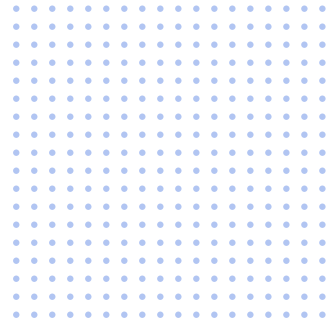




FOR KUET EXAM PREPARATION

KUET 2024 Question Paper · M.Sc. Mathematics

EXAM	YEAR	TYPE	SUBJECT
KUET	2024	Question Paper	M.Sc. Mathematics

Sr. No. **352****ENTRANCE TEST-2024****SCHOOL OF PHYSICAL & MATHEMATICAL SCIENCES****MATHEMATICS**

Question Booklet Series

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Total Questions : 60

Time Allowed : 70 Minutes

Roll No. :

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Instructions for Candidates :

1. Write your Entrance Test Roll Number in the space provided at the top of this page of Question Booklet and fill up the necessary information in the spaces provided on the OMR Answer Sheet.
2. OMR Answer Sheet has an Original Copy and a Candidate's Copy glued beneath it at the top. While making entries in the Original Copy, candidate should ensure that the two copies are aligned properly so that the entries made in the Original Copy against each item are exactly copied in the Candidate's Copy.
3. All entries in the OMR Answer Sheet, including answers to questions, are to be recorded in the Original Copy only.
4. Choose the correct / most appropriate response for each question among the options A, B, C and D and darken the circle of the appropriate response completely. The incomplete darkened circle is not correctly read by the OMR Scanner and no complaint to this effect shall be entertained.
5. Use only blue/black ball point pen to darken the circle of correct/most appropriate response. In no case gel/ink pen or pencil should be used.
6. Do not darken more than one circle of options for any question. A question with more than one darkened response shall be considered wrong.
7. There will be '**Negative Marking**' for wrong answers. Each wrong answer will lead to the deduction of 0.25 marks from the total score of the candidate.
8. Only those candidates who would obtain positive score in Entrance Test Examination shall be eligible for admission.
9. Do not make any stray mark on the OMR sheet.
10. Calculators and mobiles shall not be permitted inside the examination hall.
11. Rough work, if any, should be done on the blank sheets provided with the question booklet.
12. OMR Answer Sheet must be handled carefully and it should not be folded or mutilated in which case it will not be evaluated.
13. Ensure that your OMR Answer Sheet has been signed by the Invigilator and the candidate himself/herself.
14. At the end of the examination, hand over the OMR Answer Sheet to the invigilator who will first tear off the original OMR sheet in presence of the Candidate and hand over the Candidate's Copy to the candidate.

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1. The solution of the linear partial differential equation $2p + 3q = 1$ is given by :
 - (A) $f(2x - y, 3y - z) = 0$
 - (B) $f(3x - 2y, y - 3z) = 0$
 - (C) $f(2x, 3y, z) = 0$
 - (D) None of these
2. The 2nd order partial differential equation $x \frac{\partial^2 z}{\partial x^2} + y \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 3$, is elliptic if:
 - (A) $y^2 > 12x$
 - (B) $y^2 < 12x$
 - (C) $y^2 < 4x$
 - (D) $y^2 > 4x$
3. Choose the correct statement with regard to two bounded sets A and B such that $A \subset B$
 - I. $\sup A \leq \sup B$
 - II. $\inf A \geq \inf B$
 - (A) Only I
 - (B) Only II
 - (C) Both I & II
 - (D) Neither I nor II
4. Any infinite subset of a countable set :
 - (A) is countable
 - (B) is uncountable
 - (C) either countable or uncountable
 - (D) does not exist
5. Choose the correct statement from the following :
 - (A) The supremum of a bounded set always belongs to the set
 - (B) The infimum of a bounded set always belongs to the set
 - (C) Both supremum and infimum of a bounded set always belong to the set
 - (D) None of these
6. For a function $f: A \rightarrow B$, if the range of f is uncountable, then domain of f is :
 - (A) finite
 - (B) countable
 - (C) uncountable
 - (D) either countable or uncountable
7. Which of the following is not a Cauchy sequence ?
 - (A) $\left\{ \frac{(-1)^n}{n} \right\}$
 - (B) $\{(-1)^n\}$
 - (C) $\left\{ \frac{1}{n!} \right\}$
 - (D) $\left\{ 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{(-1)^{n-1}}{n} \right\}$

