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Ch-2
Polynomials
~~Real Numbers~~ Exercise 2.1

(v) $p(x) = x^2 + 2\sqrt{2}x - 6$

$x^2 + 2\sqrt{2}x - 6 = 0$

$x^2 + 3\sqrt{2}x - \sqrt{2}x - 6 = 0$

$x(x + 3\sqrt{2}) - \sqrt{2}(x + 3\sqrt{2}) = 0$

$(x + 3\sqrt{2})(x - \sqrt{2}) = 0$

$x + 3\sqrt{2} = 0$ | $x - \sqrt{2} = 0$

$x = -3\sqrt{2}$ | $x = \sqrt{2}$

Sum of zeroes $= \frac{-b}{a}$

$\sqrt{2} + (-3\sqrt{2}) = -\frac{2\sqrt{2}}{1}$

$-2\sqrt{2} = -2\sqrt{2}$

Product of zeroes $= \frac{c}{a}$

$\sqrt{2} \times (-3\sqrt{2}) = \frac{-6}{1}$

$-6 = -6$

Hence, verified.

(vi) $q(x) = \sqrt{3}x^2 + 10x + 7\sqrt{3}$

$\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

$\sqrt{3}x^2 + 3x + 7x + 7\sqrt{3} = 0$

$\sqrt{3}x(x + \sqrt{3}) + 7(x + \sqrt{3}) = 0$

$(x + \sqrt{3})(\sqrt{3}x + 7) = 0$

$x + \sqrt{3} = 0$ | $\sqrt{3}x + 7 = 0$

$x = -\sqrt{3}$ | $x = \frac{-7}{\sqrt{3}}$

Sum of zeroes $= \frac{-b}{a}$

$-\sqrt{3} + \left(\frac{-7}{\sqrt{3}}\right) = \frac{-10}{\sqrt{3}}$

$\frac{-3-7}{\sqrt{3}} = \frac{-10}{\sqrt{3}}$

$\frac{-10}{\sqrt{3}} = \frac{-10}{\sqrt{3}}$

Product of zeroes $= \frac{c}{a}$

$-\sqrt{3} \times \left(\frac{-7}{\sqrt{3}}\right) = \frac{7}{\sqrt{3}}$

$\frac{7}{\sqrt{3}} = \frac{7}{\sqrt{3}}$

(vii) $f(x) = x^2 - (\sqrt{3}+1)x + \sqrt{3}$

$x^2 - (\sqrt{3}+1)x + \sqrt{3} = 0$

$x^2 - \sqrt{3}x - x + \sqrt{3} = 0$

$x(x - \sqrt{3}) - 1(x - \sqrt{3}) = 0$

$(x - \sqrt{3})(x - 1) = 0$

$x - \sqrt{3} = 0$ | $x - 1 = 0$

$\boxed{x = \sqrt{3}}$ | $\boxed{x = 1}$

Sum of zeroes $= \frac{-b}{a}$

$\sqrt{3} + 1 = \frac{-[-(\sqrt{3}+1)]}{1}$

$\boxed{\sqrt{3} + 1 = \sqrt{3} + 1}$

Product of zeroes $= \frac{c}{a}$

$\sqrt{3} \times 1 = \frac{\sqrt{3}}{1}$

$\boxed{\sqrt{3} = \sqrt{3}}$

(viii) $g(x) = a(x^2+1) - x(a^2+1)$

$a(x^2+1) - x(a^2+1) = 0$

$ax^2 + a - a^2x - x = 0$

$ax^2 - a^2x - x + a = 0$

$ax(x-a) - 1(x-a) = 0$

$(x-a)(ax-1) = 0$

$x - a = 0$ | $ax - 1 = 0$

$\boxed{x = a}$ | $\boxed{x = \frac{1}{a}}$

Sum of zeroes $= \frac{-b}{a}$

$a + \frac{1}{a} = \frac{-[-(a^2+1)]}{a}$

1(viii)

$$\frac{a^2+1}{a} = \frac{a^2+1}{a}$$

Product of zeroes $= \frac{c}{a}$

$$\frac{\alpha \times 1}{\alpha} = \frac{\alpha 1}{\alpha 1}$$

$$1 = 1$$

(ix) $h(s) = 2s^2 - (1+2\sqrt{2})s + \sqrt{2}$

$$2s^2 - (1+2\sqrt{2})s + \sqrt{2} = 0$$
$$2s^2 - s - 2\sqrt{2}s + \sqrt{2} = 0$$
$$2s^2 - 2\sqrt{2}s - s + \sqrt{2} = 0$$
$$2s(s-\sqrt{2}) - 1(s-\sqrt{2}) = 0$$
$$(s-\sqrt{2})(2s-1) = 0$$

$s - \sqrt{2} = 0$ | $2s - 1 = 0$

$s = \sqrt{2}$ | $s = \frac{1}{2}$

Sum of zeroes $= \frac{-b}{a}$

$$\sqrt{2} + \frac{1}{2} = \frac{-[-(1+2\sqrt{2})]}{2}$$

$$\frac{2\sqrt{2}+1}{2} = \frac{2\sqrt{2}+1}{2}$$

Product of zeroes $= \frac{c}{a}$

$$\sqrt{2} \times \frac{1}{2} = \frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$$

