

Q1. Find the cube of each of the following binomial expression

(a) $(\frac{1}{x} + \frac{y}{3})$

(b) $(\frac{3}{x} - \frac{2}{x^2})$

(c) $(2x + \frac{3}{x})$

(d) $(4 - \frac{1}{3x})$

Sol :

(a) $(\frac{1}{x} + \frac{y}{3})^3$

Given, $(\frac{1}{x} + \frac{y}{3})^3$

The above equation is in the form of $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

We know that, $a = \frac{1}{x}$, $b = \frac{y}{3}$

By using $(a + b)^3$ formula

$$(\frac{1}{x} + \frac{y}{3})^3 = (\frac{1}{x})^3 + (\frac{y}{3})^3 + 3(\frac{1}{x})(\frac{y}{3})(\frac{1}{x} + \frac{y}{3})$$

$$= \frac{1}{x^3} + \frac{y^3}{27} + 3 * \frac{1}{x} * \frac{y}{3} (\frac{1}{x} + \frac{y}{3})$$

$$= \frac{1}{x^3} + \frac{y^3}{27} + \frac{y}{x} (\frac{1}{x} + \frac{y}{3})$$

$$= \frac{1}{x^3} + \frac{y^3}{27} + (\frac{y}{x} * \frac{1}{x}) +$$

$$(\frac{y}{x} * \frac{y}{3})$$

$$= \frac{1}{x^3} + \frac{y^3}{27} + \frac{y}{x^2} + \frac{y^2}{3x}$$

Hence,

$$(\frac{1}{x} + \frac{y}{3})^3 = \frac{1}{x^3} + \frac{y^3}{27} + \frac{y}{x^2} + \frac{y^2}{3x}$$

(b) $((\frac{3}{x} - \frac{2}{x^2}))^3$

Given, $((\frac{3}{x} - \frac{2}{x^2}))^3$

The above equation is in the form of $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

We know that, $a = \frac{3}{x}$, $b = \frac{2}{x^2}$

By using $(a - b)^3$ formula

$$((\frac{3}{x} - \frac{2}{x^2}))^3 = (\frac{3}{x})^3 - (\frac{2}{x^2})^3 - 3(\frac{3}{x})(\frac{2}{x^2})(\frac{3}{x} - \frac{2}{x^2})$$

$$= \frac{27}{x^3} - \frac{8}{x^6} - 3 * \frac{3}{x} * \frac{2}{x^2} (\frac{3}{x} - \frac{2}{x^2})$$

$$\begin{aligned}
&= \frac{27}{x^3} - \frac{8}{x^6} - \frac{18}{x^3} \left(\frac{3}{x} - \frac{2}{x^2} \right) \\
&= \frac{27}{x^3} - \frac{8}{x^6} - \left(\frac{18}{x^3} * \frac{3}{x} \right) + \left(\frac{18}{x^3} * \frac{2}{x^2} \right) \\
&= \frac{27}{x^3} - \frac{8}{x^6} - \frac{54}{x^4} + \frac{36}{x^5}
\end{aligned}$$

$$\text{Hence, } \left(\left(\frac{3}{x} - \frac{2}{x^2} \right) \right)^3 = \frac{27}{x^3} - \frac{8}{x^6} - \frac{54}{x^4} + \frac{36}{x^5}$$

$$(c) \left(2x + \frac{3}{x} \right)^3$$

$$\text{Given, } \left(2x + \frac{3}{x} \right)^3$$

The above equation is in the form of $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$$\text{We know that, } a = 2x, b = \frac{3}{x}$$

By using $(a + b)^3$ formula

$$\begin{aligned}
&= 8x^3 + \frac{27}{x^3} + \frac{18x}{x} \left(2x + \frac{3}{x} \right) \\
&= 8x^3 + \frac{27}{x^3} + \frac{18x}{x} \left(2x + \frac{3}{x} \right) \\
&= 8x^3 + \frac{27}{x^3} +
\end{aligned}$$

$$(18 * 2x) + \left(18 * \frac{3}{x} \right)$$

$$= 8x^3 + \frac{27}{x^3} + 36x \frac{54}{x}$$

Hence,

$$\text{The cube of } \left(2x + \frac{3}{x} \right)^3 = 8x^3 + \frac{27}{x^3} + 36x \frac{54}{x}$$

$$(d) \left(4 - \frac{1}{3x} \right)^3$$

$$\text{Given, } \left(4 - \frac{1}{3x} \right)^3$$

The above equation is in the form of $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$$\text{We know that, } a = 4, b = \frac{1}{3x}$$

