Institute of Actuaries of India

ACET October 2024 Indicative Solutions

Mathematics

1.	C	For a given y, x is not unique, e.g. if y = 0, x can be 1 or -1. So x is not a function of y. But for any given x, a unique y exists, namely $(1-x^{2024})^{\frac{1}{1947}}$. So y is a function of x.
2.	B	$f(g(x)) = e^{x^2}$ and $g(f(x)) = e^{2x}$. So required condition is satisfied when $x^2 < 2x$ which happens when x lies in (0,2).
3.	A	The expression $\sin^2 x + \cos^2 x + \tan^2 x + \csc^2 x + \sec^2 x + \cot^2 x$ simplifies to 3+2*($\tan^2 x + \cot^2 x$). For any positive t, the minimum value of t + 1/t is 2. So, the minimum value of the overall expression is 3+2*2 = 7. This value is attained when x = π/4.
4.	A	Note that $\alpha = \int_{20}^{24} \frac{1}{x} dx = \ln\left(\frac{24}{20}\right) = \ln(1.2)$. So $e^{\alpha} = 1.2$.
5.	B	The second quadratic is obtained by replacing x with -1/x in the original quadratic.
6.	D	$\lim_{\theta \to \frac{\pi}{4}} \frac{\cos(2\theta)}{\cos(\theta) - \sin(\theta)} = \lim_{\theta \to \frac{\pi}{4}} \frac{\cos^2(\theta) - \sin^2(\theta)}{\cos(\theta) - \sin(\theta)} = \lim_{\theta \to \frac{\pi}{4}} cos(\theta) + \sin(\theta) = \sqrt{2}$
7.	A	Let $a_i = 1*r^{i-1}$. Then, $r^{2023} = 2$. So, $4 = r^{4046} = a_{4047}$.
8.	D	If magnitude of $\vec{a}$ and $\vec{b}$ is r and angle between them is θ, then $r^2 + r^2 + 2r^2*\cos\theta = r^2$, implying cosθ = -1/2 and θ = 2π/3.
9.	C	Dot product = 20*2+2*0+4*24 = 136 = 8*17.
10.	C	If A is n x n matrix, $\lvert 2A \rvert = 2^n \lvert A \rvert$
11.	A	If $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $adj(M) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ and $adj(adj(M)) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = M$.
12.	B	$\int_0^1 e^x(x+1)dx = [xe^x]_0^1 = e - 0 = e$
13.	B	Since $e^x = \tan y$, it follows that $e^x dx = (\sec^2 y)dy$, so $\frac{dy}{dx} = \frac{e^x}{1+e^{2x}}$ which is ½ at x=0.
14.	A	Let a and d be first term and common difference. So, (23/2)*(2a+22d) = 2024. So, a+11d = 88 = 12th term, i.e. 12th term is exactly known.
15.	D	Given that each term in the expansion will have an exponent of x which is divisible by 3, x^{20} won't appear in it, leading to its coefficient being 0.
16.	C	We simplify: $\sum_{n=2}^{\infty} \frac{2024}{n^2-1} = 1012 \sum_{n=2}^{\infty} (\frac{2}{n-1} - \frac{2}{n+1}) = 1012 * \left(1 + \frac{1}{2}\right) = 1518$
17.	A	True value: $\int_0^2 x^3 dx = \left[\frac{x^4}{4}\right]_0^2 = 4$. Approximated value $= \frac{0.5}{2}(0^4 + 2*0.5^4 + 2*1^4 + 2*1.5^4 + 2^4) = 4.25$. So error = 4.25 - 4 = 0.25.
18.	A	We have $f(x) = e^x - 20x, and\ f'(x) = e^x - 20$. For x_0=1, f(1) = -17.28, f'(1) = -17.28. So $x_1 = 1 - f(1)/f'(1) = 0$. For x_1=0, f(0) = 1, f'(x) = -19. So, $x_2 = 0 - 1/(-19) = 0.053$.
19.	B	$\omega^3 - 1 = 0 \Rightarrow (\omega-1)*(\omega^2+\omega+1)=0$. But $\omega \neq 1$, so $\omega^2+\omega+1 = 0$. As $\omega^3 = 1$, $\omega^{2024} + \omega^{10} = \omega^2 + \omega^1 = -1$.
20.	C	We simplify as: $z = \frac{i-3}{i+2} = \frac{i-3}{i+2} * \frac{i-2}{i-2} = \frac{i^2-5i+6}{i^2-2^2} = \frac{5-5i}{-5} = i - 1$. So $z^2 = (i-1)^2 = i^2 + 1 - 2i = -2i$. Further, $z^8 = (-2i)^4 = 16 = 2^4$. So, $z^{2024} = 2^{1012}$