(x) $f(v) = v^2 + 4\sqrt{3}v - 15$

$$v^2 + 4\sqrt{3}v - 15 = 0$$
$$v^2 + 5\sqrt{3}v - \sqrt{3}v - 15 = 0$$
$$v(v + 5\sqrt{3}) - \sqrt{3}(v + 5\sqrt{3}) = 0$$
$$(v + 5\sqrt{3})(v - \sqrt{3}) = 0$$

$v + 5\sqrt{3} = 0$ | $v - \sqrt{3} = 0$

$v = -5\sqrt{3}$ | $v = \sqrt{3}$

Sum of zeroes $= \frac{-b}{a}$

$$-5\sqrt{3} + \sqrt{3} = \frac{-4\sqrt{3}}{1}$$
$$-4\sqrt{3} = -4\sqrt{3}$$

Product of zeroes $= \frac{c}{a}$

$$-5\sqrt{3} \times \sqrt{3} = \frac{-15}{1}$$
$$-15 = -15$$

Hence, verified.

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Q.1 (xi) $p(y) = y^2 + \frac{3\sqrt{5}}{2}y - 5$

$$y^2 + \frac{3\sqrt{5}}{2}y - 5 = 0$$

$$2y^2 + 3\sqrt{5}y - 10 = 0$$

$$2y^2 + 4\sqrt{5}y - \sqrt{5}y - 10 = 0$$

$$2y(y + 2\sqrt{5}) - \sqrt{5}(y + 2\sqrt{5}) = 0$$

$$(y + 2\sqrt{5})(2y - \sqrt{5}) = 0$$

$y + 2\sqrt{5} = 0$

$\boxed{y = -2\sqrt{5}}$

$2y - \sqrt{5} = 0$

$\boxed{y = \frac{\sqrt{5}}{2}}$

Sum of zeroes $= \frac{-b}{a}$

$$-2\sqrt{5} + \frac{\sqrt{5}}{2} = \frac{-3\sqrt{5}}{2}$$

$$\frac{-4\sqrt{5} + \sqrt{5}}{2} = \frac{-3\sqrt{5}}{2}$$

$$\boxed{\frac{-3\sqrt{5}}{2} = \frac{-3\sqrt{5}}{2}}$$

Product of zeroes $= \frac{c}{a}$

$$-2\sqrt{5} \times \frac{\sqrt{5}}{2} = \frac{-5}{1}$$

4(xi)

$$\boxed{-5 = -5}$$

Hence, verified

(xii) $q(y) = 7y^2 - \frac{11}{3}y - \frac{2}{3}$

$$7y^2 - \frac{11}{3}y - \frac{2}{3} = 0$$

$$21y^2 - 11y - 2 = 0$$

$$21y^2 - 14y + 3y - 2 = 0$$

$$7y(3y - 2) + 1(3y - 2) = 0$$

$$(3y - 2)(7y + 1) = 0$$

$3y - 2 = 0$

$$\boxed{y = \frac{2}{3}}$$

$7y + 1 = 0$

$$\boxed{y = -\frac{1}{7}}$$

Sum of zeroes $= \frac{-b}{a}$

$$\frac{2}{3} - \frac{1}{7} = \frac{-(-11)}{3 \times 7}$$

$$\frac{14 - 3}{21} = \frac{11}{21}$$

$$\boxed{\frac{11}{21} = \frac{11}{21}}$$

Product of zeroes $= \frac{c}{a}$

$$\frac{2}{3} \times \frac{(-1)}{7} = \frac{-2}{3 \times 7}$$

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A(iii)

$$\boxed{\frac{-2}{21} = \frac{-2}{21}}$$

Hence, verified

Q.2 (i) $\frac{-8}{3}, \frac{4}{3}$

$$\alpha+\beta = \frac{-8}{3}, \quad \alpha\beta = \frac{4}{3}$$

$$k[x^2 - (\alpha+\beta)x + \alpha\beta]$$
$$= k\left[x^2 - \left(\frac{-8}{3}\right)x + \frac{4}{3}\right]$$
$$= k\left[x^2 + \frac{8}{3}x + \frac{4}{3}\right]$$

Let $k = 3$

$$= 3\left[x^2 + \frac{8}{3}x + \frac{4}{3}\right]$$
$$= \boxed{3x^2 + 8x + 4}$$

$$3x^2 + 8x + 4 = 0$$
$$3x^2 + 6x + 2x + 4 = 0$$
$$3x(x+2) + 2(x+2) = 0$$
$$(x+2)(3x+2) = 0$$

$x+2 = 0$ | $3x+2 = 0$

$\boxed{x = -2}$ | $\boxed{x = \frac{-2}{3}}$

(ii) $\frac{21}{8}, \frac{5}{16}$

$$\alpha+\beta = \frac{21}{8}, \quad \alpha\beta = \frac{5}{16}$$

$$k[x^2 - (\alpha+\beta)x + \alpha\beta]$$
$$= k\left[x^2 - \left(\frac{21}{8}\right)x + \frac{5}{16}\right]$$
$$= k\left[x^2 - \frac{42}{16}x + \frac{5}{16}\right]$$