By using $(a - b)^3$ formula

$$\begin{aligned}
\left(4 - \frac{1}{3x} \right)^3 &= 4^3 - \left(\frac{1}{3x} \right)^3 - 3(4) \left(\frac{1}{3x} \right) \left(4 - \frac{1}{3x} \right) \\
&= 64 - \frac{1}{27x^3} - \frac{12}{3x} \left(4 - \frac{1}{3x} \right) \\
&= 64 - \frac{1}{27x^3} - \frac{4}{x} \left(4 - \frac{1}{3x} \right) \\
&= 64 - \frac{1}{27x^3} - \left(\frac{4}{3x} * 4 \right) + \left(\frac{4}{3x} * \frac{1}{3x} \right) \\
&= 64 - \frac{1}{27x^3} - \frac{16}{x} + \left(\frac{4}{3x^2} \right)
\end{aligned}$$

Hence ,

The cube of $(4 - \frac{1}{3x})^3 = 64 - \frac{1}{27x^3} - \frac{16}{x} + (\frac{4}{3x^2})$

Q2. Simplify each of the following

(a) $(x + 3)^3 + (x - 3)^3$

(b) $(\frac{x}{2} + \frac{y}{3})^3 - (\frac{x}{2} - \frac{y}{3})^3$

(c) $(x + \frac{2}{x})^3 + (x - \frac{2}{x})^3$

(d) $(2x - 5y)^3 - (2x + 5y)^3$

Sol :

(a) $(x + 3)^3 + (x - 3)^3$

The above equation is in the form of $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

We know that, $a = (x + 3)$, $b = (x - 3)$

By using $(a^3 + b^3)$ formula

$$\begin{aligned} &= (x + 3 + x - 3)[(x + 3)^3 + (x - 3)^3 - (x + 3)(x - 3)] \\ &= 2x[(x^2 + 3^2 + 2 \cdot x \cdot 3) + (x^2 + 3^2 - 2 \cdot x \cdot 3) - (x^2 - 3^2)] \\ &= 2x[(x^2 + 9 + 6x) + (x^2 + 9 - 6x) - x^2 + 9] \\ &= 2x[(x^2 + 9 + 6x + x^2 - 9 - 6x - x^2 + 9)] \\ &= 2x(x^2 + 27) \\ &= 2x^3 + 54x \end{aligned}$$

Hence, the result of $(x + 3)^3 + (x - 3)^3$ is $2x^3 + 54x$

(b) $(\frac{x}{2} + \frac{y}{3})^3 - (\frac{x}{2} - \frac{y}{3})^3$

The above equation is in the form of $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

We know that, $a = (\frac{x}{2} + \frac{y}{3})^3$, $b = (\frac{x}{2} - \frac{y}{3})^3$

By using $(a^3 - b^3)$ formula

$$\begin{aligned} &= [((\frac{x}{2} + \frac{y}{3})^3 - ((\frac{x}{2} - \frac{y}{3})^3)][((\frac{x}{2} + \frac{y}{3})^2)((\frac{x}{2} - \frac{y}{3})^3)^2 - ((\frac{x}{2} + \frac{y}{3})^3)((\frac{x}{2} - \frac{y}{3})^3)] \\ &= (\frac{x}{3} + \frac{y}{3} - \frac{x}{2} + \frac{y}{3})[(((\frac{x}{2})^2 + (\frac{y}{3})^2 + (\frac{2xy}{6})) + ((\frac{x}{2})^2 + (\frac{y}{3})^2 - (\frac{2xy}{6})) + ((\frac{x}{2})^2 - (\frac{y}{3})^2)] \\ &= \frac{2y}{3}[(\frac{x^2}{4} + \frac{y^2}{9} + \frac{2xy}{6}) + (\frac{x^2}{4} + \frac{y^2}{9} - \frac{2xy}{6}) + \frac{x^2}{4} - \frac{y^2}{9}] \\ &= \frac{2y}{3}[\frac{x^2}{4} + \frac{y^2}{9} + \frac{2xy}{6} + \frac{x^2}{4} + \frac{y^2}{9} - \frac{2xy}{6} + \frac{x^2}{4} - \frac{y^2}{9}] \\ &= \frac{2y}{3}[\frac{x^2}{4} + \frac{y^2}{9} + \frac{x^2}{4} + \frac{x^2}{4}] \\ &= \frac{2y}{3}[\frac{3x^2}{4} + \frac{y^2}{9}] \\ &= \frac{x^2y}{2} + \frac{2y^3}{27} \end{aligned}$$

Hence, the result of $(\frac{x}{2} + \frac{y}{3})^3 - (\frac{x}{2} - \frac{y}{3})^3 = \frac{x^2y}{2} + \frac{2y^3}{27}$

