes is

(A) ${}^nC_k$ (B) ${}^nC_{(k-1)}$ (C) ${}^{(n+k-1)}C_{(k-1)}$

(D) ${}^{(n-1)}C_{(k-1)}$ (E) ${}^{(n+k)}C_n$

49. Suppose there are 5 alike dogs, 6 alike monkeys and 7 alike horses. The number of ways of selecting one or more animals from these is

(A) 362 (B) 363 (C) 336 (D) 335 (E) 337

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50. Consider the following Linear Programming Problem (LPP) :

Maximize $Z = 60x_1 + 50x_2$

subject to

$x_1 + 2x_2 \leq 40$

$3x_1 + 2x_2 \leq 60$

$x_1, x_2 \geq 0$

Then, the

(A) LPP has a unique optimal solution.

(B) LPP is infeasible.

(C) LPP is unbounded.

(D) LPP has multiple optimal solutions.

(E) LPP has no solution.

51. Consider the linear programming problem :

Minimize $3x_1 + 4x_2 + 2x_3$

subject to

$x_1 + x_2 + x_3 \leq 6$

$x_1 + 2x_2 + x_3 \leq 10$

$x_1, x_2, x_3 \geq 0$

Then, the number of basic solutions are

(A) 7 (B) 9 (C) 10 (D) 8 (E) 3

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52. In a linear programming problem, the restrictions under which the objective function is to be optimised are called as

(A) decision variables (B) objective function (C) constraints

(D) integer solutions (E) optimal solutions

53. Which of the following is the correct formulation of linear programming problem

(A) Max $Z = 2x_1 + x_2$; subject to $x_1 + x_2 \leq 10$; $x_1 \leq 3$; $x_1 \geq 0$; $x_2 \leq 0$

(B) Max $Z = 3x_1 + 2x_2$; subject to $x_1 + 2x_2 \geq 11$; $3x_1 + x_2 \geq 24$; $x_1, x_2 \leq 0$

(C) Min $Z = x_1 + 5x_2$; subject to $2x_1 + 5x_2 \leq 10$; $x_1 + 3x_2 \leq 9$; $x_1, x_2 \geq 0$

(D) Min $Z = 4x_1 + 3x_2$; subject to $x_1 + 9x_2 \geq 8$; $2x_1 + 5x_2 \leq 9$; $x_1 \leq 0, x_2 \geq 0$

(E) Max $Z = 2x_1 + 5x_2$; subject to $4x_1 + 9x_2 \leq 8$; $2x_1 + 3x_2 \leq 9$; $x_1, x_2 \leq 0$

54. Let A and B be two independent events such that the odds in favour of A and B are 1:1 and 3:2, respectively. Then the probability that only one of the two occurs is

(A) 0.6 (B) 0.7 (C) 0.8 (D) 0.5 (E) 0.4

55. A six faced fair die is rolled for a large number of times. Then, the mean value of the outcomes is

(A) 4.5 (B) 2.5 (C) 3.5 (D) 1.5 (E) 3

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56. Let the probability distribution of random variable X be

X	–2	–1	1	2	3
P(X = x)	k	$2k$	$2k$	k	$3k$

Then, the value of $E(X^2)$ is

(A) $\frac{19}{9}$ (B) $\frac{13}{3}$ (C) $\frac{35}{9}$ (D) $\frac{11}{3}$ (E) $\frac{7}{3}$

57. Let the standard deviation of x_1, x_2 and x_3 be 9. Then, the variance of $3x_1 + 4$, $3x_2 + 4$ and $3x_3 + 4$ is

(A) 243 (B) 81 (C) 729 (D) 9 (E) 733

58. If the median of the observations $4, 6, 7, x, x + 2, 12, 12, 13$ arranged in an increasing order is 9, then the variance of these observations is

(A) $\frac{37}{4}$ (B) $\frac{38}{4}$ (C) 8 (D) 9 (E) 10

59. Let $\bar{x}$ denote the mean of the observations $1, 3, 5, a, 9$ and $\bar{y}$ denote the mean of the observations $2, 4, b, 6, 8$ where $a, b > 0$. If $\bar{x} = \bar{y}$, the value of $2(a - b)$ is

