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Ch-7
Triangles
Exercise 7.7

Q.2

(i) $a = 7$ cm, $b = 24$ cm, $c = 25$ cm

(Longest side)^2 $= c^2 = 25^2 = 625$ cm ——— ①

Sum of square of other sides $= a^2 + b^2 = 7^2 + 24^2$

$= 49 + 576 = 625$ cm ——— ②

From ① & ②

$c^2 = a^2 + b^2$

∴ It is a right Δ. (By inverse of Pythagoras theorem)

(iv) $a = 8$ cm, $b = 10$ cm, $c = 6$ cm

(Longest side)^2 $= b^2 = 10^2 = 100$ cm ——— ①

Sum of square of other sides $= a^2 + c^2 = 8^2 + 6^2$

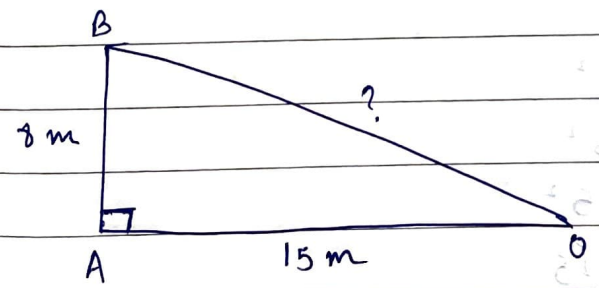
$= 64 + 36 = 100$ cm ——— ②

From ① & ②

$b^2 = a^2 + c^2$

∴ It is a right Δ. (By inverse of Pythagoras theorem)

Q.3



AO = 15 m, AB = 8 m

As AB is in North direction, ∴ $\angle BAO = 90°$

∴ ΔAOB is a right Δ.

In right ΔAOB,

$OB^2 = OA^2 + AB^2$ (Pythagoras theorem)

$OB^2 = 15^2 + 8^2$

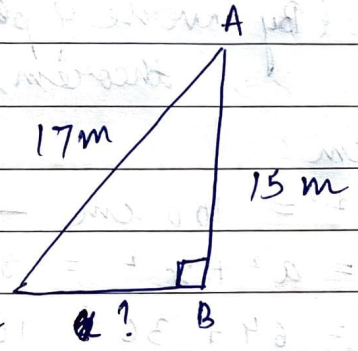
$OB^2 = 225 + 64$

$OB = \sqrt{289}$

$\boxed{OB = 17\ m}$

Hence, the man is 17 m away from the starting point.

Q.4



~~Let the distance of foot of the ladder from the building be x.~~

Length of ladder = 17 m

Height of window = 15 m

In right ΔABC,

$AC^2 = AB^2 + BC^2$

$BC^2 = AC^2 - AB^2$

$BC^2 = 17^2 - 15^2$

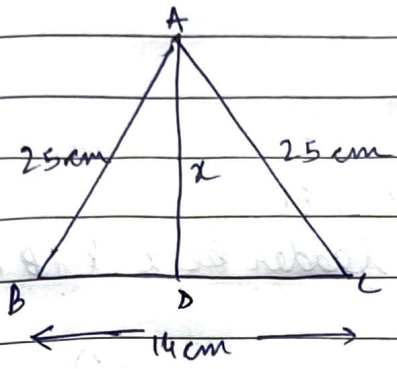
$BC^2 = 289 - 225$

$BC = \sqrt{64}$

$\boxed{BC = 8\ m}$

$\therefore$ Distance of foot of the ladder from building = $\boxed{8\ m}$

Q.6



Given – In $\triangle ABC$,

- $AB = AC = 25\,cm$
- $BC = 14\,cm$
- $AD \perp BC$

To find – AD

Solution – Let AD be x.

As AD bisects BC,

$\therefore BD = DC = 7\,cm$

In $\triangle ADC$,

$AC^2 = AD^2 + DC^2$

$x^2 = AC^2 - DC^2$

$x^2 = 25^2 - 7^2$

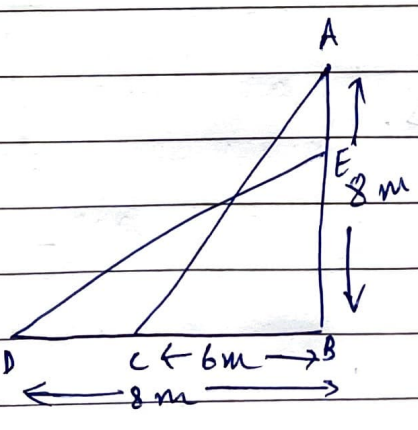
$x^2 = 625 - 49$

$x = \sqrt{576}$

$\boxed{x = 24\,cm}$

$\therefore AD = \boxed{24\,cm}$

Q.7



Given – • AB = 8 m
• BC = 6 m
• BD = 8m

To find – Height of ladder & EB

Solution – Let the height of ladder be x & EB be h

In ΔABC

$AC^2 = AB^2 + BC^2$

$x^2 = 8^2 + 6^2$

$x^2 = 64 + 36$

$x = \sqrt{100}$

$x = 10\ m$ ——— (1)

