$$= \frac{360^\circ}{\pi}\left(\frac{25}{r} - 1\right) \times \frac{\pi r^2}{360^\circ} \quad \text{(Putting ① of } \theta\text{)}$$

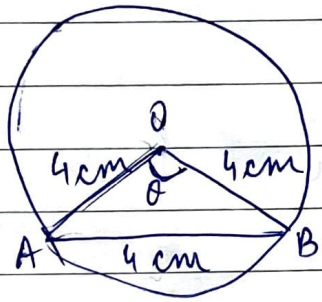
$$= \left(\frac{25 - r}{r}\right) r^{\cancel{2}}$$

$$= \boxed{25r - r^2}$$

Hence, proved ✓

Exercise 13.3

Q.1



$r = OA = OB = 4\text{ cm}$

$AB = 4\text{ cm}$

As, $OA = OB = AB = 4\text{cm}$

$\therefore \Delta OAB$ is an equilateral Δ.

$\therefore \angle AOB = 60^\circ$

Area of minor segment = Area of sector − Area of Δ

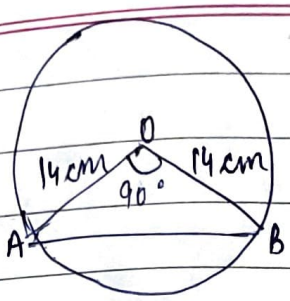
$$= \left[\frac{\theta}{360^\circ} \times \pi r^2\right] - \left[\frac{\sqrt{3}}{4}(\text{side})^2\right]$$

$$= \left(\frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 4 \times 4\right) - \left(\frac{\sqrt{3}}{4} \times 4 \times 4\right)$$

$$= \frac{176}{21} - 4\sqrt{3} = 8.38 - 6.92$$

$$= \boxed{1.46\text{ cm}^2}$$ ✓

Q.3



$r = 14$ cm

$\angle AOB = 90°$

Area of minor segment = Area of sector − Area of Δ

$$= \left(\frac{\theta}{360°} \times \pi r^2\right) - \left(\frac{1}{2} \times OA \times OB\right)$$

$$= \left(\frac{90°}{360°} \times \frac{22}{7} \times 14 \times 14\right) - \left(\frac{1}{2} \times 14 \times 14\right)$$

$$= 154 - 98$$

$$= \boxed{56 \text{ cm}^2}$$

Area of major segment = Area of circle − Area of minor segment

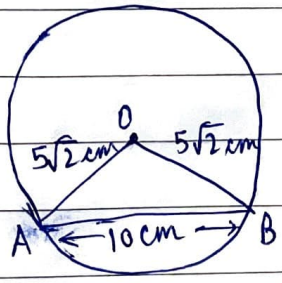
$$= \pi r^2 - 56$$

$$= \left(\frac{22}{7} \times 14 \times 14\right) - 56$$

$$= 616 - 56$$

$$= \boxed{560 \text{ cm}^2}$$

Q.4



$OA = OB = 5\sqrt{2}$ cm } given

$AB = 10$ cm

$OA^2 = OB^2 = (5\sqrt{2})^2 = 50\ cm^2$

$AB^2 = 10^2 = 100\ cm^2$

Since, $OA^2 + OB^2 = AB^2$

∴ By converse of pythagoras theorem.

$\angle AOB = 90°$

Area of minor segment = Area of sector − Area of Δ.

$$= \left(\frac{\theta}{360°} \times \pi r^2\right) - \left(\frac{1}{2} \times OB \times OA\right)$$

$$= \left(\frac{90}{360_4} \times 3.14 \times 5\sqrt{2} \times 5\sqrt{2}\right) - \left(\frac{1}{2} \times 5\sqrt{2} \times 5\sqrt{2}\right)$$

$$= \left(\frac{3.14 \times 50^{25}}{4_2}\right) - \left(\frac{1}{2} \times {}^{25}50\right)$$

$$= 39.25 - 25$$

$$= \boxed{14.25\ cm^2}$$

Area of major segment = Area of circle − Area of minor segment

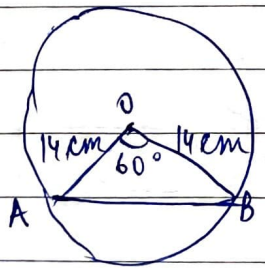
$$= \pi r^2 - 14.25$$

$$= (3.14 \times 5\sqrt{2} \times 5\sqrt{2}) - 14.25$$

$$= 157 - 14.25$$

$$= \boxed{142.75\ cm^2}$$

Q.5



$r = 14\ cm$

$\angle AOB = 60°$

} given

As OA = OB,

$\therefore \angle OAB = \angle OBA = 60°$

$\therefore$ ΔAOB is an equilateral Δ.