Let $k = 16$

$$= 16\left[x^2 - \frac{42}{16}x + \frac{5}{16}\right]$$
$$= \boxed{16x^2 - 42x + 5}$$

$$16x^2 - 42x + 5 = 0$$
$$16x^2 - 40x - 2x + 5 = 0$$
$$8x(2x-5) - 1(2x-5) = 0$$
$$(2x-5)(8x-1) = 0$$

$2x - 5 = 0$

$$\boxed{x = \frac{5}{2}}$$

$8x - 1 = 0$

$$\boxed{x = \frac{1}{8}}$$

(iii) $-2\sqrt{3}, -9$

$$\alpha+\beta = -2\sqrt{3}, \quad \alpha\beta = -9$$

$$k[x^2 - (\alpha+\beta)x + \alpha\beta]$$
$$= k[x^2 - (-2\sqrt{3})x + (-9)]$$

2(iii)

$= k[x^2 + 2\sqrt{3}x - 9]$

Let $k = 1$

$= 1[x^2 + 2\sqrt{3}x - 9]$

$= \boxed{x^2 + 2\sqrt{3}x - 9}$

$x^2 + 2\sqrt{3}x - 9 = 0$

$x^2 + 3\sqrt{3}x - \sqrt{3}x - 9 = 0$

$x(x + 3\sqrt{3}) - \sqrt{3}(x + 3\sqrt{3}) = 0$

$(x + 3\sqrt{3})(x - \sqrt{3}) = 0$

$x + 3\sqrt{3} = 0$ | $x - \sqrt{3} = 0$

$\boxed{x = -3\sqrt{3}}$ | $\boxed{x = \sqrt{3}}$

(iv) $\frac{-3}{2\sqrt{5}}, \frac{-1}{2}$

$\alpha + \beta = \frac{-3}{2\sqrt{5}}, \quad \alpha\beta = \frac{-1}{2}$

$k[x^2 - (\alpha + \beta)x + \alpha\beta]$

$= k\left[x^2 - \left(\frac{-3}{2\sqrt{5}}\right)x + \left(\frac{-1}{2}\right)\right]$

$= k\left[x^2 + \frac{3}{2\sqrt{5}}x - \frac{1}{2}\right]$

$= k\left[x^2 + \frac{3}{2\sqrt{5}}x - \frac{\sqrt{5}}{2\sqrt{5}}\right]$

Let $k = 2\sqrt{5}$

$= 2\sqrt{5}\left[x^2 + \frac{3}{2\sqrt{5}}x - \frac{\sqrt{5}}{2\sqrt{5}}\right]$