(A) 2 (B) 38 (C) 8 (D) – 4 (E) 4

60. Consider two independent events E and F such that $P(E) = \frac{1}{4}$, $P(E \cup F) = \frac{2}{5}$ and $P(F) = a$. Then, the value of a is

(A) $\frac{13}{20}$ (B) $\frac{1}{20}$ (C) $\frac{1}{4}$ (D) $\frac{1}{5}$ (E) $\frac{3}{5}$

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61. There are two cash counters A and B for placing orders in a college canteen. Let E_A be the event that there is a queue at counter A and E_B denotes the event that there is a queue at counter B. If $P(E_A) = 0.45$, $P(E_B) = 0.55$ and $P(E_A \cap E_B) = 0.25$, then the probability that there is no queue at both the counters is

(A) 0.75 (B) 0.15 (C) 0.25 (D) 0.20 (E) 1.75

62. Let $S = \{a, b, c\}$ be the sample space with the associated probabilities satisfying $P(a) = 2P(b)$ and $P(b) = 2P(c)$. Then the value of $P(a)$ is

(A) $\frac{1}{5}$ (B) $\frac{2}{7}$ (C) $\frac{1}{7}$ (D) $\frac{1}{6}$ (E) $\frac{4}{7}$

63. A coin is tossed thrice. The probability of getting a head on the second toss given that a tail has occurred in at least two tosses is

(A) $\frac{1}{2}$ (B) $\frac{1}{16}$ (C) $\frac{1}{8}$ (D) $\frac{1}{4}$ (E) $\frac{1}{3}$

64. Let X be a random variable following Binomial distribution; $Bin(n, p)$, where n is the number of independent Bernoulli trials and p is the probability of success. If $E(X) = 1$ and $Var(X) = \frac{4}{5}$, then the values of n and p are

(A) $n = 5, p = \frac{4}{5}$
(B) $n = 1, p = \frac{1}{5}$
(C) $n = 1, p = 1$
(D) $n = 5, p = \frac{1}{5}$
(E) $n = 1, p = \frac{4}{5}$

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65. A box contains 10 coupons, labelled as 1, 2, ...10. Three coupons are drawn at random and without replacement. Let X_1, X_2 and X_3 denote the numbers on the coupons. Then the probability that $\max\{X_1, X_2, X_3\} < 7$ is

(A) $\frac{{}^{3}C_1}{{}^{10}C_3}$ (B) $\frac{{}^{7}C_3}{{}^{10}C_3}$ (C) $\frac{{}^{3}C_3}{{}^{10}C_3}$

(D) $\frac{{}^{3}C_1}{{}^{10}C_7}$ (E) $\frac{{}^{6}C_3}{{}^{10}C_3}$

66. An electric bulb manufacturing company manufactures three types of electric bulbs A, B and C. In a room containing these three types of electric bulbs, it is known that 6% of type A electric bulbs are defective, 4% of type B electric bulbs are defective and 2% of type C electric bulbs are defective. An electric bulb is selected at random from a lot containing 50 type A electric bulbs, 30 type B electric bulbs and 20 type C electric bulbs. The selected electric bulb is found to be defective. Then the probability that the selected electric bulb was type A is

(A) $\frac{2}{23}$ (B) $\frac{23}{500}$ (C) $\frac{12}{23}$ (D) $\frac{15}{23}$ (E) $\frac{6}{115}$

67. For four observations x_1, x_2, x_3, x_4, it is given that $\sum_{i=1}^{4} x_i^2 = 656$ and $\sum_{i=1}^{4} x_i = 32$. Then, the variance of these four observations is

(A) 144 (B) 730 (C) 120 (D) 248 (E) 182.5

68. An urn contains 8 black marbles and 4 white marbles. Two marbles are chosen at random and without replacement. Then the probability that both marbles are black is

