[This question paper contains 4 printed pages.]

4478 *Your Roll No.* ....

B.A. (Prog.) / I **G-II**

Paper Code : A-155

MATHEMATICS – Paper I

(Algebra and Calculus)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :– The maximum marks printed on the question paper are applicable for the students of Category 'A'. These marks will, however, be scaled up proportionately in respect of the students of SOL at the time of posting of awards for compilation of result.

All questions are compulsory and have equal marks.

Attempt any **two parts** *from each question.*

1. (a) Show that vectors (1, 2, 1), (2, 1, 0), (1, -1, 2) form a basis of R^3.

   (b) Is the following system of equations

   $2x + 3y + 4z = 11$

$x + 5y + 7z = 15$

$3x + 11y + 13z = 25$

consistent? If yes, then solve it.

(c) Verify Cayley Hamilton Theorem for the matrix

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ 2 & 1 & 4 \end{bmatrix}$$

Hence compute A^{-1}, the inverse of A.

2. (a) If

$\sin\theta + \sin\phi + \sin\psi = 0,$

$\cos\theta + \cos\phi + \cos\psi = 0,$

show that

$\cos 3\theta + \cos 3\phi + \cos 3\psi = 3\cos(\theta + \phi + \psi)$ and

$\sin 3\theta + \sin 3\phi + \sin 3\psi = 3\sin(\theta + \phi + \psi).$

(b) Solve the equation:

$3x^3 + 11x^2 + 12x + 4 = 0$

the roots being in H.P.(Harmonic Progression).

(c) If α, β, λ are the roots of the equation:

$x^3 + qx + r = 0$, such that no two of them are equal in magnitude but opposite in sign. Find the values of:

(i) $\sum \frac{1}{\beta+\gamma}$

(ii) $\sum \frac{\beta^2+\gamma^2}{\beta+\gamma}$

3. (a) Examine the continuity of the function f defined by

$$f(x) = \begin{cases} \frac{x-|x|}{x} & x \neq 0 \\ x & x = 0 \end{cases}$$

at the point $x = 0$.

(b) If $y = e^{m} sin^{-1}x$, then

$(1 - x^2)y_{n+2} - (2n + 1)x\, y_{n+1} - (n^2 + m^2)y_n = 0.$

(c) If u = $\tan^{-1} \frac{x+y}{\sqrt{x}+\sqrt{y}}$, prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{4}\sin 2u$$

4. (a) Find the equation of the tangent to the parabola $y^2 = 4x + 5$, which is parallel to the line $y - 2x + 3 = 0$.

P.T.O.

(b) Show that the position and nature of the double points on the following curve

$x^3 - y^3 - 7x^2 + 4y + 15x - 13 = 0.$

(c) Trace the curve

$y^2(2a - x) = x^3.$

5. (a) Verify the Lagrange's mean value theorem for the function

$$f(x) = x(x-1)(x-2) \text{ in } \left[0, \frac{1}{2}\right].$$

(b) Define Maclaurin's infinite series expansion with Taylor's remainder and hence find the expansion of $y = \sin x$.

(c) Evaluate $\lim_{x \to 0} \frac{xe^x - \log(1+x)}{x^2}$

6. (a) Evaluate $\int \frac{x^2 + 4x}{\sqrt{x^2 + x + 2}}\, dx$

(b) Find the area common to the parabolas $y^2 = 4ax$ and $x^2 = 4by$.

(c) Find the volume of the solid generated by rotating the ellipse $4x^2 + y^2 = 4$ about the x-axis.

(1400)