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21.	C	Note that the CDF is $F(n) = n^2 / 36$; it crosses ½ at n = 5 since $F(4) < ½ < F(5)$.
22.	B	Since mode is unique, n must be one of 2 or 3. If n=2, mode=2 and median=2.5 are unequal. If n=3, median=mode=3.
23.	A	Total possibilities = 7!. Favourable possibilities = 2*6!. Probability = 2/7.
24.	C	Depending on the ratio of boys to girls, the average can lie anywhere between 52 and 60, but cannot fall outside that range.
25.	B	$P(Y\|X)=P(X\cap Y)/P(X) = P(X\|Y)*P(Y)/P(X) = (1/3)(3/10)/(2/7) = 7/20$
26.	B	There are 7 ways to choose actuaries {A, B, C, A&B, A&C, B&C, all three} and likewise 7 to choose data scientists, leading to a total of 49 ways.
27.	A	20X+24 follows a U[24,44] distribution with $Q_1 = 29$ and $Q_3 = 39$. So IQR = 39-29 = 10.
28.	B	For P, equally likely possibilities are BG, GB and BB, so p = 1/3. For Q, similarly, there are two possibilities – BG and BB, so q = ½.
29.	D	2N only takes even integer values, so it doesn't follow a Poisson distribution. 2X, however, does follow an exponential distribution with mean 2μ, as can be checked from its PDF/CDF.
30.	A	Let B follow Bin(n,p). Then np = 12 and $npq = 2^2 = 4$. So q = 1/3, p = 2/3 and n = 18, which is the maximum value for a Binomial distribution.
31.	C	We compute probability mass functions as: p(2) = 0.1, p(3) = 0.2, p(5) = 0.1. Then, the required probability = p(2) / (p(2)+p(3)+p(5)) = 0.25 = ¼.
32.	D	Firstly, all probabilities must sum up to 1, so p must equal 0.25. E(X) = 0.45*0+0.3*1+0.2*2+0.05*3 = 0.85, E(Y) = 0.5*0+0.5*1 = 0.5, E(XY) = 0.75*0+0.15*1+0.1*2 = 0.35. So, cov(X,Y) = E(XY)-E(X)E(Y) = -0.0725
33.	D	Given that $s_y = 4s_x$, $b_{yx}/b_{xy} = (rs_y/s_x)/(rs_x/s_y) = (s_y/s_x)^2 = 16$.
34.	B	$Cov(X^{2024}, X^{1947}) = E(X^{2024*}X^{1947})-E(X^{2024})E(X^{1947})$. Since X is symmetric about 0, so are X^{1947} and X^{3972}, implying $E(X^{1947})$ and $E(X^{3972})$ are both zero. Hence, the covariance (and hence the correlation coefficient) will be zero.
35.	C	Expected area = $(1^2+2^2+…+6^2)*1/6 = 91/6$
36.	C	Let the probability of increase and decrease be u and d respectively. Since u=1.5d and u+d=1, u=0.6 and d = 0.4. After 2 months, the possible stock prices are: • $100*(1.1)^2 = 121$ with a probability of $(0.6)^2 = 0.36$ • 100*(1.1)(0.9) = 99 with a probability of 2*0.6*0.4 = 0.48 • $100*(0.9)^2 = 81$ with a probability of $(0.4)^2 = 0.16$ The probability of stock being less than 100 is 0.48+0.16 = 0.64
37.	A	The given distribution is exponential, which has the memorylessness property. So the conditional probability is same as the unconditional one, which is 1/e. (Alternatively, the conditional probability can be computed from first principles.)
38.	D	Since A and B are disjoint, $P(A\|B) = 0 < P(A)$. So they are definitely not independent.
39.	A	This will follow a binomial distribution with n = 100 and p = 0.2. So mean = np = 20. Standard deviation = √(np(1-p)) = 4
40.	B	No. of handshakes by each participant = 10. But each handshake will be counted twice. So total handshakes = 10*20/2 = 100. No. of hugs by each participant = 9. Again, each hug will be counted twice. So total hugs = 9*20/2 = 90.