(A) $\frac{7}{33}$ (B) $\frac{2}{3}$ (C) $\frac{7}{11}$ (D) $\frac{14}{33}$ (E) $\frac{21}{143}$

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69. A box contains 100 tickets numbered $00, 01, 02, \ldots 99$ and a ticket is drawn at random. Let X denote the sum of the digits on that ticket and Y denote the product of those digits. Then the value of $P(X = 2|Y = 0)$ is

(A) $\frac{3}{19}$ (B) $\frac{6}{19}$ (C) $\frac{1}{19}$ (D) $\frac{2}{19}$ (E) $\frac{1}{100}$

70. Let the coefficient of variation of two datasets be 50 and 75, respectively and the corresponding variances be 25 and 36, respectively. Also let $\bar{x}_1$ and $\bar{x}_2$ denote the corresponding sample means. Then $\bar{x}_1 + \bar{x}_2$ is

(A) 2 (B) 10 (C) 18 (D) 20 (E) 16

71. The mean deviation about the median for the data 3, 5, 9, 3, 8, 10, 7 is

(A) $\frac{23}{7}$ (B) $\frac{4}{7}$ (C) $-\frac{4}{7}$ (D) $\frac{16}{7}$ (E) $\frac{17}{7}$

72. A biased die is rolled such that the probability of getting k dots, $1 \leq k \leq 6$, on the upper face of the die is proportional to k. Then the probability that five dots appear on the upper face of the die is

(A) $\frac{16}{21}$ (B) $\frac{2}{21}$ (C) $\frac{1}{21}$ (D) $\frac{3}{21}$ (E) $\frac{5}{21}$

73. Let $\Omega = \{1, 2, 3, 4, 5\}$ be the sample space with the events $A = \{1, 2, 5\}$, $B = \{1, 3, 5\}$ and $C = \{2, 3, 5\}$. Let E^c denote the complement of an event E. Then $P((A \cap B)^c \cup C^c)$ is

(A) $\frac{1}{5}$ (B) $\frac{3}{5}$ (C) $\frac{2}{5}$ (D) $\frac{4}{5}$ (E) 1

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74. For any real number x, the least value of $4\cos x - 3\sin x + 5$ is

(A) 10 (B) 2 (C) 0 (D) 8 (E) 4

75. Let $P(x) = \cos^2 x + \sin^4 x$, for any $x \in \mathbb{R}$. Then which of the following options is correct for all x ?

(A) $\frac{1}{6} \leq P(x) \leq \frac{3}{4}$ (B) $0 \leq P(x) \leq \frac{1}{2}$ (C) $0 \leq P(x) \leq 1$

(D) $\frac{1}{2} \leq P(x) \leq \frac{3}{2}$ (E) $\frac{3}{4} \leq P(x) \leq 1$

76. Let α and β be such that $\alpha + \beta = \pi$. If $\cos\alpha = \frac{1}{\sqrt{2}}$, then the value of $\cot(\beta - \alpha)$ is

(A) ∞ (B) 1 (C) $\frac{1}{2}$ (D) $\frac{1}{4}$ (E) 0

77. The value of $\operatorname{cosec} 20^\circ \tan 60^\circ - \sec 20^\circ$ is

(A) 0 (B) 1 (C) 2 (D) 4 (E) 6

78. If $\alpha + \beta + \gamma = 2\pi$, then the value of $\cot\frac{\alpha}{2}\cot\frac{\beta}{2} + \cot\frac{\alpha}{2}\cot\frac{\gamma}{2} + \cot\frac{\beta}{2}\cot\frac{\gamma}{2}$ is

(A) 0 (B) 1 (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{3}$ (E) $\frac{1}{2}$

79. Let p, q and r be real numbers such that $|r| > \sqrt{p^2 + q^2}$. Then the equation $p\cos\theta + q\sin\theta = r$ has

(A) exactly one real solution.
(B) exactly two real solutions.
(C) infinite number of real solutions.
(D) no real solution.
(E) integer solutions.