$= \boxed{2\sqrt{5}x^2 + 3x - \sqrt{5}}$

$$2\sqrt{5}x^2 + 3x - \sqrt{5} = 0$$

$$2\sqrt{5}x^2 + 5x - 2x - \sqrt{5} = 0$$

$$\sqrt{5}x(2x + \sqrt{5}) - 1(2x + \sqrt{5}) = 0$$

$$(2x + \sqrt{5})(\sqrt{5}x - 1) = 0$$

$2x + \sqrt{5} = 0$

$$\boxed{x = \frac{-\sqrt{5}}{2}}$$

$\sqrt{5}x - 1 = 0$

$$\boxed{x = \frac{1}{\sqrt{5}}}$$

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Exercise 2.1

Q.3 $f(x) = x^2 - 5x + 4$

$a = 1, b = -5, c = 4$

α & β are the zeroes of $f(x)$

$\alpha + \beta = \frac{-b}{a} = \frac{-(-5)}{1} = 5$

$\alpha\beta = \frac{c}{a} = \frac{4}{1} = 4$

$\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$

$= \frac{\beta + \alpha}{\alpha\beta} - 2\alpha\beta$

$= \frac{5}{4} - 2 \times 4$

$= \frac{5}{4} - 8$

$= \frac{5 - 32}{4} = \boxed{\frac{-27}{4}}$ ✓

Q.4 $p(y) = 5y^2 - 7y + 1$

$a = 5, b = -7, c = 1$

α & β are the zeroes of $p(y)$

$\alpha + \beta = \frac{-b}{a} = \frac{-(-7)}{5} = \frac{7}{5}$

$\alpha\beta = \frac{c}{a} = \frac{1}{5}$

$$\frac{1}{\alpha} + \frac{1}{\beta}$$

$$= \frac{\beta + \alpha}{\alpha\beta} = \frac{\frac{7}{5}}{\frac{1}{5}}$$

$$= \frac{7}{\cancel{5}} \times \frac{\cancel{5}}{1} = \boxed{7}$$ ✓

Q.5 $f(x) = x^2 - x - 4$

$a = 1, b = -1, c = -4$

α & β are the zeroes of $f(x)$

$$\alpha + \beta = \frac{-b}{a} = \frac{-(-1)}{1} = 1$$

$$\alpha\beta = \frac{c}{a} = \frac{-4}{1} = -4$$

$$\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$$

$$= \frac{\beta + \alpha}{\alpha\beta} - \alpha\beta = \frac{1}{-4} - (-4)$$

$$= \frac{-1}{4} + 4 = \frac{-1 + 16}{4} = \boxed{\frac{15}{4}}$$ ✓

Q.6 $f(x) = x^2 + x - 2$

$a = 1, b = 1, c = -2$

α & β are the zeroes of $f(x)$

$$\alpha + \beta = \frac{-b}{a} = \frac{-1}{1} = -1$$

$$\alpha\beta = \frac{c}{a} = \frac{-2}{1} = -2$$

$\frac{1}{\alpha} - \frac{1}{\alpha}$

q.6 $f(x) = x^2 + x - 2$

$x^2 + x - 2 = 0$

$x^2 + 2x - x - 2 = 0$

$x(x+2) - 1(x+2) = 0$

$(x+2)(x-1) = 0$

$x + 2 = 0$	$x - 1 = 0$
$x = -2$	$x = 1$

$\alpha = -2$

$\beta = 1$

$\frac{1}{\alpha} - \frac{1}{\beta}$

$= \frac{1}{-2} - \frac{1}{1} = \frac{-1-2}{2}$

$= \boxed{-\frac{3}{2}}$

q.7 $f(x) = 4x^2 - 8kx - 9$

$a = 4, b = -8k, c = -9$

α and β are the zeroes.

$\alpha + \beta = \frac{-b}{a} = \frac{-(-8k)}{4} = \frac{8k}{4} = 2k$

$\alpha + \beta = 2k$

$\alpha + (-\alpha) = 2k$ (One zero is -ve of the other)

$$\alpha - \alpha = 2k$$

$$0 = 2k$$

$$k = \frac{0}{2}$$

$$\boxed{k = 0}$$

Q.8 $f(t) = kt^2 + 2t + 3k$

$a = k, \; b = 2, \; c = 3k$

α & β are the zeroes.

$$\alpha + \beta = \frac{-b}{a} = \frac{-2}{k}$$

$$\alpha\beta = \frac{c}{a} = \frac{3k}{k} = 3$$

ATQ,

$$\alpha + \beta = \alpha\beta$$

$$\frac{-2}{k} = 3$$

$$-2 = 3k$$

$$\boxed{k = \frac{-2}{3}}$$

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Q.9 $f(x) = 4x^2 - 5x - 1$

$a = 4, b = -5, c = -1$

α & β are the zeroes.

$$\alpha + \beta = \frac{-b}{a} = \frac{-(-5)}{4} = \frac{5}{4}$$

$$\alpha\beta = \frac{c}{a} = \frac{-1}{4}$$

$$\alpha^2\beta + \alpha\beta^2$$
$$= \alpha\beta(\alpha + \beta)$$
$$= \frac{-1}{4} \times \frac{5}{4} = \boxed{\frac{-5}{16}}$$

Q.10 $f(t) = t^2 - 4t + 3$

$a = 1, b = -4, c = 3$

α & β are the zeroes.

$$\alpha + \beta = \frac{-b}{a} = \frac{-(-4)}{1} = 4$$

$$\alpha\beta = \frac{c}{a} = \frac{3}{1} = 3$$

$$\alpha^4\beta^3 + \alpha^3\beta^4$$
$$= (\alpha\beta)^3(\alpha + \beta) = (3)^3(4)$$
$$= 24 \times 4 = \boxed{108}$$

Q-11 $f(x) = 6x^2 + x - 2$

$6x^2 + x - 2 = 0$

$6x^2 + 4x - 3x - 2 = 0$

$2x(3x+2) - 1(3x+2) = 0$

$(3x+2)(2x-1) = 0$

$3x + 2 = 0$ | $2x - 1 = 0$

$x = \frac{-2}{3}$ | $x = \frac{1}{2}$

$\alpha = \frac{-2}{3}, \quad \beta = \frac{1}{2}$

$\therefore \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

$= \frac{\frac{-2}{3}}{\frac{1}{2}} + \frac{\frac{1}{2}}{\frac{-2}{3}} = \left(\frac{-2}{3} \times \frac{2}{1}\right) + \left(\frac{1}{2} \times \frac{3}{-2}\right)$

$= \frac{-4}{3} + \frac{(-3)}{4}$

$= \frac{-16 - 9}{12} = \boxed{\frac{-25}{12}}$

Q-12 $p(s) = 3s^2 - 6s + 4$

$a = 3, \quad b = -6, \quad c = 4$

$b^2 - 4ac = (-6)^2 - (4)(3)(4)$

$= 36 - 48$

$= -12$

α and β are the zeroes.