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Data Interpretation

<table>
<tr><td>41.</td><td>A</td><td>Total MF investment = Rs. 1.2 crore = 60% of total. So, total investment = Rs. 2 crore.
Fixed income investments are 20%+5% = 25%, so in monetary terms it is Rs. 50 lakh.</td></tr>
<tr><td>42.</td><td>B</td><td>For total MF to reduce to 50%, equity MF should become 30% and direct stocks should become 20% which is same as real estate.</td></tr>
<tr><td>43.</td><td>C</td><td>Equity investments of Rs. 1 crore will become Rs. 1.15 crore.
Real estate of Rs. 40 lakh will become Rs. 45 lakh.
Gold of Rs. 10 lakh will become Rs. 11 lakh.
Fixed income investment of Rs. 50 lakh will remain unchanged.
So, total value = Rs. 2.21 crore.</td></tr>
<tr><td>44.</td><td>B</td><td>Returns > 1cr: 49k, total returns with positive salary = 58713k – 29680k = 29033k; ratio = 0.17%</td></tr>
<tr><td>45.</td><td>C</td><td>For returns > 1 cr, total salary income = 110594cr; ratio = 110594cr/2004069cr = 5.5%</td></tr>
<tr><td>46.</td><td>A</td><td>For returns > 25 cr, sum of salary income = 3505+2276+1154 cr = 6935 crore.
Count of returns = 106+35+9 = 150. Average = 6935 crore / 150 = 46.2 crore</td></tr>
<tr><td>47.</td><td>D</td><td>Since more than half the returns have filed a zero salary income, the median is zero.</td></tr>
<tr><td>48.</td><td>B</td><td rowspan="4">The given information can be analysed to produce the following table:
<table>
<tr><td>Year \ Unit</td><td>A</td><td>B</td><td>C</td><td>Total</td></tr>
<tr><td>2021</td><td>(500+x)/2</td><td>(700-x)/2</td><td>600</td><td>1200</td></tr>
<tr><td>2022</td><td>750</td><td>x</td><td>450</td><td>1200+x</td></tr>
<tr><td>2023</td><td>700-x</td><td>x</td><td>700</td><td>1400</td></tr>
<tr><td>Total</td><td>1700-x/2</td><td>(700+3x)/2</td><td>1750</td><td>3800+x</td></tr>
</table>
Here, x can be any even integer between 0 and 700.

For Q48, the proportion is 50% in 2021 and 2023 and definitely less than 50% in 2022.
For Q49, the information is insufficient for a unique answer. It could be any of the options depending on x.
For Q50, the information is insufficient for a unique answer. It could be A or D, but not B or C (as the corresponding values of x won’t be integer).
For Q51, the maximum value that any of the values contingent on x can take is 700 (this happens when x = 0 or x = 700). However, production at A in 2022 is 750 which is clearly the highest.</td></tr>
<tr><td>49.</td><td>A,B,C,D</td></tr>
<tr><td>50.</td><td>A,D</td></tr>
<tr><td>51.</td><td>B</td></tr>
</table>

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52.	D	
53.	B	
54.	B	
55.	C	
56.	A	The judges disposed of …
57.	D	
58.	D	‘Astonished’ means ‘surprised’; others mean ‘happy’.
59.	B	
60.	C	“*Political strife and violent conflicts … while attracting little focus in others.*”
61.	D	
62.	D	

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Logical Reasoning

63.	D	2024 being a leap year, if the birthday was in January or February, it'll be Monday; else it'll be Tuesday.
64.	A	The gap between each consecutive overlap is 12/11 hours. So there are 11 such intervals over a 12 hour period, leading to 10 overlap points (excluding the two ends).
65.	B	Of the 12 edged, 8 of them have red on either side, leading to 8 times 5 = 40 cubes. However, 4 of the cubes (which have red on 3 sides) have been counted thrice, so we subtract 4 times 2 = 8. The final count is 40-8 = 32.
66.	C	X's mother and Y are siblings.
67.	D	2,4,5,1,3 is a valid sequence where A, B and C are false.
68.	B	B is the contrapositive of "All alphas are betas."
69.	B	Min. no. of boys studying Sanskrit = 20+24-40 = 4.
70.	C	Other three are increasingly bigger.

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