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80. If $x \in (0, \pi)$ satisfies the equation $6^{1+\sin x+\sin^2 x+\cdots} = 36$, then the value of x is

(A) 0 (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$ (E) $\frac{\pi}{4}$

81. The value(s) of $a\ (\neq 0)$ for which the equation $\frac{1}{2}(x-2)^2 + 1 = \sin\left(\frac{a}{x}\right)$ holds is/are

(A) $(4n+1)\pi, n \in \mathbb{Z}$ (B) $2(n-1)\pi, n \in \mathbb{Z}$

(C) $n\pi, n \in \mathbb{N}$ (D) $\frac{n\pi}{2}, n \in \mathbb{N}$

(E) 1

82. If x is a real number such that $\tan x + \cot x = 2$, then $x =$

(A) $\left(n+\frac{1}{4}\right)\pi, n \in \mathbb{Z}$ (B) $(n+1)\pi, n \in \mathbb{Z}$

(C) $\left(n+\frac{1}{2}\right)\pi, n \in \mathbb{Z}$ (D) $n\pi, n \in \mathbb{Z}$

(E) $\frac{2}{3}n\pi, n \in \mathbb{Z}$

83. If $\frac{1+\sin x}{1-\sin x} = \frac{(1+\sin y)^3}{(1-\sin y)^3}$ for some real values x and y, then $\frac{\sin x}{\sin y} =$

(A) $\frac{3+\sin^2 y}{1+3\sin^2 y}$ (B) $\frac{3+\cos^2 y}{1+3\cos^2 y}$

(C) $\frac{3+\sin^2 y}{1-3\sin^2 y}$ (D) $\frac{3+\sin^2 y}{1-3\cos^2 y}$

(E) $\frac{1+3\sin^2 y}{1-3\cos^2 y}$

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84. Let k be a real number such that $\sin\frac{3\pi}{14}\cos\frac{3\pi}{14} = k\cos\frac{\pi}{14}$. Then the value of $4k$ is

(A) 1 (B) 2 (C) 3 (D) 4 (E) 0

85. In a triangle ABC, if $\cos^2 A - \sin^2 B + \cos^2 C = 0$, then the value of $\cos A \cos B \cos C$ is

(A) $\frac{1}{4}$ (B) 1 (C) $\frac{\pi}{2}$ (D) $\frac{1}{2}$ (E) 0

86. The value of $\cos^{-1}\left(\cos\left(\frac{7\pi}{4}\right)\right)$ is

(A) 0 (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$ (E) $\frac{\pi}{6}$

87. The value of $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{2}{5}\right)$ is

(A) $\tan^{-1}(5)$ (B) $\tan^{-1}\left(\frac{1}{5}\right)$ (C) $\tan^{-1}\left(\frac{2}{3}\right)$ (D) $\tan^{-1}\left(\frac{8}{9}\right)$

(E) $\tan^{-1}\left(\frac{9}{8}\right)$

88. The value of $\tan^{-1}\left(\sqrt{3}\right) - \sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$ (E) $\frac{\pi}{6}$

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89. Let $\vec{a} = \hat{\imath} - \hat{\jmath} + 2\hat{k}$. Then the vector in the direction of $\vec{a}$ with magnitude 5 units is

(A) $5\hat{\imath} - 5\hat{\jmath} + 10\hat{k}$ (B) $-5\hat{\imath} - 5\hat{\jmath} + 10\hat{k}$ (C) $\frac{1}{\sqrt{6}}(5\hat{\imath} - 5\hat{\jmath} + 10\hat{k})$

(D) $\frac{1}{\sqrt{6}}(-5\hat{\imath} - 5\hat{\jmath} + 10\hat{k})$ (E) $\frac{1}{\sqrt{6}}(10\hat{\imath} - 5\hat{\jmath} + 5\hat{k})$

90. Let $\vec{a} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\vec{b} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ be two vectors. Then the unit vector in the direction of $\vec{a} - \vec{b}$ is

(A) $\frac{1}{\sqrt{10}}(2\hat{\jmath} - 3\hat{k})$ (B) $\frac{1}{\sqrt{10}}(3\hat{\jmath} - \hat{k})$ (C) $3\hat{\jmath} - \hat{k}$