(c) $(x + \frac{2}{x})^3 + (x - \frac{2}{x})^3$

The above equation is in the form of $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

We know that, $a = (x + \frac{2}{x})^3$, $b = (x - \frac{2}{x})^3$

By using $(a^3 + b^3)$ formula

$$= (x + \frac{2}{x} + x - \frac{2}{x})[(x + \frac{2}{x})^2 + (x - \frac{2}{x})^2 - ((x + \frac{2}{x})(x - \frac{2}{x}))]$$

$$= (2x)[(x^2 + \frac{4}{x^2} + \frac{4x}{x}) + (x^2 + \frac{4}{x^2} - \frac{4x}{x}) - (x^2 - \frac{4}{x^2})]$$

$$= (2x)[(x^2 + \frac{4}{x^2} + \frac{4x}{x} + x^2 + \frac{4}{x^2} - \frac{4x}{x} - x^2 + \frac{4}{x^2})]$$

$$= (2x)[(x^2 + \frac{4}{x^2} + \frac{4}{x^2} + \frac{4}{x^2})]$$

$$= (2x)[(x^2 + \frac{12}{x^2})]$$

$$= 2x^3 + \frac{24}{x}$$

Hence, the result of $(x + \frac{2}{x})^3 + (x - \frac{2}{x})^3 = (2x)[(x^2 + \frac{12}{x^2})]$

(d) $(2x - 5y)^3 - (2x + 5y)^3$

Given, $(2x - 5y)^3 - (2x + 5y)^3$

The above equation is in the form of $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

We know that, $a = (2x - 5y)$, $b = (2x + 5y)$

By using $(a^3 - b^3)$ formula

$$= (2x - 5y - 2x - 5y)[(2x - 5y)^2 + (2x + 5y)^2 + ((2x - 5y) * (2x + 5y))]$$

$$= (-10y)[(4x^2 + 25y^2 - 20xy) + (4x^2 + 25y^2 + 20xy) + 4x^2 - 25y^2]$$

$$= (-10y)[4x^2 + 25y^2 - 20xy + 4x^2 + 25y^2 + 20xy + 4x^2 - 25y^2]$$

$$= (-10y)[4x^2 + 4x^2 + 4x^2 + 25y^2]$$

$$= (-10y)[12x^2 + 25y^2]$$

$$= -120x^2y - 250y^3$$

Hence, the result of $(2x - 5y)^3 - (2x + 5y)^3 = -120x^2y - 250y^3$

Q3. If $a + b = 10$ and $ab = 21$, Find the value of $a^3 + b^3$

Sol :

Given,

$$a + b = 10, ab = 21$$

we know that, $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$ ---- 1

substitute $a + b = 10$, $ab = 21$ in eq 1

$$\Rightarrow (10)^3 = a^3 + b^3 + 3(21)(10)$$

$$\Rightarrow 1000 = a^3 + b^3 + 630$$

$$\Rightarrow 1000 - 630 = a^3 + b^3$$

$$\Rightarrow 370 = a^3 + b^3$$

Hence, the value of $a^3 + b^3 = 370$

Q4. If $a - b = 4$ and $ab = 21$, Find the value of $a^3 - b^3$

Sol :

Given,

$$a - b = 4, ab = 21$$

$$\text{we know that, } (a - b)^3 = a^3 - b^3 - 3ab(a - b) \quad \text{---- 1}$$

substitute $a - b = 4$, $ab = 21$ in eq 1

$$\Rightarrow (4)^3 = a^3 - b^3 - 3(21)(4)$$

$$\Rightarrow 64 = a^3 - b^3 - 252$$

$$\Rightarrow 64 + 252 = a^3 - b^3$$

$$\Rightarrow 316 = a^3 - b^3$$

Hence, the value of $a^3 - b^3 = 316$

Q5. If $(x + \frac{1}{x}) = 5$, Find the value of $x^3 + \frac{1}{x^3}$

Sol :

$$\text{Given, } (x + \frac{1}{x}) = 5$$

$$\text{We know that, } (a + b)^3 = a^3 + b^3 + 3ab(a + b) \quad \text{-- 1}$$

Substitute $(x + \frac{1}{x}) = 5$ in eq 1

$$(x + \frac{1}{x})^3 = x^3 + \frac{1}{x^3} + 3(x * \frac{1}{x})(x + \frac{1}{x})$$

$$5^3 = x^3 + \frac{1}{x^3} + 3(x * \frac{1}{x})(x + \frac{1}{x})$$

$$125 = x^3 + \frac{1}{x^3} + 3(x + \frac{1}{x})$$

$$125 = x^3 + \frac{1}{x^3} + 3(5)$$

$$125 = x^3 + \frac{1}{x^3} + 15$$

$$125 - 15 = x^3 + \frac{1}{x^3}$$

$$x^3 + \frac{1}{x^3} = 110$$

hence, the result is $x^3 + \frac{1}{x^3} = 110$

Q6. If $(x - \frac{1}{x}) = 7$, Find the value of $x^3 - \frac{1}{x^3}$

Sol :