8. For the given statements, choose the correct statement(s) from the following :
- I. A monotonic increasing sequence which is not bounded above, diverges to infinity
- II. A monotonic decreasing sequence which is not bounded below, diverges to minus infinity
- (A) only I
(B) only II
(C) both I & II
(D) neither I nor II
9. The sequence $\{S_n\}$, with $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$;
- (A) converges to some finite number
(B) diverges finitely
(C) converges to 2
(D) diverges to infinity
10. For what value of a , the value of $\lim_{n \rightarrow \infty} 3 \frac{n^{1/n}}{2} = a^{-1}$?
- (A) 0
(B) 1
(C) $\frac{2}{3}$
(D) $\frac{3}{2}$
11. If the series $\sum (2a_n - 1)$ is convergent, then the $\lim_{n \rightarrow \infty} a_n = ?$
- (A) 0
(B) $1/2$
(C) 1
(D) none of these
12. If $\sum a_n$ is a convergent series of positive terms, then $\sum \frac{a_n}{1+a_n}$ is :
- (A) oscillatory
(B) divergent
(C) convergent
(D) none of these
13. The $\lim_{n \rightarrow \infty} \frac{1^2 + 2^2 + \dots + n^2}{n^3} = ?$
- (A) $\frac{1}{3}$
(B) $\frac{2}{3}$
(C) $\frac{1}{2}$
(D) None of these
14. For the series with n th term $a_n = \frac{1}{n(n+1)}$, the sum $\sum a_n = ?$
- (A) 0
(B) 1
(C) $1/2$
(D) Infinity

15. Which of the following correctly defines Leibnitz rule of a function given by $f(\alpha) = \int_a^b f(x, \alpha) dx$, where a, b are constants?
- (A) $f'(\alpha) = \frac{\partial}{\partial \alpha} \int_a^b f(x, \alpha) dx$
- (B) $f'(\alpha) = \frac{d}{d\alpha} \int_a^b f(\alpha) dx$
- (C) $f'(\alpha) = \int_a^b \frac{\partial}{\partial \alpha} f(x, \alpha) dx$
- (D) $f'(\alpha) = \int_a^b \frac{d}{d\alpha} f(x, \alpha) dx$
16. Given $f(a) = \int_a^b \frac{\sin ax}{x} dx$, then what is the value of $f'(a)$?
- (A) $\frac{\sin 3a}{a}$
- (B) $\frac{3 \sin a^3 - 2 \sin a^2}{a}$
- (C) $\frac{3 \sin a^2 - 2 \sin^2 a}{a}$
- (D) $\frac{3 \sin a^3 - 3 \sin^2 a}{6a}$
17. The power series $\sum 3^{-n} (z-1)^{2n}$ converges if:
- (A) $|z| \leq 3$
- (B) $|z| < \sqrt{3}$
- (C) $|z-1| < \sqrt{3}$
- (D) $|z-1| \leq \sqrt{3}$
18. Which of the following is the radius of convergence of the series $\sum z^{n!}$?
- (A) 0
- (B) ∞
- (C) 1
- (D) a real number > 1
19. If the number of generators in a cyclic group of infinite order is k , then $k = ?$
- (A) 1
- (B) 2
- (C) finite
- (D) infinite
20. If α and β are the smallest positive integers which respectively denote the orders of non-cyclic and non-abelian groups, then
- (A) $\alpha = 4$ and $\beta = 4$
- (B) $\alpha = 4$ and $\beta = 6$
- (C) $\alpha = 6$ and $\beta = 4$
- (D) None of these
21. The number of even permutations in a symmetric group S_6 is:
- (A) 3
- (B) 6
- (C) 360
- (D) 720

22. Which of the following is not a group under multiplication ? (Given $\omega^3 = 1, i^2 = -1$):
- (A) $\left\{1, 5, \frac{1}{5}\right\}$
 (B) $\{1, -1, i, -i\}$
 (C) $\{1, \omega, \omega^2\}$
 (D) Non-zero rational numbers
23. Which of the following is true for a group of prime order p ?
- I. It has no non-trivial subgroups
 II. It is always cyclic as well as abelian
- (A) Only I
 (B) Only II
 (C) Both I & II
 (D) Neither I nor II
24. In the group of integer modulo 6, $\langle \mathbb{Z}_6, + \rangle$, the inverse of an element $\bar{2} \in \mathbb{Z}_6$ is :
- (A) $\bar{1}$
 (B) $\bar{2}$
 (C) $\bar{3}$
 (D) $\bar{4}$
25. The alternating group A_n is not simple for :
- (A) $n = 3$
 (B) $n = 4$
 (C) $n = 5$
 (D) $n \leq 5$
26. Which of the following is not a normal subgroup ?
- (A) Kernel of a homomorphism
 (B) Alternating group in S_n
 (C) Centralizer of a group
 (D) Normalizer of an element in a group
27. Choose the correct statement(s) :
- I. Every ideal is a subring
 II. The ring of complex numbers is a field
- (A) only I
 (B) only II
 (C) neither I nor II
 (D) both I & II
28. The polynomial $p(x) = a_0 + a_1x + \dots + a_nx^n$; over a UFD is said to be primitive if the gcd of $a_0, a_1, a_2, \dots, a_n$ is (Given $a_0, a_2, \dots, a_n$ are integers)
- (A) 1
 (B) 2
 (C) 3
 (D) 4
29. Given two ideals I and J of a ring R such that $I \subset J$, then union of I and J :
- (A) is an ideal
 (B) need not be an ideal
 (C) is a maximal ideal of R
 (D) is a prime ideal of R