(D) $\frac{1}{\sqrt{5}}(2\hat{\jmath} - 3\hat{k})$ (E) $\frac{-1}{\sqrt{5}}(2\hat{\jmath} - 3\hat{k})$

91. The direction cosines of the vector $\vec{a} = -2\hat{\imath} + \hat{\jmath} - \hat{k}$ are

(A) $\left(\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$ (B) $\left(\frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\right)$ (C) $\left(\frac{-2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\right)$

(D) $\left(\frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$ (E) $\left(\frac{-2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

92. The value of λ for which the vectors $\hat{\imath} + \hat{\jmath} - \hat{k}$ and $\lambda\hat{\imath} + 3\hat{\jmath} + \hat{k}$ are perpendicular is

(A) –2 (B) 2 (C) 0 (D) 1 (E) –1

93. The position vectors of two points P and Q are given by $\overrightarrow{OP} = 2\vec{a} - \vec{b}$ and $\overrightarrow{OQ} = \vec{a} + 3\vec{b}$, respectively. If a point R divides the line joining P and Q internally in the ratio 1:2, then the position vector of the point R is

(A) $\frac{1}{3}(5\vec{a} - \vec{b})$ (B) $\frac{1}{3}(5\vec{a} + \vec{b})$ (C) $\frac{1}{3}(\vec{a} - 5\vec{b})$

(D) $\frac{1}{3}(\vec{a} + 5\vec{b})$ (E) $\frac{1}{3}(\vec{a} + \vec{b})$

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94. Let $\vec{a}$ and $\vec{b}$ be perpendicular vectors such that $|\vec{a}| = \sqrt{104}$ and $|\vec{b}| = 6$. Then the value of $|\vec{a} - \vec{b}|$ is

(A) $\sqrt{110}$ (B) $\sqrt{140}$ (C) $\sqrt{98}$ (D) $\sqrt{55}$ (E) $\sqrt{70}$

95. Let x be a real number and $\vec{a}$ be any non-zero vector such that $|(4-x)\vec{a}| < |3\vec{a}|$. Then which of the following options is correct ?

(A) $0 < x < 6$ (B) $0 < x < 7$ (C) $1 < x < 7$

(D) $1 \leq x \leq 7$ (E) $0 \leq x \leq 6$

96. The value of λ for which the vectors $2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $-4\hat{\imath} + \lambda\hat{\jmath} - 8\hat{k}$ are collinear is

(A) 0 (B) 1 (C) 3 (D) 6 (E) 4

97. The projection of the vector $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ on the vector $\vec{b} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ is

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) $\frac{2}{3}$ (D) $\frac{1}{3}$ (E) 0

98. Let $f(x) = \begin{cases} -5 & , \ x \leq 0 \\ x - 5 & , \ x > 0 \end{cases}$ and $g(x) = |f(x)| + 2f(|x|)$. Then $g(-2)$ will be

(A) -1 (B) -15 (C) 1 (D) 0 (E) -11

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99. Let $[\cdot]$ denote the greatest integer function and $f(x) = [x] + |2 - x|, \ -1 \leq x \leq 4$. Then

(A) f is continuous at $x = 2$.

(B) f is not continuous at $x = 1$.

(C) f is continuous at $x = 0$.

(D) f is differentiable at $x = 3$.

(E) f is not differentiable at $x = \frac{3}{2}$.

100. $\lim_{x \to 0} \frac{e^x - 1}{3(1 - e^{2x})} =$

(A) $\frac{1}{6}$ (B) $-\frac{1}{6}$ (C) 3 (D) 0 (E) $-\frac{1}{3}$

101. Let $f(x) = \left(1 - \frac{1}{x}\right)^2$, $x > 0$. Then

(A) f is increasing in $(0, 2)$ and decreasing in $(2, \infty)$.

(B) f is decreasing in $(0, 2)$ and increasing in $(2, \infty)$.

(C) f is increasing in $(0, 1)$ and decreasing in $(1, \infty)$.