Area of minor segment = Area of sector − Area of Δ

$$= \left[\frac{\theta}{360°} \times \pi r^2\right] - \left[\left(\frac{\sqrt{3}}{4}\right) \times (\text{side})^2\right]$$

$$= \left[\frac{60}{360} \times \frac{22}{7} \times 14 \times 14\right] - \left[\frac{\sqrt{3}}{4} \times 14 \times 14\right]$$

$$= \left(\frac{22 \times 14}{3}\right) - \left(\sqrt{3} \times 49\right)$$

$$= 102.66 - 84.77$$

$$= \boxed{17.89 \text{ cm}^2}$$

Ch-13

Exercise 13.3

Q.6 Same as Q.5

Q.7 $r = 10\ cm$

$\theta = 90°$

Area of minor segment = Area of sector − Area of Δ

$$= \left(\frac{\theta}{360°} \times \pi r^2\right) - \left(\frac{1}{2} \times b \times h\right)$$

$$= \left(\frac{90}{360} \times 3.14 \times 10 \times 10\right) - \left(\frac{1}{2} \times 10 \times 10\right)$$

$$= 78.5 - 50$$

$$= 28.5\ cm^2$$

Area of major segment = Area of circle − Area of minor segment

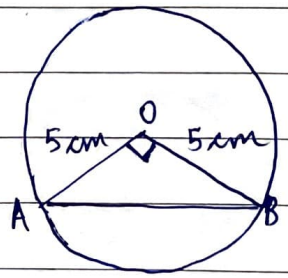
$$= \pi r^2 - 28.5$$

$$= (3.14 \times 10 \times 10) - 28.5$$

$$= 314 - 28.5$$

$$= \boxed{285.5\ cm^2}$$

Q.8



$r = 5\ cm$
$\theta = 90°$
} given

Area of minor segment = Area of sector − Area of Δ

$= \left(\frac{\theta}{360^\circ} \times \pi r^2\right) - \left(\frac{1}{2} \times OA \times OB\right)$

$= \left(\frac{90^\circ}{360^\circ_4} \times \frac{3.14 \times 5 \times 5}{}\right) - \left(\frac{1}{2} \times 5 \times 5\right)$

$= 19.625 - 12.5$

$= \boxed{7.125 \text{ cm}^2}$

Area of major segment = Area of circle − Area of minor segment

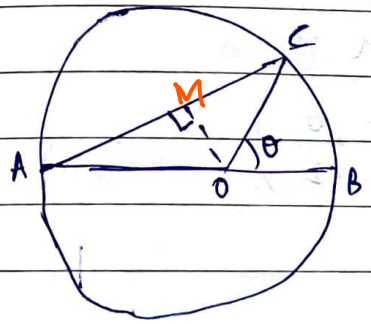
$= \pi r^2 - 7.125 \text{ cm}^2$

$= (3.14 \times 5 \times 5) - 7.125$

$= 78.5 - 7.125$

$= \boxed{71.375 \text{ cm}^2}$

Q.9



AB is the diameter
$\angle COB = \theta$
} given

ATQ,

Area of minor segment cut off by AC = 2 (Area of sector BOC)

To prove − $\sin\frac{\theta}{2} \cos\frac{\theta}{2} = \pi\left(\frac{1}{2} - \frac{\theta}{120^\circ}\right)$

Cons. − Draw a perpendicular bisector OM which meets AC at M.

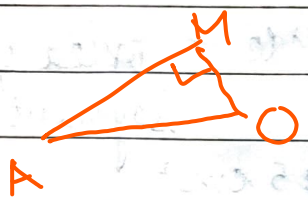
Proof – Area of sector BOC $= \frac{\theta}{360^\circ} \times \pi r^2$

$\angle AOC = 180^\circ - \theta$ (Linear pair)

Area of sector AOC $= \frac{(180^\circ - \theta)}{360^\circ} \times \pi r^2$

$= \frac{180\,\pi r^2}{360^\circ\,2} - \frac{\pi\theta r^2}{360^\circ}$

$= \frac{\pi r^2}{2} - \frac{\pi\theta r^2}{360^\circ}$



In right $\triangle AMO$,

$\sin \angle AOM = \frac{AM}{OA}$

$\Rightarrow \sin\left(\frac{180^\circ - \theta}{2}\right) = \frac{AM}{r}$ $\left[\angle AOM = \frac{\angle AOC}{2}\right]$

$\Rightarrow AM = r \sin\left(90^\circ - \frac{\theta}{2}\right)$

$\Rightarrow AM = r \cos\frac{\theta}{2}$ ✓

$\cos \angle AOM = \frac{OM}{OA}$

$\Rightarrow \cos\left(\frac{90^\circ - \theta}{2}\right) = \frac{OM}{r}$

$\Rightarrow OM = r \cos\left(90^\circ - \frac{\theta}{2}\right)$

$\Rightarrow OM = r \sin\frac{\theta}{2}$

Area of segment = Area of sector AOC - Area of Δ

$= \left(\frac{\pi r^2}{2} - \frac{\pi\theta r^2}{360^\circ}\right) - \left(\frac{1}{2} \times AC \times OM\right)$ [AC = 2AM]