$\alpha + \beta = \frac{-b}{a} = \frac{-(-6)}{3} = \frac{6}{3} = 2$

Q12

$$\alpha\beta = \frac{c}{a} = \frac{4}{3}$$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$$

$$= \frac{\alpha^2 + \beta^2}{\alpha\beta} + 2\left(\frac{\beta + \alpha}{\alpha\beta}\right) + 3\alpha\beta$$

$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} + 2\left(\frac{\beta + \alpha}{\alpha\beta}\right) + 3\alpha\beta$$

$$= \frac{(2)^2 - 2\left(\frac{4}{3}\right)}{\frac{4}{3}} + 2\left(\frac{x}{\frac{x_2}{3}}\right) + 3\left(\frac{4}{3}\right)$$

$$= \frac{4 - \frac{8}{3}}{\frac{4}{3}} + \frac{6_3}{x} + \frac{12_4}{3}$$

$$= \frac{12 - 8}{x} \times \frac{3}{4} + 3 + 4$$

$$= \frac{4}{4} + 3 + 4$$

$$= 1 + 3 + 4 = \boxed{8}$$

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$-(b+a)$
$+(b+a)$
$-(b-a)$

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Q-13 $x^2 + \beta x + 45$

$a = 1, b = \beta, c = 45$

α and β are the zeroes

$$\alpha + \beta = \frac{-b}{a} = \frac{-\beta}{1} = -\beta$$

$$\alpha\beta = \frac{c}{a} = \frac{45}{1} = 45$$

~~$\alpha^2 + \beta^2 = 144$~~

~~$(\alpha+\beta)(\alpha-\beta) = 144$~~

~~$\alpha^2 - \beta^2 = 144$~~

~~$-\beta^2 + \alpha^2 = 144$~~

~~$(-\beta + \alpha)^2 - 2\alpha(-\beta) = 144$~~

$$(a-b)^2 = 144$$

$$(a+b)^2 - 4ab = (a-b)^2$$

$$(\alpha+\beta)^2 - 4\alpha\beta = 144$$

$$(-\beta)^2 - 4(45) = 144$$

$$\beta^2 - 180 = 144$$

$$\beta^2 = 144 + 180$$

$$\beta^2 = 324$$

$$\beta = \sqrt{324}$$

$$\boxed{\beta = \pm 18}$$

Q.14 $f(x) = x^2 - px + q$

To prove $\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{p^4}{q^2} - \frac{4p^2}{q} + 2$

$x^2 - px + q$

$a = 1, b = -p, c = q$

$\alpha + \beta = \frac{-b}{a} = \frac{-(-p)}{1} = p$

$\alpha\beta = \frac{c}{a} = \frac{q}{1} = q$

$LHS = \frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2}$

$= \frac{(\alpha^2)^2 + (\beta^2)^2}{\alpha^2\beta^2}$

$= \frac{\alpha^4 + \beta^4 + 2\alpha^2\beta^2 - 2\alpha^2\beta^2}{\alpha^2\beta^2}$

$= \frac{(\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2}{\alpha^2\beta^2}$

$= \frac{[(\alpha+\beta)^2 - 2\alpha\beta]^2 - 2\alpha^2\beta^2}{\alpha^2\beta^2}$

$= \frac{[p^2 - 2q]^2 - 2q^2}{\alpha^2\beta^2}$

$= \frac{p^4 + 4q^2 - 4p^2q - 2q^2}{q^2}$

$= \frac{p^4 + 2q^2 - 4p^2q}{q^2}$

$= \frac{p^4}{q^2} + \frac{2q^2}{q^2} - \frac{4p^2q}{q^2}$

$= \boxed{\frac{p^4}{q^2} - \frac{4p^2}{q} + 2} = RHS$

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Q.15 $f(x) = x^2 - px - p - c$

$= x^2 - px - (p + c)$

$a = 1, b = -p, c = -(p + c)$

α and β are the zeroes.

$\alpha + \beta = \frac{-b}{a} = \frac{-(-p)}{1} = p$

$\alpha\beta = \frac{c}{a} = \frac{-(p+c)}{1} = -(p+c)$

LHS $= (\alpha + 1)(\beta + 1)$

$= \alpha\beta + \alpha + \beta + 1$

$= \alpha\beta + (\alpha + \beta) + 1$

$= -(p+c) + p + 1$

$= -p - c + p + 1$

$= -c + 1$

$= \boxed{1 - c} = $ RHS

Hence, proved.

Q.16 α and β are the zeroes

$\alpha + \beta = 24$ and $\alpha - \beta = 8$

$\quad$ ① $\quad$ ②

Add ① & ②

$\alpha + \beta = 24$

$(+)\ \alpha - \beta = 8$

$2\alpha = 32$

$\alpha = 16$ —— ③

Put ③ in ①

$\alpha + \beta = 24$

Q16

$$16 + \alpha = 24$$
$$\alpha = 24 - 16$$
$$\alpha = 8 \quad \text{---} \quad (4)$$

$$\alpha\beta = 16 \times 8 = 128$$

$$k\,[x^2 - (\alpha+\beta)x + \alpha\beta]$$
$$= k\,[x^2 - 24x + 128]$$

Let $k = 1$

$$= 1\,[x^2 - 24x + 128]$$
$$= \boxed{x^2 - 24x + 128}$$ ✓

Q.17 $f(x) = x^2 - 1$

$$= x^2 + 0x - 1$$

$a = 1, \quad b = 0, \quad c = -1$

$$\alpha + \beta = \frac{-b}{a} = 0$$

$$\alpha\beta = \frac{c}{a} = \frac{-1}{1} = -1$$

Zeroes of another polynomial are $\frac{2\alpha}{\beta}$ & $\frac{2\beta}{\alpha}$.