Given, If $(x - \frac{1}{x}) = 7$

We know that, $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$ ---- 1

Substitute $(x - \frac{1}{x}) = 7$ in eq 1

$$(x - \frac{1}{x})^3 = x^3 - \frac{1}{x^3} - 3(x * \frac{1}{x})(x - \frac{1}{x})$$

$$7^3 = x^3 - \frac{1}{x^3} - 3(x - \frac{1}{x})$$

$$343 = x^3 - \frac{1}{x^3} - (3 * 7)$$

$$343 = x^3 - \frac{1}{x^3} - 21$$

$$343 + 21 = x^3 - \frac{1}{x^3}$$

$$x^3 - \frac{1}{x^3} = 364$$

hence, the result is $x^3 - \frac{1}{x^3} = 364$

Q7. If $(x - \frac{1}{x}) = 5$, Find the value of $x^3 - \frac{1}{x^3}$

Sol :

Given, If $(x - \frac{1}{x}) = 5$

We know that, $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$ ---- 1

Substitute $(x - \frac{1}{x}) = 5$ in eq 1

$$(x - \frac{1}{x})^3 = x^3 - \frac{1}{x^3} - 3(x * \frac{1}{x})(x - \frac{1}{x})$$

$$5^3 = x^3 - \frac{1}{x^3} - 3(x - \frac{1}{x})$$

$$125 = x^3 - \frac{1}{x^3} - (3 * 5)$$

$$125 = x^3 - \frac{1}{x^3} - 15$$

$$125 + 15 = x^3 - \frac{1}{x^3}$$

$$x^3 - \frac{1}{x^3} = 140$$

hence, the result is $x^3 - \frac{1}{x^3} = 140$

Q8. If $(x^2 + \frac{1}{x^2}) = 51$, Find the value of $x^3 - \frac{1}{x^3}$

Sol :

$$\text{Given, } (x^2 + \frac{1}{x^2}) = 51$$

$$\text{We know that, } (x - y)^2 = x^2 + y^2 - 2xy \quad \text{--- 1}$$

$$\text{Substitute } (x^2 + \frac{1}{x^2}) = 51 \text{ in eq 1}$$

$$(x - \frac{1}{x})^2 = x^2 + \frac{1}{x^2} - 2 * x * \frac{1}{x}$$

$$(x - \frac{1}{x})^2 = x^2 + \frac{1}{x^2} - 2$$

$$(x - \frac{1}{x})^2 = 51 - 2$$

$$(x - \frac{1}{x})^2 = 49$$

$$(x - \frac{1}{x}) = \sqrt{49}$$

$$(x - \frac{1}{x}) = \pm 7$$

$$\text{We need to find } x^3 - \frac{1}{x^3}$$

$$\text{So, } a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$$

$$x^3 - \frac{1}{x^3} = (x - \frac{1}{x})(x^2 + \frac{1}{x^2} + x * \frac{1}{x})$$

We know that ,

$$(x - \frac{1}{x}) = 7 \text{ and } (x^2 + \frac{1}{x^2}) = 51$$

$$x^3 - \frac{1}{x^3} = 7(51 + 1)$$

$$x^3 - \frac{1}{x^3} = 7(52)$$

$$x^3 - \frac{1}{x^3} = 364$$

$$\text{Hence, the value of } x^3 - \frac{1}{x^3} = 364$$

$$\textbf{Q9. If } (x^2 + \frac{1}{x^2}) = 98 \text{ , Find the value of } x^3 + \frac{1}{x^3}$$

Sol :

$$\text{Given, } (x^2 + \frac{1}{x^2}) = 98$$

$$\text{We know that, } (x + y)^2 = x^2 + y^2 + 2xy \quad \text{--- 1}$$

$$\text{Substitute } (x^2 + \frac{1}{x^2}) = 98 \text{ in eq 1}$$

$$(x + \frac{1}{x})^2 = x^2 + \frac{1}{x^2} + 2 * x * \frac{1}{x}$$

$$(x + \frac{1}{x})^2 = x^2 + \frac{1}{x^2} + 2$$

$$(x + \frac{1}{x})^2 = 98 + 2$$

$$\left(x + \frac{1}{x}\right)^2 = 100$$

$$\left(x + \frac{1}{x}\right) = \sqrt{100}$$

$$\left(x + \frac{1}{x}\right) = \pm 10$$

We need to find $x^3 + \frac{1}{x^3}$

$$\text{So, } a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$$

$$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} - \left(x * \frac{1}{x}\right)\right)$$