$= \frac{\pi r^2}{2} - \frac{\pi\theta r^2}{360^\circ} - \frac{1}{2}\left(2r\cos\frac{\theta}{2} \times r\sin\frac{\theta}{2}\right)$

$= \boxed{r^2\left(\frac{\pi}{2} - \frac{\pi\theta}{360^\circ} - \cos\frac{\theta}{2}\sin\frac{\theta}{2}\right)}$

ATQ,

Area of segment by AC = 2 (Area of sector AOCO)

$r^2\left(\frac{\pi}{2} - \frac{\pi\theta}{360^\circ} - \cos\frac{\theta}{2}\sin\frac{\theta}{2}\right) = 2\left(\frac{\theta}{360^\circ} \times \pi r^2\right)$

$\left(\frac{\pi}{2} - \frac{\pi\theta}{360^\circ} - \cos\frac{\theta}{2}\sin\frac{\theta}{2}\right) = 2\left(\frac{\pi\theta}{360^\circ}\right)$

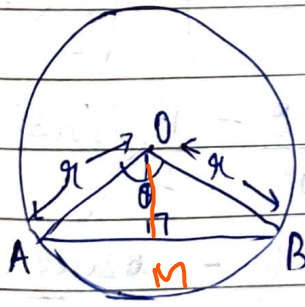
$\cos\frac{\theta}{2}\sin\frac{\theta}{2} = \frac{\pi}{2} - \frac{\pi\theta}{360^\circ} - \frac{2\pi\theta}{360^\circ}$

$\cos\frac{\theta}{2}\sin\frac{\theta}{2} = \frac{\pi}{2} - \frac{3\pi\theta}{360^\circ}$ (÷2 → 120°)

$\boxed{\cos\frac{\theta}{2}\sin\frac{\theta}{2} = \pi\left(\frac{1}{2} - \frac{\theta}{120^\circ}\right)}$

Hence, proved

Q.10



$\angle AOB = \theta$
$AO = BO = r$ (radius) } given

~~ATQ,~~

~~Area of minor segment~~

To prove - $8 \sin\frac{\theta}{2} \cos\frac{\theta}{2} + \pi = \frac{\pi\theta}{45^\circ}$

Cons. - Draw a perpendicular bisector OM which bisects AB at M.

Proof - $\angle AOM = \angle BOM = \frac{\theta}{2}$

In right $\triangle AMO$,

$\sin\frac{\theta}{2} = \frac{AM}{AO}$

$\Rightarrow \sin\frac{\theta}{2} = \frac{AM}{r}$

$\Rightarrow AM = r\sin\frac{\theta}{2}$

$\cos\frac{\theta}{2} = \frac{OM}{AO}$

$\Rightarrow \cos\frac{\theta}{2} = \frac{OM}{r}$

$\Rightarrow OM = r\cos\frac{\theta}{2}$

$AB = AM + AM$

$= 2r\sin\frac{\theta}{2}$

Area of segment cut by AB = Area of sector - Area of $\triangle$

$= \left(\frac{\theta}{360^\circ} \times \pi r^2\right) - \left(\frac{1}{2} \times AB \times OM\right)$

$= \frac{r^2\pi\theta}{360^\circ} - \frac{1}{2} \times \left(2r\sin\frac{\theta}{2} \; r\cos\frac{\theta}{2}\right)$

$= r^2\left(\frac{\pi\theta}{360^\circ} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)$

ATQ,

Area of segment cut by AB = $\frac{1}{8}$ (Area of circle)

$r^2\left(\frac{\pi\theta}{360^\circ} - \frac{\sin\theta}{2}\,\frac{\cos\theta}{2}\right) = \frac{1}{8}\pi r^2$

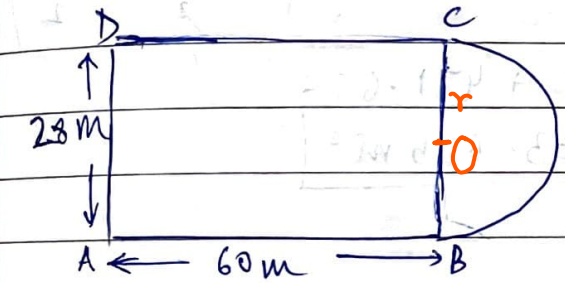
$\pi = \frac{8\pi\theta}{360^\circ 45} - 8\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$$8\sin\frac{\theta}{2}\cos\frac{\theta}{2} + \pi = \frac{\pi\theta}{45^\circ}$$

Hence, proved

Exercise 13.4

Q.1



AB = CD = 60 m
AD = BC = 28 m
ABCD is a rectangle

$r = \frac{BC}{2} = \frac{28}{2} 14 = 14$ m

Area of plot = Area of rectangle + Area of semicircle

$= (l \times B) + \left(\frac{\pi r^2}{2}\right)$

$= (60 \times 28) + \left(\frac{1}{2} \times \frac{22}{7} \times 14 \times 14\right)$

$= 1680 + 308$

$= 1988\ m^2$