$$\text{Sum} = \frac{2\alpha}{\beta} + \frac{2\beta}{\alpha}$$

$$= \frac{2\alpha^2 + 2\beta^2}{\alpha\beta}$$

$$= \frac{2(\alpha^2 + \beta^2)}{\alpha\beta} = \frac{2[(\alpha+\beta)^2 - 2\alpha\beta]}{\alpha\beta}$$

$$= \frac{2[(0)^2 - 2(-1)]}{-1} = \frac{2[2]}{-1} = -4$$

Product $= \frac{2\alpha}{\beta} \times \frac{2\beta}{\alpha}$

$= 4$

$k[x^2 - (\alpha+\beta)x + \alpha\beta]$

$= k[x^2 - (-4)x + 4]$

$= k[x^2 + 4x + 4]$

Let $k = 1$

$= 1[x^2 + 4x + 4]$

$= \boxed{x^2 + 4x + 4}$ ✓

Q.13 $f(x) = x^2 - 3x - 2$

$a = 1, b = -3, c = -2$

$\alpha + \beta = \frac{-b}{a} = \frac{-(-3)}{1} = 3$

$\alpha\beta = \frac{c}{a} = \frac{-2}{1} = -2$

Zeros of another polynomial are $\frac{1}{2\alpha+\beta}$ and $\frac{1}{2\beta+\alpha}$

Sum $= \frac{1}{2\alpha+\beta} + \frac{1}{2\beta+\alpha}$

$= \frac{2\beta+\alpha+2\alpha+\beta}{(2\alpha+\beta)(2\beta+\alpha)} = \frac{3\alpha+3\beta}{4\alpha\beta+2\alpha^2+2\beta^2+\alpha\beta}$

$= \frac{3(\alpha+\beta)}{2(\alpha^2+\beta^2+2\alpha\beta)+\alpha\beta} = \frac{3(\alpha+\beta)}{2(\alpha+\beta)^2+\alpha\beta}$

$= \frac{3 \times 3}{2(3)^2 - 2} = \frac{9}{18-2} = \frac{9}{16}$

Q.18

$$\text{Product} = \frac{1}{2\alpha+\beta} \times \frac{1}{2\beta+\alpha}$$

$$= \frac{1}{(2\alpha+\beta)(2\beta+\alpha)}$$

$$= \frac{1}{4\alpha\beta + 2\alpha^2 + 2\beta^2 + \alpha\beta}$$

$$= \frac{1}{2(\alpha^2+\beta^2+2\alpha\beta)+\alpha\beta}$$

$$= \frac{1}{2(\alpha+\beta)^2 + \alpha\beta}$$

$$= \frac{1}{2(3)^2 + (-2)}$$

$$= \frac{1}{18-2} = \frac{1}{16}$$

$$k[x^2 - (\alpha+\beta)x + \alpha\beta]$$

$$= k\left[x^2 - \frac{9}{16}x + \frac{1}{16}\right]$$

Let $k = 16$

$$= 16\left[x^2 - \frac{9}{16}x + \frac{1}{16}\right]$$

$$= \boxed{x^2 - 9x + 1}$$

Q.19 $f(x) = x^2 + px + q$

$a = 1, b = p, c = q$

$$\alpha + \beta = \frac{-b}{a} = \frac{-p}{1} = -p$$

$$\alpha\beta = \frac{c}{a} = \frac{q}{1} = q$$

Q19

Zeros of another polynomial are $(\alpha+\beta)^2$ and $(\alpha-\beta)^2$

$$\text{Sum} = (\alpha+\beta)^2 + (\alpha-\beta)^2$$
$$= (-\beta)^2 + \alpha^2 + \beta^2 - 2\alpha\beta$$
$$= \beta^2 + (\alpha+\beta)^2 - 2\alpha\beta - 2\alpha\beta$$
$$= \beta^2 + \beta^2 - 4(q)$$
$$= 2\beta^2 - 4q$$

$$\text{Product} = (\alpha+\beta)^2 \times (\alpha-\beta)^2$$
$$= (\beta^2) \times (\beta^2 - 4q)$$
$$= (\beta^2 - 4q)\beta^2$$

$$k[x^2 - (\alpha+\beta)x + \alpha\beta]$$
$$= k[x^2 - (2\beta^2 - 4q)x + \beta^2(\beta^2 - 4q)]$$
$$= k[x^2 - 2(\beta^2 - 2q)x + \beta^2(\beta^2 - 4q)]$$

Let $k = 1$

$$= 1[x^2 - 2(\beta^2 - 2q)x + \beta^2(\beta^2 - 4q)]$$
$$= [x^2 - 2(\beta^2 - 2q)x + \beta^2(\beta^2 - 4q)]$$