(D) f is decreasing in $(0, 1)$ and increasing in $(1, \infty)$.

(E) f is increasing in $(0, \infty)$.

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102. Let $f: \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} 3e^x & \text{if } x < 0 \\ x^2 + 3x + 3 & \text{if } 0 \le x < 1 \\ x^2 - 3x - 3 & \text{if } x \ge 1 \end{cases}$$

(A) f is continuous on $\mathbb{R}$.

(B) f is not continuous on $\mathbb{R}$.

(C) f is continuous on $\mathbb{R} \setminus \{0\}$.

(D) f is continuous on $\mathbb{R} \setminus \{1\}$.

(E) f is not continuous on $\mathbb{R} \setminus \{0, 1\}$.

103. Let $f(x) = \pi \cos x + x^2$. The value of $c \in (0, \pi)$ where f attains its local maximum / minimum is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{3\pi}{4}$ (D) $\frac{\pi}{3}$ (E) $\frac{\pi}{6}$

104. The minimum of $f(x) = \sqrt{10 - x^2}$ in the interval $[-3, 2]$ is

(A) $\sqrt{4}$ (B) $\sqrt{6}$ (C) 1 (D) 0 (E) $\sqrt{10}$

105. The equation of the line passing through origin which is parallel to the tangent of the curve $y = \frac{x-2}{x-3}$ at $x = 4$ is

(A) $y = 2x$ (B) $y = -2x + 1$ (C) $y = -x$

(D) $y = x + 2$ (E) $y = 4x$

106. Let $f(x) = \alpha \sin^2 3x$. If $f'\left(\frac{\pi}{12}\right) = -3$, then the value of α is

(A) -1 (B) $-\pi$ (C) π (D) $\frac{\pi}{2}$ (E) 1

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107. Let $f: \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} 2x+3 & , \quad x \leq 5 \\ 3x+a & , \quad x > 5 \end{cases}$$

Then the value of a so that f is continuous on $\mathbb{R}$ is

(A) 2 (B) -2 (C) 3 (D) -3 (E) 0

108. If $y = x^{e^x} + x^e$ for $x > 0$, then $\frac{dy}{dx}$ is equal to

(A) $x^{e^x}\left[\frac{1}{x} + \ln x\right] + e^x$

(B) $x^{e^x} e^x \left[\frac{1}{x} + \ln x\right] + ex^{e-1}$

(C) $e^x \cdot x^{e^x - 1} + e\, x^e$

(D) $x^{e^x} e^{-x}\left[\frac{1}{x} - \ln x\right] + ex^{e-1}$

(E) $x^{e^x} e^x \left[\frac{1}{x} - \ln x\right] + ex^{e-1}$

109. $\lim\limits_{x \to 0} \frac{\ln(1+(\ln 5)x)}{5^x - 1} =$

(A) 1 (B) $\ln 5$ (C) -1 (D) 5 (E) $\frac{1}{5}$

110. $\int \frac{1}{x^2 - 2x + 2}\, dx =$

(A) $\tan^{-1}(x-1) + C$

(B) $\sin^{-1}(2x-1) + C$

(C) $\sin^{-1}(x-1) + C$

(D) $\tan^{-1}(2x-1) + C$

(E) $\frac{1}{(2x-1)^3} + C$

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111. $\int \sin^2 \pi x \, dx =$

(A) $\frac{x}{2} - \frac{1}{4\pi} \sin 2\pi x + C$

(B) $\frac{x}{2} + \frac{1}{8\pi} \sin 4\pi x + C$

(C) $\frac{x}{8} - \frac{1}{4\pi} \cos 2\pi x + C$

(D) $x + \frac{1}{2\pi} \sin 2\pi x + C$

(E) $\frac{x}{2} - \frac{1}{2\pi} \cos 2\pi x + C$

112. $\int \frac{x+5}{x^2-1} dx =$

(A) $3 \ln|x-1| - 2 \ln|x+1| + C$

(B) $2 \ln|x-1| - 3 \ln|x+1| + C$

(C) $\ln|x-2| + \ln|x+1| + C$

(D) $\ln|x+2| + \ln|x-1| + C$

(E) $2 \ln|x-1| + 3 \ln|x+1| + C$

113. $\int \frac{2 \tan x + 3}{\sin^2 x + 2\cos^2 x} dx =$

(A) $\frac{3}{\sqrt{2}} \sin^{-1}\left(\frac{\sin x}{\sqrt{2}}\right) + ln|\sin^2 x + 2| + C$

