Exercise 21.4

Q.1 Find the longest rod that can be placed in a room 12 m long, 9 m broad and 8 m high.

Sol: $\because$ $l = 12m$, $b = 9m$, $h = 8m$.

$\therefore$ length of longest rod = length of diagonal of room

$$= \sqrt{l^2+b^2+h^2}$$

$$= \sqrt{(12)^2+(9)^2+(8)^2}$$

$$= \sqrt{144+81+64} = \sqrt{289}$$

$$= 17m.$$

Q.2 If V is the volume of a cuboid of dimensions a, b, c and S is its surface area, then prove that

$$\frac{1}{V} = \frac{2}{S}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

Sol: $\because$ $S = 2(ab+bc+ac)$

$$V = abc$$

$$\therefore \frac{S}{V} = \frac{2(ab+bc+ac)}{abc}$$

$$= 2\left(\frac{ab}{abc}+\frac{bc}{abc}+\frac{ac}{abc}\right)$$

$$\Rightarrow \frac{1}{V} = \frac{2}{S}\left(\frac{1}{c}+\frac{1}{a}+\frac{1}{b}\right)$$

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hence $\frac{1}{V} = \frac{2}{S}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

Q.3 The areas of three adjacent faces of a cuboid are x, y and z. If the volume is V, prove that $V^2 = xyz$.

Sol: Area of three adjacent faces $lb = x$, $bh = y$, $lh = z$

Multiplying them

$lb \times bh \times lh = xyz$

$\Rightarrow l^2b^2h^2 = xyz$

$\Rightarrow (lbh)^2 = xyz$

$\Rightarrow V^2 = xyz$

Q.4 A rectangular water reservoir contains 105 m^3 of water. Find the depth of the water in the reservoir if its base measures 12m by 3.5 m.

Sol: volume of water = 105 m^3

$l = 12$ m

$b = 3.5$ m.

$$\text{depth} = \frac{\text{volume}}{l \times b} = \frac{105}{12 \times 3.5} = 2.5 \text{ m}.$$

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Q.5 : Cubes A, B and C having edges 18 cm, 24 cm and 30 cm respectively are melted and moulded into a new cube D. Find the edge of the bigger cube D.

Sol: volume of bigger cube = V of cube A + Vol. of cube B + Vol. of cube C

Let edge of bigger cube = a cm

$\Rightarrow a^3 = (18)^3 + (24)^3 + (30)^3$

$= 5832 + 13824 + 27000$

$= 46656.$

$\Rightarrow a^3 = (36)^3 = 36$ cm.

Q.6 The breadth of a room is twice its height, one half of its length and the volume of the room is 512 cu. dm. Find its dimensions

Sol. Let length $= x$ dm, breadth $= 2x$ dm, height $= \frac{l}{2} = \frac{4x}{2}$

Volume $= 512\ dm^3$

$\Rightarrow l \times b \times h = 512$

$\Rightarrow 4x \times 2x \times x = 512$

$\Rightarrow 8x^3 = 512$

$\Rightarrow x^3 = \frac{512}{8} = 64$

$\Rightarrow x^3 = (4)^3 \Rightarrow x = 4$

$\therefore h = 4$ dm, $b = 2 \times 4 = 8$ dm, $l = 4 \times 4 = 16$ dm

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Q.7 A closed iron tank 12 m long, 9 m wide and 4 m deep is to be made. Determine the cost of iron sheet used at the rate of Rs 5 per metre sheet, sheet being 2 m wide.

Sol: Length of tank = 12 m, b = 9 m, h = 4 m.

$\Rightarrow$ Area of sheet required $= 2(lb + bh + lh)$

$= 2(12 \times 9 + 9 \times 4 + 12 \times 4)$

$= 2(108 + 36 + 48) = 2(192)$

$= 384 m^2$

Let length of sheet $= l_1$ m

breadth $= 2$ m

$\therefore l_1 \times 2 = 384 m^2$

$\Rightarrow l_1 = \frac{384}{2} = 192$ m.

$\therefore$ Cost of iron sheet at the rate of Rs 5/m

$= 5 \times 192 = Rs\ 960$

Q.8 A tank open at the top is made of iron sheet 4 m wide If the dimension of the tank are 12 m x 8 m x 6 m, find the cost of iron sheet at Rs. 17.5 ~~Rs 20~~ per metre.