We know that,

$$\left(x + \frac{1}{x}\right) = 10 \text{ and } \left(x^2 + \frac{1}{x^2}\right) = 98$$

$$x^3 + \frac{1}{x^3} = 10(98 - 1)$$

$$x^3 + \frac{1}{x^3} = 10(97)$$

$$x^3 + \frac{1}{x^3} = 970$$

Hence, the value of $x^3 + \frac{1}{x^3} = 970$

Q10. If $2x + 3y = 13$ and $xy = 6$, Find the value of $8x^3 + 27y^3$

Sol :

$$\text{Given, } 2x + 3y = 13, xy = 6$$

We know that,

$$(2x + 3y)^3 = 13^3$$

$$\Rightarrow 8x^3 + 27y^3 + 3(2x)(3y)(2x + 3y) = 2197$$

$$\Rightarrow 8x^3 + 27y^3 + 18xy(2x + 3y) = 2197$$

$$\text{Substitute } 2x + 3y = 13, xy = 6$$

$$\Rightarrow 8x^3 + 27y^3 + 18(6)(13) = 2197$$

$$\Rightarrow 8x^3 + 27y^3 + 1404 = 2197$$

$$\Rightarrow 8x^3 + 27y^3 = 2197 - 1404$$

$$\Rightarrow 8x^3 + 27y^3 = 793$$

Hence, the value of $8x^3 + 27y^3 = 793$

Q11. If $3x - 2y = 11$ and $xy = 12$, Find the value of $27x^3 - 8y^3$

Sol :

$$\text{Given, } 3x - 2y = 11, xy = 12$$

$$\text{We know that } (a - b)^3 = a^3 - b^3 - 3ab(a + b)$$

$$(3x - 2y)^3 = 11^3$$

$$\Rightarrow 27x^3 - 8y^3 - (18 * 12 * 11) = 1331$$

$$\Rightarrow 27x^3 - 8y^3 - 2376 = 1331$$

$$\Rightarrow 27x^3 - 8y^3 = 1331 + 2376$$

$$\Rightarrow 27x^3 - 8y^3 = 3707$$

Hence, the value of $27x^3 - 8y^3 = 3707$

Q12. If $x^4 + (\frac{1}{x^4}) = 119$, Find the value of $x^3 - (\frac{1}{x^3})$

Sol:

$$\text{Given, } x^4 + (\frac{1}{x^4}) = 119 \quad \text{---- 1}$$

We know that $(x + y)^2 = x^2 + y^2 + 2xy$

Substitute $x^4 + (\frac{1}{x^4}) = 119$ in eq 1

$$(x^2 + (\frac{1}{x^2}))^2 = x^4 + (\frac{1}{x^4}) + (2 \cdot x^2 \cdot \frac{1}{x^2})$$

$$= x^4 + (\frac{1}{x^4}) + 2$$

$$= 119 + 2$$

$$= 121$$

$$(x^2 + (\frac{1}{x^2}))^2 = 121$$

$$x^2 + (\frac{1}{x^2}) = \sqrt{121}$$

$$x^2 + (\frac{1}{x^2}) = \pm 11$$

Now, find $(x - \frac{1}{x})$

We know that $(x - y)^2 = x^2 + y^2 - 2xy$

$$(x - \frac{1}{x})^2 = x^2 + \frac{1}{x^2} - (2 \cdot x \cdot \frac{1}{x})$$

$$= x^2 + \frac{1}{x^2} - 2$$

$$= 11 - 2$$

$$= 9$$

$$(x - \frac{1}{x}) = \sqrt{9}$$

$$= \pm 3$$

We need to find $x^3 - (\frac{1}{x^3})$

We know that, $a^3 - b^3 = (a - b)(a^2 + b^2 - ab)$

$$x^3 - (\frac{1}{x^3}) = (x - \frac{1}{x})(x^2 + (\frac{1}{x^2}) + x \cdot \frac{1}{x})$$

Here, $x^2 + (\frac{1}{x^2}) = 11$ and $(x - \frac{1}{x}) = 3$

$$x^3 - \left(\frac{1}{x^3}\right) = 3(11 + 1)$$

$$= 3(12)$$

$$= 36$$

$$\text{Hence, the value of } x^3 - \left(\frac{1}{x^3}\right) = 36$$

Q13. Evaluate each of the following

(a) $(103)^3$

(b) $(98)^3$

(c) $(9.9)^3$

(d) $(10.4)^3$

(e) $(598)^3$

(f) $(99)^3$

Sol :