Pythagoras theorem

In ΔEBD,

$ED^2 = EB^2 + BD^2$

$EB^2 = x^2 - BD^2$

$h^2 = 10^2 - 8^2$ (From (1))

$h^2 = 100 - 64$

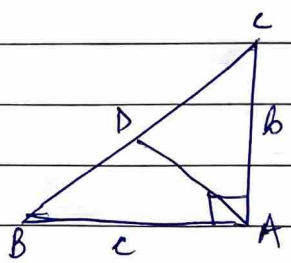
$h = \sqrt{36}$

$\boxed{h = 6\ m}$

Height of ladder = $\boxed{10\ m}$

$EB = \boxed{6m}$

Q.9



Given – • ΔABC is a right Δ.
• AC = b, AB = c
• AD ⊥ BC

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To Find – Length of AD in terms of b and c

Solution – In ΔABC,

$BC^2 = AC^2 + AB^2$ (Pythagoras theorem)

$BC^2 = b^2 + c^2$

$BC = \sqrt{b^2+c^2}$

Area of $\Delta = \frac{1}{2} \times AB \times AC = \frac{1}{2} bc$ ——— (1)

Also, Area of $\Delta = \frac{1}{2} \times BC \times AD$ ——— (2)

From (1) & (2),

$\frac{1}{2} bc = \frac{1}{2} \times BC \times AD$

$bc = \sqrt{b^2+c^2} \times AD$ (Putting value of BC)

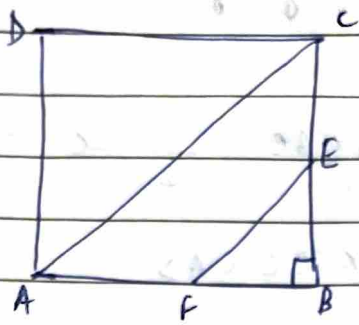
$$\boxed{AD = \frac{bc}{\sqrt{b^2+c^2}}}$$

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Ch - 7

Exercise 7.7

Q. 11



Given - • $FB = 1/2\ AB$
• $BE = 1/3\ BC = 1/3\ AB$
• $\Delta FBE = 108\ cm^2$
• ABCD is a square.

To find - AC

Solution - Area of $\Delta FBE = \frac{1}{2} \times FB \times BE$

$$108\ cm^2 = \frac{1}{2} \times \frac{1}{2} AB \times \frac{1}{3} AB$$

$$108 = \frac{AB^2}{12}$$

$$AB = \sqrt{108 \times 12}$$

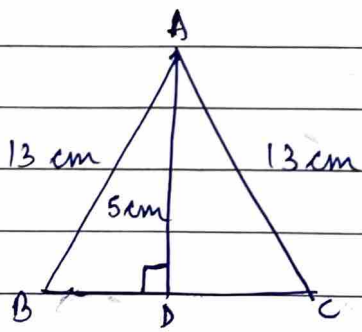
$$AB = \sqrt{1296}$$

$$AB = 36\ cm$$

$$AC = \sqrt{2} \times side$$

$$= \boxed{36\sqrt{2}\ cm}$$

Q. 12



Given - In $\triangle ABC$,

- $AB = AC$
- $AD \perp BC$
- $AD = 5$ cm
- $BD = DC = 1/2\ BC$

To find - BC

Solution - In right $\triangle ABD$,

$AB^2 = AD^2 + BD^2$

$13^2 = 5^2 + BD^2$ (Pythagoras theorem)

$BD^2 = 169 - 25$

$BD = \sqrt{144}$

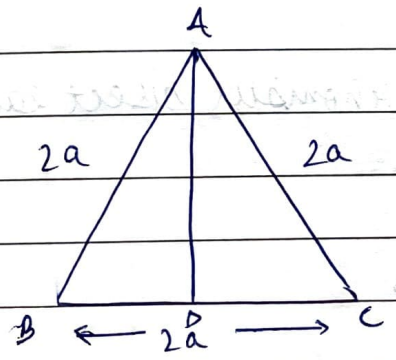
$BD = 12$ cm

$BD = DC = 1/2\ BC$ (Given)