Sol: Dimension of tank = 12 m x 8 m x 6 m

Where $l = 12$ m, $b = 8$ m, $h = 6$ m

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Area of sheet required = Total surface area of tank with one top open

$= l \times b + 2(l+b)h$

$= 12 \times 8 + 2(12+8)6$

$= 96 + 240 = 336\ m^2$

let length of iron sheet $= l_1$

$l_1 \times 4 = 336$

$l_1 = \frac{336}{4} = 84m.$

$\therefore$ Cost of iron sheet $= 84 \times 17.5 = Rs\ 1470$

Q.9 Three equal cubes are placed adjacently in a row. Find the ratio of total surface area of the new cuboid to that of the sum of the surface areas of the three cubes.

Sol! Let edge of three equal cubes $= a$

Then,

sum of surface areas of 3 cubes $= 3 \times 6a^2$

$= 18a^2$

When these cubes are placed in a row,

then, length of cuboid $= a+a+a = 3a$

breadth $= a$

height $= a$

Total surface area of cuboid $= 2(lb+bh+lh)$

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$$= 2(3a \times a + a \times a + 3a \times a)$$

$$= 2(3a^2 + a^2 + 3a^2) = 14a^2$$

Now, $\frac{\text{TSA of new cuboid}}{\text{Sum of area of 3 cuboids}} = \frac{14}{18} = \frac{7}{9}$

$= 7:9$

Q.10 The dimension of a room are 12.5 m by 9 m by 7 m. There are 2 doors and 4 windows in the room, each door measures 2.5 m by 1.2 m and each window 1.5 m by 1 m. Find the cost of painting the walls at Rs 3.50 per square metre.

Sol: Dimension of room = 12.5 m × 9 m × 7 m.

Dimension of door = 2.5 m × 1.2 m.

Dimension of window = 1.5 m × 1 m.

∴ Area of four walls = $2(l+b)h$

$= 2(12.5+9)7$

$= 14 \times 21.5 = 301\ m^2$

Area of 2 doors and 4 windows = $2(2.5 \times 1.2) + 4(1.5 \times 1$

$= 6 + 6 = 12\ m^2$

∴ Area of only walls = $301 - 12 = 289\ m^2$

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$\therefore$ Cost of painting the walls = Rs (3·50 × 289)

= Rs 1011·50

Q.11 A field is 150 m long and 100 m wide. A plot (outside the field) 50 m long and 30 m wide is dug to a depth of 8 m and the earth taken out from the plot is spread evenly in the field. By how much is the level of field raised?

Sol: length of field = 150 m

breadth = 100 m.

Area of field = 150 m × 100 m

= 15000 m^2

Now, length of plot = l = 50 m,

b = 30 m

h = 8 m.

$\therefore$ volume of earth dug out = 50 × 30 × 8

= 12000 m^3.

$\therefore$ level of field raised = $\frac{12000}{15000}$ = 0.8 m

= 80 cm.

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Q.12 Two cubes each of volume $512\ cm^3$ are joined end to end. Find the surface area of the resulting cuboid.

Sol. Volume of cube $= 512\ cm^3$

$$a^3 = 512 = (8)^3$$

$$\Rightarrow a = 8\ cm.$$

length of resulting cuboid $= 8+8 = 16\ cm$

$$b = 8\ cm$$

$$h = 8\ cm.$$

$\therefore$ Surface area $= 2(lb + bh + lh)$

$$= 2(16\times 8 + 8\times 8 + 16\times 8)$$

$$= 2(128 + 64 + 128)$$

$$= 2(320) = 640\ cm^2$$

Q.13 Three cubes whose edges measures 3cm, 4cm and 5cm respectively are melted to form a new cube. Find the surface area of the new cube formed.

Sol: Vol. of new cube = Sum of vol. of 3 cubes

$$a^3 = (3)^3 + (4)^3 + (5)^3$$

$$= 27 + 64 + 125 = 216$$

$$a^3 = (6)^3 \Rightarrow a = 6\ cm.$$

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$\therefore$ Surface area of cube $= 6a^2$

$= 6 \times 6^2 = 6 \times 36$

$= 216\ cm^2$

Q.14 The cost of preparing the walls of a room 12 m long at the rate of Rs 1.35 per square metre is Rs. 340.20 and the cost of matting the floor at 85 paise per square metre is Rs 91.80. Find the height of the room.