Q20 $f(x) = x^2 - 2x + 3$

$a = 1, b = -2, c = 3$

$$\alpha+\beta = \frac{-b}{a} = \frac{-(-2)}{1} = 2$$

$$\alpha\beta = \frac{c}{a} = \frac{3}{1} = 3$$

(i) Zeros are $\alpha+2, \beta+2$

$$\text{Sum} = \alpha+2+\beta+2$$
$$= (\alpha+\beta) + 4$$
$$= 2 + 4 = 6$$

Q20 (i)

Product $= (\alpha+2)(\beta+2)$

$= \alpha\beta + 2\alpha + 2\beta + 4$

$= \alpha\beta + 2(\alpha+\beta) + 4$

$= 3 + (2 \times 2) + 4$

$= 3 + 4 + 4$

$= 11$

$k[x^2 - (\alpha+\beta)x + \alpha\beta]$

$= k[x^2 - 6x + 11]$

Let $k = 1$

$= 1[x^2 - 6x + 11]$

$= \boxed{x^2 - 6x + 11}$ ✓

(ii) Zeros are $\frac{\alpha-1}{\alpha+1}$, $\frac{\beta-1}{\beta+1}$

Sum $= \frac{\alpha-1}{\alpha+1} + \frac{\beta-1}{\beta+1}$

$= \frac{(\alpha-1)(\beta+1) + (\beta-1)(\alpha+1)}{(\alpha+1)(\beta+1)}$

$= \frac{\alpha\beta + \alpha - \beta - 1 + \alpha\beta + \beta - \alpha - 1}{\alpha\beta + \alpha + \beta + 1}$

$= \frac{2\alpha\beta - 2}{\alpha\beta + (\alpha+\beta) + 1}$

$= \frac{(2 \times 3) - 2}{3 + 2 + 1} = \frac{6-2}{6} = \frac{4}{6} = \frac{2}{3}$

Product $= \frac{\alpha-1}{\alpha+1} \times \frac{\beta-1}{\beta+1}$

$= \frac{(\alpha-1)(\beta-1)}{(\alpha+1)(\beta+1)}$

$= \frac{\alpha\beta - \alpha - \beta + 1}{\alpha\beta + \alpha + \beta + 1} = \frac{\alpha\beta - (\alpha+\beta) + 1}{\alpha\beta + (\alpha+\beta) + 1}$

Q2 d(ii)

$$= \frac{(3-2)+1}{3+2+1} = \frac{2}{6}\,1 = \frac{1}{3}$$

$$k[x^2 - (\alpha+\beta)x + \alpha\beta]$$
$$= k\left[x^2 - \frac{2}{3}x + \frac{1}{3}\right]$$

Let $k = 3$

$$= 3\left[x^2 - \frac{2}{3}x + \frac{1}{3}\right]$$
$$= \boxed{3x^2 - 2x + 1}$$ ✓

Q-2) $f(x) = ax^2 + bx + c$

$$\alpha + \beta = \frac{-b}{a}$$

$$\alpha\beta = \frac{c}{a}$$

(i) $\alpha - \beta$

$$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$$
$$\alpha - \beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$$
$$= \sqrt{\left(\frac{-b}{a}\right)^2 - \frac{4c}{a}} = \sqrt{\frac{b^2}{a^2} - \frac{4c}{a}}$$
$$= \sqrt{\frac{b^2 - 4ac}{a^2}} = \boxed{\frac{\sqrt{b^2-4ac}}{a}}$$

(ii) $\frac{1}{\alpha} - \frac{1}{\beta}$

$$\frac{1}{\alpha} - \frac{1}{\beta} = \frac{\beta - \alpha}{\alpha\beta} = \frac{-(\alpha-\beta)}{\alpha\beta}$$
$$= \frac{\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}}{\alpha\beta}$$

Q21(ii)

$$= \frac{\sqrt{\left(\frac{-b}{a}\right)^2 - \frac{4c}{a}}}{\frac{c}{a}} = \frac{\sqrt{\frac{b^2}{a^2} - \frac{4c}{a}}}{\frac{c}{a}}$$

$$= \frac{\sqrt{\frac{b^2-4ac}{a^2}}}{\frac{c}{a}} = \sqrt{\frac{b^2-4ac}{\alpha}} \times \frac{\alpha}{c}$$

$$= \boxed{\frac{\sqrt{b^2-4ac}}{c}}$$

(iii) $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$

$$\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta = \frac{\beta+\alpha}{\alpha\beta} - 2\alpha\beta$$

$$= \frac{\frac{-b}{a} - \frac{2c}{a}}{\frac{c}{a}}$$

$$= \left(\frac{-b}{a} \times \frac{\alpha}{c}\right) - \frac{2c}{a}$$

$$= \frac{\frac{-b}{c} - \frac{2c}{a}}{c}$$

$$= \boxed{-\left(\frac{b}{c} + \frac{2c}{a}\right)}$$

(iv) $\alpha^2\beta + \alpha\beta^2$.

$$\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha+\beta)$$

$$= \frac{c}{a} \times \frac{-b}{a} = \boxed{\frac{-bc}{a^2}}$$

Q2)