Given,

(a) $(103)^3$

$$\text{we know that } (a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

$$\Rightarrow (103)^3 \text{ can be written as } (100 + 3)^3$$

$$\text{Here , } a = 100 \text{ and } b = 3$$

$$(103)^3 = (100 + 3)^3$$

$$= (100)^3 + (3)^3 + 3(100)(3)(100 + 3)$$

$$= 1000000 + 27 + (900 \times 103)$$

$$= 1000000 + 27 + 92700$$

$$= 1092727$$

$$\text{The value of } (103)^3 = 1092727$$

(b) $(98)^3$

$$\text{we know that } (a - b)^3 = a^3 - b^3 - 3ab(a - b)$$

$$\Rightarrow (98)^3 \text{ can be written as } (100 - 2)^3$$

$$\text{Here , } a = 100 \text{ and } b = 2$$

$$(98)^3 = (100 - 2)^3$$

$$= (100)^3 - (2)^3 - 3(100)(2)(100 - 2)$$

$$= 1000000 - 8 - (600 \times 102)$$

$$= 1000000 - 8 - 58800$$

$$= 941192$$

$$\text{The value of } (98)^3 = 941192$$

(c) $(9.9)^3$

we know that $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$\Rightarrow (9.9)^3$ can be written as $(10 - 0.1)^3$

Here , $a = 10$ and $b = 0.1$

$$(9.9)^3 = (10 - 0.1)^3$$

$$= (10)^3 - (0.1)^3 - 3(10)(0.1)(10 - 0.1)$$

$$= 1000 - 0.001 - (3 \times 9.9)$$

$$= 1000 - 0.001 - 29.7$$

$$= 1000 - 29.701$$

$$= 970.299$$

The value of $(9.9)^3 = 970.299$

(d) $(10.4)^3$

we know that $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$\Rightarrow (10.4)^3$ can be written as $(10 + 0.4)^3$

Here , $a = 10$ and $b = 0.4$

$$(10.4)^3 = (10 + 0.4)^3$$

$$= (10)^3 + (0.4)^3 + 3(10)(0.4)(10 + 0.4)$$

$$= 1000 + 0.064 + (12 \times 10.4)$$

$$= 1000 + 0.064 + 124.8$$

$$= 1000 + 124.864$$

$$= 1124.864$$

The value of $(10.4)^3 = 1124.864$

(e) $(598)^3$

we know that $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$\Rightarrow (598)^3$ can be written as $(600 - 2)^3$

Here , $a = 600$ and $b = 2$

$$(598)^3 = (600 - 2)^3$$

$$= (600)^3 - (2)^3 - 3(600)(2)(600 - 2)$$

$$= 216000000 - 8 - (3600 \times 598)$$

$$= 216000000 - 8 - 2152800$$

$$= 216000000 - 2152808$$

$$= 213847192$$

The value of $(598)^3 = 213847192$

(f) $(99)^3$

we know that $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$\Rightarrow (99)^3$ can be written as $(100 - 1)^3$

Here , $a = 100$ and $b = 1$

$$(99)^3 = (100 - 1)^3$$

$$= (100)^3 - (1)^3 - 3(100)(1)(100 - 1)$$

$$= 1000000 - 1 - (300 \times 99)$$

$$= 1000000 - 1 - 29700$$

$$= 1000000 - 29701$$

$$= 970299$$

The value of $(99)^3 = 970299$

Q14. Evaluate each of the following

(a) $111^3 - 89^3$

(b) $46^3 + 34^3$

(c) $104^3 + 96^3$

(d) $93^3 - 107^3$

Sol:

