AISSCE-2020 Class-XII Mathematics

SET-3 (SECTION-A)

1. $\vec{OA} \times \vec{OB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -3 & -2 & 1 \end{vmatrix} = 8\hat{i} - 10\hat{j} + 4\hat{k}$, $ar(\Delta OAB) = \frac{1}{2}|\vec{OA} \times \vec{OB}|$

$= \frac{1}{2}\sqrt{64+100+16}$

$= \frac{1}{2}\sqrt{180} = \frac{1}{2} \cdot 6\sqrt{5} = 3\sqrt{5}$

Ans: (A) $3\sqrt{5}$ sq. units

2. $-1 \le 2x \le 1 \Rightarrow -\frac{1}{2} \le x \le \frac{1}{2}$

Ans: (C) $[-\frac{1}{2}, \frac{1}{2}]$

3. $f'(x) = -x^2 e^{-x} + 2x e^{-x} = -x e^{-x}(x-2)$

$x = 0, 2$ $\quad$ $-$ $\;$ $+$ $\;$ $-$ (0, 2)

Ans (D) $(0, 2)$

4 $LHL = RHL = 0 \quad \therefore k = 0$

Ans (A) 0.

5. $y = \log x$, $z = \frac{1}{x}$ $\therefore \frac{dy}{dx} = \frac{1}{x}$, $\frac{dz}{dx} = -\frac{1}{x^2}$ $\therefore \frac{dy}{dz} = \frac{1/x}{-1/x^2} = -x$

Ans (C) $-x$

6. Ans. (C) $(0, -3, 0)$

7. Ans (A) Symmetric and transitive but not reflexive

8. Ans (D) $6\sqrt{3}$ $\quad$ $(\vec{a}\cdot\vec{b})^2 = |\vec{a}|^2|\vec{b}|^2 - |\vec{a}\times\vec{b}|^2 = 9 \times 16 - 36$

$= 144 - 36 = 108$

$\therefore (\vec{a}\cdot\vec{b}) = \pm\sqrt{108} = 6\sqrt{3}$

9 Ans (D) 27 $\quad$ $\because |A^2| = |3A|$

$\Rightarrow |A|^2 = 3^3|A| \Rightarrow |A| = 27$

10. Ans: (A) $[0, 12]$

11 Ans: π cm/s $\quad$ $C = 2\pi r \therefore \frac{dC}{dt} = 2\pi\frac{dr}{dt} = 2\pi \times 0.5 = \pi$ cm/sec

12 Ans: $k = 3$ $\quad$ $[6 - 2k] = [0] \Rightarrow 6 - 2k = 0 \Rightarrow k = 3$

13. Ans: $(5, 4)$ $\therefore P = 3\times5 + 2\times4 = 15 + 8 = 23$

14 Ans: $[0, \pi] - \{\frac{\pi}{2}\}$

OR

Ans: $\cos^{-1}(-\frac{1}{2}) = \pi - \cos^{-1}(\frac{1}{2}) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ $\therefore$ Ans: $\frac{2\pi}{3}$

15: $4x+2y-4z-12=0$; $4x+2y-4z=0$ $d = \left|\frac{0-(-12)}{\sqrt{16+4+16}}\right| = \frac{12}{6} = 2$

Ans: 2 units

OR

$1(x-1)+0-3(z+3)=0 \Rightarrow x-1-3z-9=0 \Rightarrow x-3z-10=0$

Ans: $x-3z-10=0$

16

$f(x)=x\cos^2 x$ is an odd function $\therefore \int_{-\pi/2}^{\pi/2} x\cos^2 x\,dx=0$

Ans: 0

17. Let $P(3\lambda+1, 7\lambda-4, 2\lambda-4)$ be the point of intersection.

It cuts xy-plane $\therefore$ $2\lambda-4=0 \Rightarrow \lambda=2$

$\therefore P(3\times2+1, 7\times2-4, 0) = P(7,10,0)$ Ans

18. $K(1)^2+5=2 \Rightarrow \boxed{K=-3}$ Ans

19. $y=mx \Rightarrow \frac{dy}{dx}=m$

$\therefore m=\frac{y}{x} \Rightarrow \frac{dy}{dx}=\frac{y}{x} \Rightarrow \boxed{x\frac{dy}{dx}-y=0}$ Ans

20

$\frac{d}{dx^2}[\sec^2(x^2)] = 2\sec(x^2).\sec(x^2)\tan(x^2)$

$= 2\sec^2(x^2).\tan x^2$

OR

$y=f(x^2) \Rightarrow \frac{dy}{dx}=f'(x^2).2x = e^x.2x$ Ans

$\boxed{\frac{dy}{dx}=2xe^x}$

SECTION-B

21. $\hat{r}=\frac{1}{\sqrt3}\hat{i}+\frac{1}{\sqrt3}\hat{j}+\frac{1}{\sqrt3}\hat{k}$ $\therefore \vec{r}=3\sqrt3.\hat{r}=3\sqrt3.\frac{1}{\sqrt3}(\hat{i}+\hat{j}+\hat{k})$

$\boxed{\vec{r}=3\hat{i}+3\hat{j}+3\hat{k}}$

OR

$|\sqrt3\vec{a}-\vec{b}|^2=1 \Rightarrow 3|\vec{a}|^2-2\sqrt3\vec{a}\cdot\vec{b}+|\vec{b}|^2=1$