30. For what value of k , $\frac{z}{\langle k \rangle}$ is a field?
- (A) $k = 1$
 (B) $k = 2$
 (C) $k = 4$
 (D) Any value of k
31. Given a differentiable function $f(x)$ in a closed interval $[2, 7]$ with $f(2) = 3$ and $f'(x) \leq 5$ for all values of x in $(2, 7)$, the maximum possible value of $f(x)$ at $x = 7$ is:
- (A) 7
 (B) 14
 (C) 15
 (D) 28
32. Let $f(x)$ be continuous and differentiable function for all real's such that $f(x + y) = f(x) - 3xy + f(y)$ and $\lim_{h \rightarrow 0} \frac{f(h)}{h} = 7$, then the value of $f'(x)$ is:
- (A) $-3x + 7$
 (B) $3x - 7$
 (C) $2f(x) + 7$
 (D) 7
33. If $u = \log xy$ where $x^2 + y^2 = 1$, then $\frac{du}{dx} = ?$
- (A) $\frac{1}{x}$
 (B) $\frac{1}{x} + \frac{1}{y}$
 (C) $\frac{1}{x} - \frac{x}{y}$
 (D) $\frac{1}{x} - \frac{x}{y^2}$
34. The n th derivative of the function $f(x) = x^n e^x$ at $x = 0$ is given by:
- (A) $n! e^n$
 (B) $n!$
 (C) $(n + 1)!$
 (D) None of these
35. In a Cycloid, $x = a(\theta + \sin \theta)$ and $y = a(1 - \cos \theta)$, the angle ψ the tangent makes to the horizontal is:
- (A) 2θ
 (B) θ
 (C) $\frac{\theta}{2}$
 (D) $\frac{\theta}{3}$
36. For $r = ae^{\theta \cot \alpha}$, the tangent is inclined at—angle to the radius vector.
- (A) 0
 (B) π
 (C) $\frac{\pi}{2}$
 (D) a constant
37. The curvature of the function $f(x) = x^3 - x + 1$ at $x = 1$ is given by:
- (A) $\frac{6}{5}$
 (B) $\frac{3}{5}$
 (C) $\frac{6}{5^2}$
 (D) $\frac{3}{5^2}$

38. A double point on a curve is said to be node, if two tangents of the curves :
- Are real and coincident
 - Have no real tangents
 - Are real and distinct
 - None of these
39. The value of k in Rolle's theorem for $f(x) = x^3 - 3x$ in the interval $[0, \sqrt{3}]$ holds :
- 1
 - $\frac{1}{3}$
 - 1
 - $\frac{2}{3}$
40. If the Rolle's theorem holds for the function $f(x) = x^4 + ax^3 + bx$, in $-1 \leq x \leq 1$ and $f'\left(\frac{1}{2}\right) = 0$, then the value of $ab = ?$
- 64
 - 8
 - 4
 - 1
41. Using Cauchy's Mean value theorem for the functions $f(x) = e^x$, $g(x) = e^{-x}$ in the interval $[2, 3]$, the value of c is:
- 1.5
 - 2.5
 - 3
 - 3.5
42. Given $f(x) = x^3 - 3x - 1$ is continuous in the closed interval $\left[\frac{13}{7}, -\frac{11}{7}\right]$ and $f'(x)$ exists in the interval, $\left(\frac{13}{7}, -\frac{11}{7}\right)$ the value of k such that it lies in $\left(\frac{13}{7}, -\frac{11}{7}\right)$ is :
- 0
 - 1
 - 1
 - ± 1
43. The value of the integral $\int \frac{dx}{x(x+1)}$ is :
- $\ln\left|\frac{x+1}{x}\right| + c$
 - $\ln\left|\frac{x}{x+1}\right| + c$
 - $\ln\left|\frac{x-1}{x}\right| + c$
 - $\ln\left|\frac{x}{x+1}\right| + c$
44. The reduction formula for $\int \tan^n x \, dx$ is :
- $\frac{\tan^{n-1} x}{n-1} - \int \tan^{n-1} x \, dx$
 - $\frac{\tan^{n-1} x}{n} - \int \tan^{n-2} x \, dx$
 - $\frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \, dx$
 - $\frac{\tan^{n-1} x}{n-1} + \int \tan^{n-2} x \, dx$