Given,

(a) $111^3 - 89^3$

the above equation can be written as $(100 + 11)^3 - (100 - 11)^3$

we know that, $(a + b)^3 - (a - b)^3 = 2[b^3 + 3ab^2]$

here, $a = 100$ $b = 11$

$$(100 + 11)^3 - (100 - 11)^3 = 2[11^3 + 3(100)^2(11)]$$

$$= 2[1331 + 330000]$$

$$= 2[331331]$$

$$= 662662$$

The value of $111^3 - 89^3 = 662662$

(b) $46^3 + 34^3$

the above equation can be written as $(40 + 6)^3 + (40 - 6)^3$

we know that, $(a + b)^3 + (a - b)^3 = 2[a^3 + 3ab^2]$

here, $a = 40$, $b = 6$

$$(40 + 6)^3 + (40 - 6)^3 = 2[40^3 + 3(6)^2(40)]$$

$$= 2[64000 + 4320]$$

$$= 2[68320]$$

$$= 1366340$$

The value of $46^3 + 34^3 = 1366340$

(c) $104^3 + 96^3$

the above equation can be written as $(100 + 4)^3 + (100 - 4)^3$

we know that, $(a + b)^3 + (a - b)^3 = 2[a^3 + 3ab^2]$

here, $a = 100$ $b = 4$

$$(100 + 4)^3 - (100 - 4)^3 = 2[100^3 + 3(4)^2(100)]$$

$$= 2[1000000 + 4800]$$

$$= 2[1004800]$$

$$= 2009600$$

The value of $104^3 + 96^3 = 2009600$

(a) $93^3 - 107^3$

the above equation can be written as $(100 - 7)^3 - (100 + 7)^3$

we know that, $(a - b)^3 - (a + b)^3 = -2[b^3 + 3ba^2]$

here, $a = 93$, $b = 107$

$$(100 - 7)^3 - (100 + 7)^3 = -2[7^3 + 3(100)^2(7)]$$

$$= -2[343 + 210000]$$

$$= -2[210343]$$

$$= -420686$$

The value of $93^3 - 107^3 = -420686$

Q15. If $x + \frac{1}{x} = 3$, calculate $x^2 + \frac{1}{x^2}$, $x^3 + \frac{1}{x^3}$, $x^4 + \frac{1}{x^4}$

Sol :

Given, $x + \frac{1}{x} = 3$

We know that $(x + y)^2 = x^2 + y^2 + 2xy$

$$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + \left(2 * x * \frac{1}{x}\right)$$

$$3^2 = x^2 + \frac{1}{x^2} + 2$$

$$9 - 2 = x^2 + \frac{1}{x^2}$$

$$x^2 + \frac{1}{x^2} = 7$$

squaring on both sides

$$\left(x^2 + \frac{1}{x^2}\right)^2 = 7^2$$

$$x^4 + \frac{1}{x^4} + 2 * x^2 * \frac{1}{x^2} = 49$$

$$x^4 + \frac{1}{x^4} + 2 = 49$$

$$x^4 + \frac{1}{x^4} = 49 - 2$$

$$x^4 + \frac{1}{x^4} = 47$$

again , cubing on both sides

$$\left(x + \frac{1}{x}\right)^3 = 3^3$$

$$x^3 + \frac{1}{x^3} + 3x \cdot \frac{1}{x} \left(x + \frac{1}{x}\right) = 27$$

$$x^3 + \frac{1}{x^3} + (3 \cdot 3) = 27$$

$$x^3 + \frac{1}{x^3} + 9 = 27$$

$$x^3 + \frac{1}{x^3} = 27 - 9$$

$$x^3 + \frac{1}{x^3} = 18$$

hence, the values are $x^2 + \frac{1}{x^2} = 7$, $x^4 + \frac{1}{x^4} = 47$, $x^3 + \frac{1}{x^3} = 18$

Q16. If $x^4 + \frac{1}{x^4} = 194$, calculate $x^2 + \frac{1}{x^2}$, $x^3 + \frac{1}{x^3}$, $x + \frac{1}{x}$

Sol :

Given,

$$x^4 + \frac{1}{x^4} = 194 \quad \text{---- 1}$$

add and subtract $(2 \cdot x^2 \cdot \frac{1}{x^2})$ on left side in above given equation

$$x^4 + \frac{1}{x^4} + (2 \cdot x^2 \cdot \frac{1}{x^2}) - 2(2 \cdot x^2 \cdot \frac{1}{x^2}) = 194$$

$$x^4 + \frac{1}{x^4} + (2 \cdot x^2 \cdot \frac{1}{x^2}) - 2 = 194$$

$$\left(x^2 + \frac{1}{x^2}\right)^2 - 2 = 194$$

$$\left(x^2 + \frac{1}{x^2}\right)^2 = 194 + 2$$

$$\left(x^2 + \frac{1}{x^2}\right)^2 = 196$$

$$\left(x^2 + \frac{1}{x^2}\right) = \sqrt{196}$$

$$\left(x^2 + \frac{1}{x^2}\right) = 14 \quad \text{----- 2}$$

Add and subtract $(2 \cdot x \cdot \frac{1}{x})$ on left side in eq 2

$$\left(x^2 + \frac{1}{x^2}\right) + (2 \cdot x \cdot \frac{1}{x}) - (2 \cdot x \cdot \frac{1}{x}) = 14$$