$BC = 2BD$

$= 2 \times 12 = \boxed{24 \text{cm}}$

Q.13



Given - $\bullet\ AB = BC = CA = 2a$

$\bullet\ AD \perp BC$

To prove - (i) $AD = a\sqrt{3}$

(ii) ar $(ABC) = \sqrt{3}a^2$

Proof - (i) In $\triangle ABD$,

$AB^2 = AD^2 + BD^2$

$(2a)^2 = AD^2 + (a)^2$

$AD^2 = 4a^2 - a^2$

$AD = \sqrt{3a^2}$

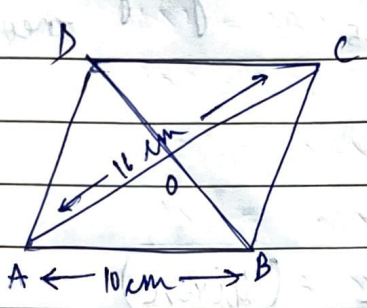
$\boxed{AD = \sqrt{3}a}$

(ii) Area of $\Delta ABC = \frac{1}{2} \times BC \times AD$

$= \frac{1}{2} \times 2a \times \sqrt{3}a$

$= \boxed{\sqrt{3}a^2}$

Q.15



Given - • $AB = BC = CD = DA = 10$ cm

• $ABCD$ is a rhombus

• $AC = 16$ cm

To find - BD

Solution - As diagonals of a rhombus bisect each other at right $\angle$s,

$\therefore AO = OC = 8$ cm

$BO = OD = \frac{1}{2} BD$

In right ΔAOB

$AB^2 = AO^2 + OB^2$

$10^2 = 8^2 + OB^2$

$OB^2 = 100 - 64$

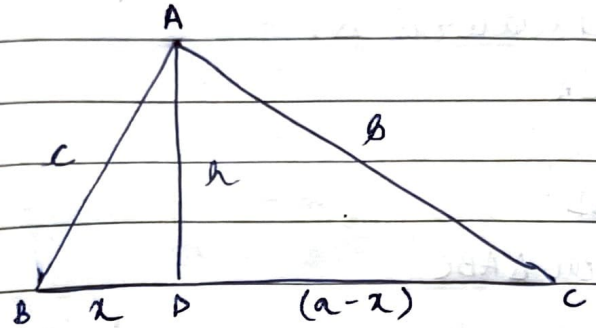
$OB = \sqrt{36}$

$OB = 6$ cm

$BD = 2OB$

$= 6 \times 2 = \boxed{12 \text{ cm}}$

Q.17



Given - • $\angle B < 90°$
• $AD \perp BC$
• $AB = c$, $AC = b$, $BC = a$

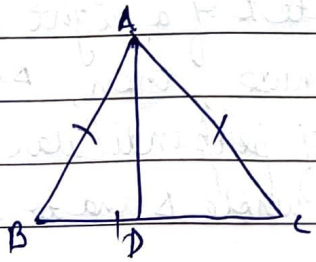
To prove - (i) $b^2 = h^2 + a^2 + x^2 - 2ax$
(ii) $B^2 = a^2 + c^2 - 2ax$

Proof - Let BD be x
$DC = (a - x)$

(i) In right ΔADC, } Pythagoras theorem
$AC^2 = AD^2 + DC^2$
$b^2 = h^2 + (a-x)^2$
$\boxed{b^2 = h^2 + a^2 + x^2 - 2ax}$ ——— ①
Hence, proved

(ii) In right ΔABD
$AB^2 = AD^2 + BD^2$
$c^2 = h^2 + x^2$ ——— ②
In eq. ①
$b^2 = h^2 + a^2 + x^2 - 2ax$ } Pythagoras theorem
$b^2 = (h^2 + x^2) + a^2 - 2ax$
$\boxed{b^2 = c^2 + a^2 - 2ax}$ (From ②)
Hence, proved

Q.18



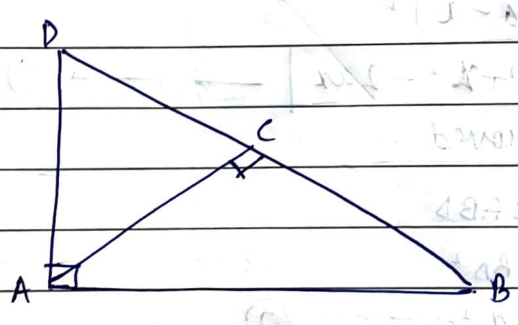
Given – • ABC is an equilateral Δ.
• AB = BC = CA
• AD ⊥ BC