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45. For what value of k , $\int \frac{dx}{x(x+2)} = k \ln \left| \frac{x}{x+2} \right| + c$? 49. The value of $\frac{1}{D^2 + a^2} \sin ax = ?$

(A) 1

(B) -1

(C) 2

(D) $\frac{1}{2}$

46. What is the value of $\int_0^{8\pi} |\sin x| dx$?

(A) 2

(B) 4

(C) 8

(D) 16

47. The particular integral of $(D^2 - 3D + 2)y = e^{5x}$ is :

(A) e^{5x}

(B) $\frac{1}{30} e^{5x}$

(C) $\frac{1}{12} e^{5x}$

(D) None of these

48. The complementary function of $y_2 - 3y_1 + 2y = \cos 3x$ is :

(A) $c_1 e^x + c_2 e^{2x}$

(B) $c_1 e^{-x} + c_2 e^{-2x}$

(C) $c_1 e^{2x} + c_2 e^{-3x}$

(D) None of these

(A) $-\frac{x}{2} \cos ax$

(B) $-\frac{x}{2a} \cos ax$

(C) $-\frac{x}{2a} \sin ax$

(D) $\frac{x}{2a} \cos ax$

50. The solution $y(x)$ of $(D^2 + 1)y = 0$ is :

(A) $A \cos x - B \sin x$

(B) $Ae^x + Be^{-x}$

(C) $A \cos x + B \sin x$

(D) $-A \cos x + B \sin x$

51. The solution of $p^2 - 9p + 18 = 0$ is given by :

(A) $(y - x - a)(y + x - a)$

(B) $(y - 3x - a)(y - 6x - a)$

(C) $(y + 3x - a)(y - x + a)$

(D) None of these

52. Which of the following is the solution of

$\frac{dy}{dx} = y (\operatorname{cosec} x - \cot x)$?

(A) $y = c \sec^2 \frac{x}{2}$

(B) $y = c \log \sin \frac{x}{2}$

(C) $y = c \log \frac{x}{2}$

(D) None of these

53. The singular solution of $y = px + p - p^2$ is :

(A) $y = \frac{x+1}{2}$

(B) $y = \frac{x+1}{4}$

(C) $y = \left(\frac{x+1}{2}\right)^2$

(D) Does not exist

54. By means of a substitution $u = y^3$, the equation $y = 3xp + 6y^2p^2$ reduces to :

(A) $u = x \frac{du}{dx} + \frac{2}{3} \left(\frac{du}{dx}\right)^2$

(B) $u = x \frac{du}{dx} + \frac{1}{3} \left(\frac{du}{dx}\right)^2$

(C) $u = x \frac{du}{dx} - \frac{2}{3} \left(\frac{du}{dx}\right)^2$

(D) None of these

55. For any positive integer n and a Bessel's function $J_n(x)$, choose the correct expression from the following:

(A) $J_{-n}(x) = (-1)^{n+1} J_n(x)$

(B) $J_{-n}(x) = (-1)^n J_n(x)$

(C) $J_{-n}(x) = -J_n(x)$

(D) $J_{-n}(x) = (-1)^n J_{n+1}(x)$

56. Which of the following determines a Bessel's differential equation ?

(A) $x^2 y_2 + xy + (x^2 - n^2) y = 0$

(B) $x y_2 + y + xy = 0$

(C) $x^2 y_2 + xy + (x^2 - 1) y = 0$

(D) All of them

57. If $P_n(x)$ is the Legendre polynomial, then $P_0(x) +$

$P_1(x) = ?$

(A) 1

(B) x

(C) $1 + x$

(D) None of these

58. The Wronskian of $y_1 = x^2$ and $y_2 = x^2 \log x$ is :

(A) 0

(B) 1

(C) x

(D) x^3

59. By eliminating arbitrary constants a , b and c from $z = a(x + y) + b(x - y) + c$, the resulting partial differential equation is :

$a(x + y) + b(x - y) + c$, the resulting partial

differential equation is :

(A) $z = \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} + \frac{\partial z}{\partial t} + \left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial z}{\partial y}\right)\left(\frac{\partial z}{\partial t}\right)$

(B) $z = \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} + \frac{\partial z}{\partial t} - \left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial z}{\partial y}\right)\left(\frac{\partial z}{\partial t}\right)$

(C) $\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 = 4\left(\frac{\partial z}{\partial t}\right)$

(D) None of these

PDE is given by :

(A) $px + qy = 0$

(B) $z = px + qy$

(C) $px - qy = 0$

(D) None of these