$$\left(x + \frac{1}{x}\right)^2 - 2 = 14$$

$$\left(x + \frac{1}{x}\right)^2 = 14 + 2$$

$$\left(x + \frac{1}{x}\right)^2 = 16$$

$$\left(x + \frac{1}{x}\right) = \sqrt{16}$$

$$\left(x + \frac{1}{x}\right) = 4 \quad \text{----- 3}$$

Now, cubing eq 3 on both sides

$$\left(x + \frac{1}{x}\right)^3 = 4^3$$

We know that, $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$$x^3 + \frac{1}{x^3} + 3 \cdot x \cdot \frac{1}{x} \left(x + \frac{1}{x}\right) = 64$$

$$x^3 + \frac{1}{x^3} + (3 \cdot 4) = 64$$

$$x^3 + \frac{1}{x^3} = 64 - 12$$

$$x^3 + \frac{1}{x^3} = 52$$

hence, the values of $\left(x^2 + \frac{1}{x^2}\right)^2 = 196$, $\left(x + \frac{1}{x}\right) = 4$, $x^3 + \frac{1}{x^3} = 52$

Q17. Find the values of $27x^3 + 8y^3$, if

(a) $3x + 2y = 14$ and $xy = 8$

(b) $3x + 2y = 20$ and $xy = \frac{14}{9}$

Sol:

(a) Given, $3x + 2y = 14$ and $xy = 8$

cubing on both sides

$$(3x + 2y)^3 = 14^3$$

We know that, $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$$27x^3 + 8y^3 + 3(3x)(2y)(3x + 2y) = 2744$$

$$27x^3 + 8y^3 + 18xy(3x + 2y) = 2744$$

$$27x^3 + 8y^3 + 18(8)(14) = 2744$$

$$27x^3 + 8y^3 + 2016 = 2744$$

$$27x^3 + 8y^3 = 2744 - 2016$$

$$27x^3 + 8y^3 = 728$$

Hence, the value of $27x^3 + 8y^3 = 728$

(b) Given, $3x + 2y = 20$ and $xy = \frac{14}{9}$

cubing on both sides

$$(3x + 2y)^3 = 20^3$$

We know that, $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$$27x^3 + 8y^3 + 3(3x)(2y)(3x + 2y) = 8000$$

$$27x^3 + 8y^3 + 18xy(3x + 2y) = 8000$$

$$27x^3 + 8y^3 + 18\left(\frac{14}{9}\right)(20) = 8000$$

$$27x^3 + 8y^3 + 560 = 8000$$

$$27x^3 + 8y^3 = 8000 - 560$$

$$27x^3 + 8y^3 = 7440$$

Hence, the value of $27x^3 + 8y^3 = 7440$

Q18. Find the value of $64x^3 - 125z^3$, if $4x - 5z = 16$ and $xz = 12$

Sol:

$$\text{Given, } 64x^3 - 125z^3$$

$$\text{Here, } 4x - 5z = 16 \text{ and } xz = 12$$

Cubing $4x - 5z = 16$ on both sides

$$(4x - 5z)^3 = 16^3$$

We know that, $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$$(4x)^3 - (5z)^3 - 3(4x)(5z)(4x - 5z) = 16^3$$

$$64x^3 - 125z^3 - 60(xz)(16) = 4096$$

$$64x^3 - 125z^3 - 60(12)(16) = 4096$$

$$64x^3 - 125z^3 - 11520 = 4096$$

$$64x^3 - 125z^3 = 4096 + 11520$$

$$64x^3 - 125z^3 = 15616$$

The value of $64x^3 - 125z^3 = 15616$

Q19. If $x - \frac{1}{x} = 3 + 2\sqrt{2}$, Find the value of $x^3 - \frac{1}{x^3}$

Sol :

$$\text{Given, } x - \frac{1}{x} = 3 + 2\sqrt{2}$$

Cubing $x - \frac{1}{x} = 3 + 2\sqrt{2}$ on both sides

We know that, $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$$\left(x - \frac{1}{x}\right)^3 = (3 + 2\sqrt{2})^3$$

$$x^3 - \frac{1}{x^3} - 3 \cdot x \cdot \frac{1}{x} \left(x - \frac{1}{x}\right) = 3^2 + (2\sqrt{2})^3 + 3 \cdot 3 \cdot 2\sqrt{2} (3 + 2\sqrt{2})$$

$$x^3 - \frac{1}{x^3} - 3(3 + 2\sqrt{2}) = 27 + 16\sqrt{2} + 18\sqrt{2}(3 + 2\sqrt{2})$$

$$x^3 - \frac{1}{x^3} = 27 + 16\sqrt{2} + 54\sqrt{2} + 72 + 9 + 6\sqrt{2}$$

$$x^3 - \frac{1}{x^3} = 108 + 76\sqrt{2}$$

hence, the value of $x^3 - \frac{1}{x^3} = 108 + 76\sqrt{2}$