To prove – $AD^2 = 3BD^2$

Proof – In equilateral Δ ABC,
AD ⊥ BC
∴ BD = DC = 1/2 BC
∴ In Δ ABD
$AB^2 = AD^2 + BD^2$
$AD^2 = AB^2 - BD^2$
$AD^2 = BC^2 - BD^2$
$AD^2 = (2BD)^2 - BD^2$
$AD^2 = 4BD^2 - BD^2$
$\boxed{AD^2 = 3BD^2}$
Hence, proved

} Pythagoras theorem

Q. 19 (i)



Given – • ABC is a right Δ.
• AC ⊥ BD

To prove – $AB^2 = BC \times BD$

Proof – In figure,
Δ BAD ~ Δ ACB

} If a perpendicular is drawn from the vertex of a right Δ to the hypotenuse, then Δs on both sides of perpendicular are similar to the whole Δ and to each other.

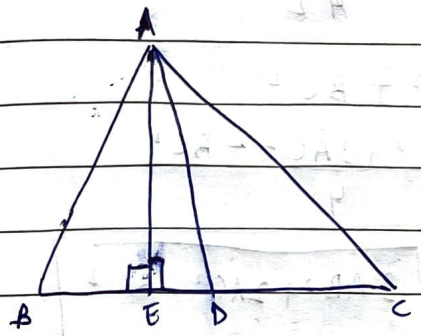
$\therefore \frac{BA}{BC} = \frac{BD}{BA}$ (CPST)

$\Rightarrow BA^2 = BD \times BC$

$\Rightarrow \boxed{AB^2 = BC \times BD}$

Hence proved

Q.22



Given - • ΔABC is an acute Δ.
• AD is the median

To find - Express median in terms of the sides of Δ.

Cons - Draw $AE \perp BC$.

Solution - In right ΔAEB,

$AB^2 = AE^2 + BE^2$

$AB^2 = AD^2 - DE^2 + (BD - DE)^2$ $\left[\begin{array}{l} AB^2 = AE^2 + ED^2 \\ AE^2 = AD^2 - DE^2 \end{array}\right]$

$AB^2 = AD^2 - DE^2 + BD^2 + DE^2 - 2BD \times DE$

$AB^2 = AD^2 + BD^2 - 2BD \times DE$

$AB^2 = AD^2 + \left(\frac{BC}{2}\right)^2 - BC \times DE$ $\left(BC = 2BD\right)$

$AB^2 = AD^2 + \frac{BC^2}{4} - BC \times DE$ ——— (1)

In right ΔAEC,

$AC^2 = AE^2 + EC^2$

$AC^2 = AD^2 - DE^2 + (DE + CD)^2$

$AC^2 = AD^2 + CD^2 + 2CD \times DE$

$AC^2 = AD^2 + \left(\frac{BC}{2}\right)^2 + BC \times DE$

$$AC^2 = AD^2 + \frac{BC^2}{4} + BC \times DE \quad \text{---} (1)$$

Add (1) & (2)

$$AB^2 + AC^2 = AD^2 + \frac{BC^2}{4} - BC \times DE + AD^2 + \frac{BC^2}{4} + BC \times DE$$

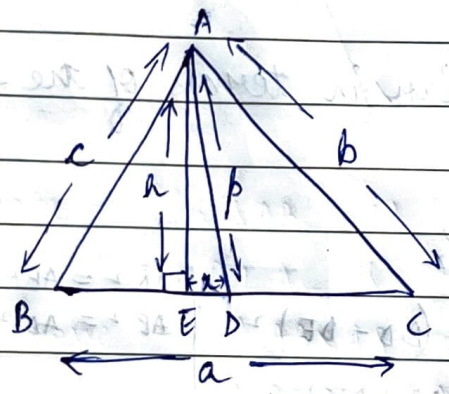
$$AB^2 + AC^2 = 2AD^2 + \frac{BC^2}{2}$$

$$2AB^2 + 2AC^2 = 4AD^2 + BC^2$$

$$AD^2 = \frac{2AB^2 + 2AC^2 - BC^2}{4}$$

$$\boxed{AD = \frac{1}{2}\sqrt{2AB^2 + 2AC^2 - BC^2}}$$

Q.24



Given - In ΔABC,

- AD is the median.
- $AE \perp BC$
- $AB = c$, $BC = a$, $CA = b$, $ED = x$, $AD = p$, $AE = h$