(v) $\alpha^4 + \beta^4$

$$\alpha^4 + \beta^4 = (\alpha^2)^2 + (\beta)^{2\,2} + 2\alpha^2\beta^2 - 2\alpha^2\beta^2$$

$$= (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$$

$$= [(\alpha+\beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2$$

$$= \left[\left(\frac{-b}{a}\right)^2 - \frac{2c}{a}\right]^2 - 2\left(\frac{c}{a}\right)^2$$

$$= \left(\frac{b^2}{a^2} - \frac{2c}{a}\right)^2 - \frac{2c^2}{a^2}$$

$$= \left(\frac{b^2 - 2ac}{a^2}\right)^2 - \frac{2c^2}{a^2}$$

$$= \frac{(b^2 - 2ac)^2}{a^4} - \frac{2c^2}{a^2}$$

$$= \boxed{\frac{(b^2 - 2ac)^2 - 2a^2c^2}{a^4}}$$

(vi) $\frac{1}{a\alpha+b} + \frac{1}{a\beta+b}$

$$\frac{1}{a\alpha+b} + \frac{1}{a\beta+b} = \frac{a\beta + b + a\alpha + b}{(a\alpha+b)(a\beta+b)}$$

$$= \frac{a(\alpha+\beta)+2b}{a^2\alpha\beta + ab\alpha + ab\beta + b^2}$$

$$= \frac{a(\alpha+\beta)+2b}{a^2\alpha\beta + ab(\alpha+\beta) + b^2}$$

$$= \frac{\cancel{\alpha}\left(\frac{-b}{\cancel{a}}\right) + 2b}{\frac{a^2c}{\cancel{\alpha}} + ab\left(\frac{-b}{\cancel{a}}\right) + b^2}$$

$$= \frac{-b + 2b}{ac - b^2 + b^2} = \boxed{\frac{b}{ac}}$$

Q2)

(vii) $\frac{\beta}{a\alpha+b} + \frac{\alpha}{a\beta+b}$

$$\frac{\beta}{a\alpha+b} + \frac{\alpha}{a\beta+b} = \frac{\beta(a\beta+b)+\alpha(a\alpha+b)}{(a\alpha+b)(a\beta+b)}$$

$$= \frac{a\beta^2+b\beta+a\alpha^2+\alpha b}{a^2\alpha\beta+ab\alpha+ab\beta+b^2}$$

$$= \frac{b(\alpha+\beta)+a(\alpha^2+\beta^2)}{a^2\alpha\beta+ab(\alpha+\beta)+b^2}$$

$$= \frac{b(\alpha+\beta)+a[(\alpha+\beta)^2-2\alpha\beta]}{a^2\alpha\beta+ab(\alpha+\beta)+b^2}$$

$$= \frac{b\left(\frac{-b}{a}\right)+a\left[\left(\frac{-b}{a}\right)^2-\frac{2c}{a}\right]}{a^2\left(\frac{c}{a}\right)+ab\left(\frac{-b}{a}\right)+b^2}$$

$$= \frac{\frac{-b^2}{a}+a\left[\frac{b^2}{a^2}-\frac{2c}{a}\right]}{ac-b^2+b^2}$$

$$= \frac{\frac{-b^2}{a}+a\left[\frac{b^2-2ac}{a^2}\right]}{ac}$$

$$= \frac{\frac{-b^2}{a}+\frac{(b^2-2ac)}{a}}{ac}$$

$$= \frac{-b^2+b^2-2ac}{a^2c}$$

$$= \frac{-2ac}{a^2c} = \boxed{\frac{-2}{a}}$$

(viii) $a\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)+b\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)$

Q21(viii)

$$a\left(\frac{\alpha^2+\beta^2}{\beta}+\frac{\beta}{\alpha}\right)+b\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)$$

$$= a\left(\frac{\alpha^3+\beta^3}{\alpha\beta}\right)+b\left(\frac{\alpha^2+\beta^2}{\alpha\beta}\right)$$

$$= a\left[\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}\right] + b\left[\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}\right]$$

Using,
~~$a^3+b^3=(a+b)($~~
$(a+b)^3 = a^3+b^3+3ab(a+b)$
$\Rightarrow a^3+b^3 = (a+b)^3-3ab(a+b)$

$$= a\left[\frac{\left(\frac{-b}{a}\right)^3-\frac{3c}{a}\left(\frac{-b}{a}\right)}{\frac{c}{a}}\right] + b\left[\frac{\frac{b^2}{a^2}-\frac{2c}{a}}{\frac{c}{a}}\right]$$

$$= \frac{a^2}{c}\left[\frac{-b^3}{a^3}+\frac{3bc}{a^2}\right] + b\left[\frac{b^2-2ac}{ac}\right]$$

$$= \frac{a^2}{c}\left[\frac{-b^3+3abc}{a^3}\right] + \frac{b^3-2abc}{ac}$$

$$= \frac{-b^3+3abc}{ac} + \frac{b^3-2abc}{ac}$$

$$= \frac{-b^3+3abc+b^3-2abc}{ac} = \frac{abc}{ac} = \boxed{b}$$

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