(B) $\frac{3}{\sqrt{2}} \tan^{-1}\left(\frac{\tan x}{\sqrt{2}}\right) + ln|\tan^2 x + 2| + C$

(C) $\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{\tan x}{\sqrt{2}}\right) - ln|\tan^2 x + 2| + C$

(D) $\frac{3}{\sqrt{2}} \cos^{-1}\left(\frac{\cos x}{\sqrt{2}}\right) + ln|\sin^2 x + 2| + C$

(E) $\frac{1}{\sqrt{2}} \cos^{-1}\left(\frac{\cos x}{\sqrt{2}}\right) - ln|\cos^2 x + 2| + C$

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114. $\int x \log(1+x^2)\, dx =$

(A) $\frac{1}{2}(1+x^2)\log(1+x^2)+\frac{x^2}{2}+C$

(B) $\frac{1}{2}(1+x^2)\log(1+x^2)-\frac{x^2}{2}+C$

(C) $\frac{1}{2}(1+x^2)\log(2+x^2)-\frac{x^2}{2}+C$

(D) $(1+x^2)\log(1+x^2)+(1+x^2)+C$

(E) $(1-x^2)\log(1+x^2)+(1-x^2)+C$

115. Let $f:\mathbb{R}\to\mathbb{R}$ be defined by

$$f(x)=\begin{cases} x & \text{if } x\le 1 \\ -x+2 & \text{if } x>1 \end{cases}$$

Then $\int_0^2 f(x)\, dx =$

(A) $\frac{\pi}{2}$ (B) 1 (C) 2 (D) 4 (E) $\frac{\pi}{6}$

116. $\int \frac{1}{\cos x\,(\sin x+2\cos x)}\, dx =$

(A) $\ln|1-\tan x|+C$ (B) $\ln|3+\sin x|+C$ (C) $\ln|2+\tan x|+C$

(D) $\ln|1+2\sec x|+C$ (E) $\ln|2-\tan x|+C$

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117. $\int_0^1 \frac{2e^x}{1+e^{2x}}\,dx =$

(A) $4(tan^{-1}2 - \pi)$ (B) $2(tan^{-1}e - \frac{\pi}{2})$ (C) $2(tan^{-1}e + \frac{\pi}{4})$

(D) $2(tan^{-1}e - \frac{\pi}{4})$ (E) $2(tan^{-1}2 + \pi)$

118. $\int_0^1 (5xe^{2x} - \tan\frac{\pi}{4})\,dx =$

(A) $\frac{5}{4}e^2 + \frac{1}{4}$ (B) $-\frac{5}{4}e^2 - \frac{1}{4}$ (C) $\frac{5}{4}e^2 - \frac{9}{4}$

(D) $\frac{3}{4}e^2 + \frac{1}{4}$ (E) $\frac{1}{4}e^2 + \frac{5}{4}$

119. The area of the region in the first quadrant enclosed by the curves $y = \sqrt{x},\ y = -x + 6$ and the x-axis is

(A) $\frac{22}{7}$ (B) $\frac{22}{3}$ (C) 12 (D) 24 (E) 8

120. The area of the region in the first quadrant which is above the parabola $y = x^2$ and enclosed by the circle $x^2 + y^2 = 2$ and the y-axis is

(A) $\frac{1}{6} + \frac{\pi}{4}$ (B) $\frac{1}{12} + \frac{\pi}{6}$ (C) $-\frac{1}{6} + \frac{\pi}{4}$

(D) $\frac{1}{4} + \frac{\pi}{6}$ (E) $-\frac{\pi^2}{2} + 4$

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