To prove - (i) $b^2 = p^2 + ax + \frac{a^2}{4}$

(ii) $c^2 = p^2 - ax + \frac{a^2}{4}$

(iii) $b^2 + c^2 = 2p^2 + \frac{a^2}{2}$

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Proof – (i) In right ΔAEC,

$$AC^2 = AE^2 + EC^2$$
$$AC^2 = AE^2 + (ED + DC)^2$$
$$b^2 = h^2 + \left(x + \frac{1}{2}a\right)^2$$

} Pythagoras theorem

As AD is median, BD = DC = 1/2 BC

$$\boxed{b^2 = h^2 + x^2 + ax + \frac{a^2}{4}} \quad \text{——— (1)}$$

(ii) In right ΔAED

$$AD^2 = AE^2 + ED^2$$
$$p^2 = h^2 + x^2 \quad \text{——— (2)}$$

} Pythagoras theorem

Putting (2) in (1)

$$b^2 = (h^2 + x^2) + ax + \frac{a^2}{4}$$

$$\boxed{b^2 = p^2 + ax + \frac{a^2}{4}} \quad \text{——— (3)}$$

(iii) In right ΔAEB,

$$AB^2 = AE^2 + BE^2$$
$$AB^2 = AE^2 + (BD - ED)^2$$
$$c^2 = h^2 + \left(\frac{a}{2} - x\right)^2$$

} Pythagoras theorem

As AD is median, BD = DC = 1/2 BC

$$c^2 = h^2 + x^2 - ax + \frac{a^2}{4}$$

Putting value of (2)

$$c^2 = (h^2 + x^2) - ax + \frac{a^2}{4}$$

$$\boxed{c^2 = p^2 - ax + \frac{a^2}{4}} \quad \text{——— (4)}$$

(iii) Add (3) & (4),

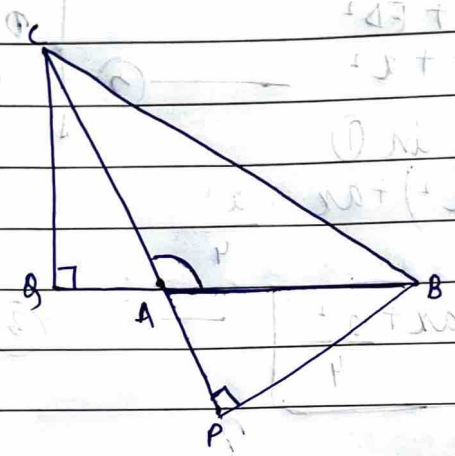
$$b^2+c^2 = p^2 + ax + \frac{a^2}{4} + p^2 - ax + \frac{a^2}{4}$$

$$b^2+c^2 = 2p^2 + \frac{2a^2}{2}$$

$$\therefore \boxed{b^2+c^2 = 2p^2 + \frac{a^2}{2}}$$

Hence, proved

Q.25



Given - • In ΔABC, $\angle A$ is obtuse
• $PB \perp AC$ & $QC \perp AB$

To prove - (i) $AB \times AQ = AC \times AP$
(ii) $BC^2 = (AC \times CP + AB \times BQ)$

Proof - (i) In ΔABP & ΔACQ,

$\angle PAB = \angle QAC$ (V.O.A)

$\angle APB = \angle AQC$ (90° each)

$\therefore \Delta ABP \sim \Delta ACQ$ (AA similarity)

$\therefore \frac{AB}{AC} = \frac{AP}{AQ}$ (CPST)

$\Rightarrow \boxed{AB \times AQ = AC \times AP}$ ——— ①

Hence, proved

(ii) In right ΔCBP,

$BC^2 = CP^2 + BP^2$ (Pythagoras theorem)

$BC^2 = (AC+AP)^2 + BP^2$

$BC^2 = AC^2 + AP^2 + 2AC \times AP + BP^2$

$BC^2 = (BP^2 + AP^2) + AC^2 + 2AC \times AP$ } In right $\triangle BPA$, $BP^2 + AP^2 = AB^2$

$BC^2 = AB^2 + AC^2 + 2AC \times AP$

$BC^2 = AB^2 + AC^2 + AB \times AQ + AC \times AP$ (from ①)

$BC^2 = AB^2 + AB \times AQ + AC^2 + AC \times AP$

$BC^2 = AB(AB + AQ) + AC(AC + AP)$

$BC^2 = AB \times BQ + AC \times CP$

$\therefore \boxed{BC^2 = (AC \times CP + AB \times BQ)}$

Hence, proved