

Inverse Trigonometric Functions Ex 4.1 Q2.

$$\text{Let } \cos^{-1}\left(\frac{1}{2}\right) = x. \text{ Then, } \cos x = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right).$$

$$\therefore \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

$$\text{Let } \sin^{-1}\left(\frac{1}{2}\right) = y. \text{ Then, } \sin y = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right).$$

$$\therefore \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

$$\therefore \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + \frac{2\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$$

$$\therefore \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$$

$$\therefore \tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right] = \tan^{-1}\left[2\cos\left(2 \times \frac{\pi}{6}\right)\right]$$

$$= \tan^{-1}\left[2\cos\frac{\pi}{3}\right] = \tan^{-1}\left[2 \times \frac{1}{2}\right]$$

$$= \tan^{-1}1 = \frac{\pi}{4}$$

Concept Insight:

Solve the innermost bracket first, so first find the principal value of $\sin^{-1}(1/2)$

$$\text{Let } \tan^{-1}(1) = x. \text{ Then, } \tan x = 1 = \tan\frac{\pi}{4}.$$

$$\therefore \tan^{-1}(1) = \frac{\pi}{4}$$

$$\text{Let } \cos^{-1}\left(-\frac{1}{2}\right) = y. \text{ Then, } \cos y = -\frac{1}{2} = -\cos\left(\frac{\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right).$$

$$\therefore \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$$

$$\text{Let } \sin^{-1}\left(-\frac{1}{2}\right) = z. \text{ Then, } \sin z = -\frac{1}{2} = -\sin\left(\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right).$$

$$\therefore \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$

$$\therefore \tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$$

$$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$$

$$= \frac{3\pi + 8\pi - 2\pi}{12} = \frac{9\pi}{12} = \frac{3\pi}{4}$$

$$\text{Let } \cos^{-1}\left(\frac{1}{2}\right) = x. \text{ Then, } \cos x = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$$

$$\therefore \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

$$\text{Let } \sin^{-1}\left(-\frac{1}{2}\right) = y. \text{ Then, } \sin y = -\frac{1}{2} = -\sin\left(\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right)$$

$$\therefore \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$

$$\therefore \cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{3} - \left(-\frac{2\pi}{6}\right) = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$$

Inverse Trigonometric Functions Ex 4.1 Q3.

$$\text{Let } \sin^{-1}\left(\frac{1}{2}\right) = x. \text{ Then, } \sin x = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$$

$$\therefore \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

$$\text{Let } \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = y. \text{ Then, } \sin y = \frac{1}{\sqrt{2}} = \sin\left(\frac{\pi}{4}\right)$$

$$\therefore \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

$$\therefore \sin^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{6} - \frac{2\pi}{4} = \frac{\pi}{6} - \frac{\pi}{2} = \frac{\pi - 3\pi}{6} = -\frac{\pi}{3}$$

$$\text{Let } \sin^{-1}\left(-\frac{1}{2}\right) = x. \text{ Then, } \sin x = -\frac{1}{2} = \sin\left(-\frac{\pi}{6}\right)$$

$$\therefore \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$

$$\text{Let } \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = y. \text{ Then, } \cos y = \frac{-\sqrt{3}}{2} = \cos\left(\pi - \frac{\pi}{6}\right)$$

$$\therefore \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \frac{5\pi}{6}$$

$$\therefore \sin^{-1}\left(-\frac{1}{2}\right) + 2\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = -\frac{\pi}{6} + \frac{10\pi}{6} = \frac{-\pi + 10\pi}{6} = \frac{9\pi}{6} = \frac{3\pi}{2}$$

$$\text{Let } \tan^{-1}(-1) = x. \text{ Then, } \tan x = -1 = -\tan\left(\frac{\pi}{4}\right) = \tan\left(\pi - \frac{\pi}{4}\right)$$

$$\therefore \tan^{-1}(-1) = \frac{3\pi}{4}$$

$$\text{Let } \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = y. \text{ Then, } \cos y = \frac{-1}{\sqrt{2}} = -\cos\left(\frac{\pi}{4}\right) = \cos\left(\pi - \frac{\pi}{4}\right)$$

$$\therefore \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = \frac{3\pi}{4}$$

$$\therefore \tan^{-1}(-1) + \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = \frac{3\pi}{4} + \frac{3\pi}{4} = \frac{6\pi}{4} = \frac{3\pi}{2}$$

$$\text{Let } \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = x. \text{ Then, } \sin x = -\frac{\sqrt{3}}{2} = -\sin\left(\frac{\pi}{3}\right) = \sin\left(\pi - \frac{\pi}{3}\right)$$

$$\therefore \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{2\pi}{3}$$

$$\text{Let } \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = y. \text{ Then, } \cos y = \frac{\sqrt{3}}{2} = \cos\left(\frac{\pi}{6}\right)$$

$$\therefore \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$$

$$\therefore \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{2\pi}{3} + \frac{\pi}{6} = \frac{4\pi + \pi}{6} = \frac{5\pi}{6}$$

$$\text{Let } \tan^{-1}(\sqrt{3}) = x. \text{ Then, } \tan x = \sqrt{3} = \tan\left(\frac{\pi}{3}\right)$$

$$\therefore \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$$

$$\text{Let } \sec^{-1}(-2) = y. \text{ Then, } \sec y = -2 = \sec\left(\pi - \frac{\pi}{3}\right)$$

$$\therefore \sec^{-1}(-2) = \frac{2\pi}{3}$$

$$\therefore \tan^{-1}(\sqrt{3}) - \sec^{-1}(-2) = \frac{\pi}{3} - \frac{2\pi}{3} = \frac{\pi - 2\pi}{3} = -\frac{\pi}{3}$$