Sol: Length of room $= 12m$,

Let breadth $= b$.

Area of floor $= 12 \times b = 12b\ m^2$

$\because$ Cost of matting the floor $=$ Rs 91.80.

Rate $=$ 85 Paisa $/ m^2$

$\Rightarrow 12b \times 0.85 = 91.80$

$\Rightarrow b = \frac{91.80}{12 \times 0.85} = 9m.$

$\therefore b = 9\ m.$

Area of four walls $= 2(l+b) \times h$

$= 2(12+9)h = 42h\ m^2$

Cost of papering walls $=$ Rs 340.20

Rate $=$ Rs $1.35/m^2$

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$$42h \times 1.35 = 340.20$$

$$\Rightarrow h = \frac{340.20}{42 \times 1.35} = \frac{252}{42} = 6m.$$

$\therefore$ height of room $= 6m$.

Q.15 The length of a hall is 18m and the width 12m. The sum of the areas of the floor and flat roof is equal to the sum of the areas of the four walls. Find the height of the wall.

Sol: length of hall $= 18m$, width $= 12m$.

$\therefore$ ~~Sum~~

Area of floor + area of roof = Area of four walls.

$$\Rightarrow lb + lb = 2(l+b)h$$

$$\Rightarrow 2lb = 2(l+b)h$$

$$\rightarrow lb = (l+b)h$$

$$\Rightarrow h = \frac{lb}{l+b} = \frac{18 \times 12}{18+12} = \frac{216}{30} = 7.2m.$$

Q.16 A metal cube of edge 12cm is melted and formed into three smaller cubes. If the edges of the two smaller cubes are 6cm and 8cm, find the edge of third smaller cubes.

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Sol: Let the edge of third cube = a cm.

$\therefore \quad (12)^3 = (6)^3 + (8)^3 + a^3$

$\Rightarrow \quad 1728 = 216 + 512 + a^3$

$\Rightarrow \quad 1728 = 728 + a^3$

$\Rightarrow \quad a^3 = 1000 = (10)^3$

$\Rightarrow \quad a = 10$ cm.

Q.17 The dimension of a cinema hall are 100 m, 50 m, and 18 m. How many persons can sit in the hall, if each person requires 150 m^3 of air?

Sol: length of cinema hall = 100 m

breadth = 50 m

height = 18 m.

Vol. of air = $l \times b \times h$

$= 100 \times 50 \times 18 = 90000\ m^3$

Vol. of air occupied by each person = 150 m^3

$\therefore$ No. of persons = $\frac{90000}{150} = 600$ persons.

Q.18 The external dimensions of a closed wooden box are 48 cm, 36cm, 30 cm. The box is made of 1.5 cm thick wood. How many bricks of size 6cm x 3cm x 0.75 cm can be put in the box?

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Sol: Internal length = 48 cm,

$b = 36$ cm

$h = 30$ cm.

Thickness = 1.5 cm.

$\therefore$ Internal length = $48 - 3 = 45$ cm

$b = 36 - 3 = 33$ cm

$h = 30 - 3 = 27$ cm.

also, Dimension of bricks = 6 cm × 3 cm × 0.75 cm.

$\therefore$ No. of bricks = $\frac{45 \times 33 \times 27}{6 \times 3 \times 0.75} = 2970$ bricks.

Q.19 The dimensions of a rectangular box are in the ratio of 2:3:4. and the difference between the cost of covering it with sheet of paper at the rates of Rs 8 and Rs 9.50 per m^2 is Rs 1248. Find dimension of box.

Sol: let length = $2x$, $b = 3x$, $h = 4x$

Area of box = $2(2x \times 3x + 3x \times 4x + 2x \times 4x)$

$= 2(6x^2 + 12x^2 + 8x^2) = 52x^2$

Now, $(52x^2 \times 9.5) - (52x^2 \times 8) = 1248$

$= 494x^2 - 416x^2 = 1248$

$\Rightarrow 78x^2 = 1248 \Rightarrow x^2 = \frac{1248}{78} = 16$

$\Rightarrow x = 4$

$\therefore l = 2x = 2 \times 4 = 8$ m

$b = 3x = 3 \times 4 = 12$ m

$h = 4x = 4 \times 4 = 16$ m.

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