$\Rightarrow 3\times1-2\sqrt3|\vec{a}||\vec{b}|\cos\theta+1=1$

$\Rightarrow -2\sqrt3\cos\theta=1-4=-3$

$\cos\theta=\frac{3}{2\sqrt3}=\frac{\sqrt3}{2} \Rightarrow \boxed{\theta=\frac{\pi}{6}}$

22. $A^2 = \begin{bmatrix}-3 & 2\\ 1 & -1\end{bmatrix}\begin{bmatrix}-3 & 2\\ 1 & -1\end{bmatrix} = \begin{bmatrix}9+2 & -6-2\\ -3-1 & 2+1\end{bmatrix} = \begin{bmatrix}11 & -8\\ -4 & 3\end{bmatrix}$

$A^2 + I = kA \Rightarrow \begin{bmatrix}11 & -8\\ -4 & 3\end{bmatrix} + \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = \begin{bmatrix}-3k & 2k\\ k & -k\end{bmatrix}$

$\Rightarrow \begin{bmatrix}12 & -8\\ -4 & 4\end{bmatrix} = \begin{bmatrix}-3k & 2k\\ k & -k\end{bmatrix} \Rightarrow \boxed{k=-4}$ Ans

23. $f(x) = \sqrt{\dfrac{\sec x - 1}{\sec x + 1}} = \sqrt{\dfrac{\frac{1}{\cos x} - 1}{\frac{1}{\cos x} + 1}} = \sqrt{\dfrac{1-\cos x}{1+\cos x}} = \sqrt{\dfrac{\cancel{2}\sin^2 x/2}{\cancel{2}\cos^2 x/2}} = \tan\dfrac{x}{2}$

$f'(x) = \frac{1}{2}\sec^2\frac{x}{2} \quad \therefore f'\left(\frac{\pi}{3}\right) = \frac{1}{2}\sec^2\left(\frac{\pi}{6}\right) = \frac{1}{2}\times\left(\frac{2}{\sqrt{3}}\right)^2 = \frac{2}{3}$ Ans

OR

$f(x) = (\tan x)^{\tan x} \Rightarrow \log[f(x)] = \tan x \cdot \log(\tan x)$

$\Rightarrow \frac{1}{f(x)}\cdot f'(x) = \cancel{\tan x}\cdot\frac{1}{\cancel{\tan x}}\sec^2 x + \log(\tan x)\cdot\sec^2 x$

$\Rightarrow f'(x) = f(x)\cdot\sec^2 x\,[1+\log(\tan x)]$

$= (\tan x)^{\tan x}\cdot\sec^2 x\,[1+\log(\tan x)]$

24. $I = \int \frac{\tan^3 x}{\cos^3 x}\,dx = \int \sec^3 x\cdot\tan^3 x\,dx$

$= \int \sec^2 x\cdot\sec x\cdot\tan x\cdot\tan^2 x\,dx$

$= \int \sec^2 x\cdot(\sec^2 x - 1)\cdot\sec x\tan x\,dx$

$\sec x = t \Rightarrow \sec x\tan x\,dx = dt$

$= \int t^2(t^2-1)\,dt = \int (t^4 - t^2)\,dt$

$= \frac{t^5}{5} - \frac{t^3}{3} + c = \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + c$

25. $\sin\theta = \dfrac{\hat{k}\cdot(\hat{j}-\hat{k})}{|\hat{k}|\,|\hat{j}-\hat{k}|} = \left|\dfrac{0-1}{\sqrt{1}\sqrt{1+1}}\right| = \left|\dfrac{-1}{\sqrt{2}}\right| \Rightarrow \theta = \frac{\pi}{4}$ or $\frac{3\pi}{4}$

$\theta = \frac{\pi}{4}$ or $\frac{3\pi}{4}$

26. p = probability of getting the sum as multiple of 6

$= \frac{6}{36} = \frac{1}{6}$ $\therefore q = 1-p = \frac{5}{6}$

$P(B) = qp + qqqp + qqqqqp + \cdots \infty$

$= \frac{qp}{1-q^2} = \frac{\frac{5}{6}\times\frac{1}{6}}{1-\frac{25}{36}} = \frac{\frac{5}{36}}{\frac{36-25}{36}} = \frac{5}{11}$ Ans

SECTION-C

27. $(1+e^{y/x})\,dy + e^{y/x}\left(1-\frac{y}{x}\right)dx = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{e^{y/x}\left(1-\frac{y}{x}\right)}{1+e^{y/x}}$ Putting $\frac{y}{x} = v$ $\therefore y = vx$

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{-e^v(1-v)}{1+e^v}$

$\Rightarrow x\frac{dv}{dx} = \frac{-e^v(1-v)}{1+e^v} - v = \frac{-e^v + \cancel{e^v v} - v - \cancel{ve^v}}{1+e^v} = -\frac{(e^v+v)}{1+e^v}$

$\Rightarrow \int \frac{1+e^v}{e^v+v}\,dv = -\int \frac{dx}{x}$

$\Rightarrow \log|e^v+v| = -\log|x| + \log|c|$

$\Rightarrow \log|e^v+v| = \log\left|\frac{c}{x}\right|$

$\Rightarrow e^v + v = \frac{c}{x}$

$\Rightarrow xe^v + vx = c$

$\Rightarrow \boxed{xe^{y/x} + y = c}$ Ans

Q.28 Let Pedestal lamp produced in a day $= x$

Wooden shades produced in a day $= y$

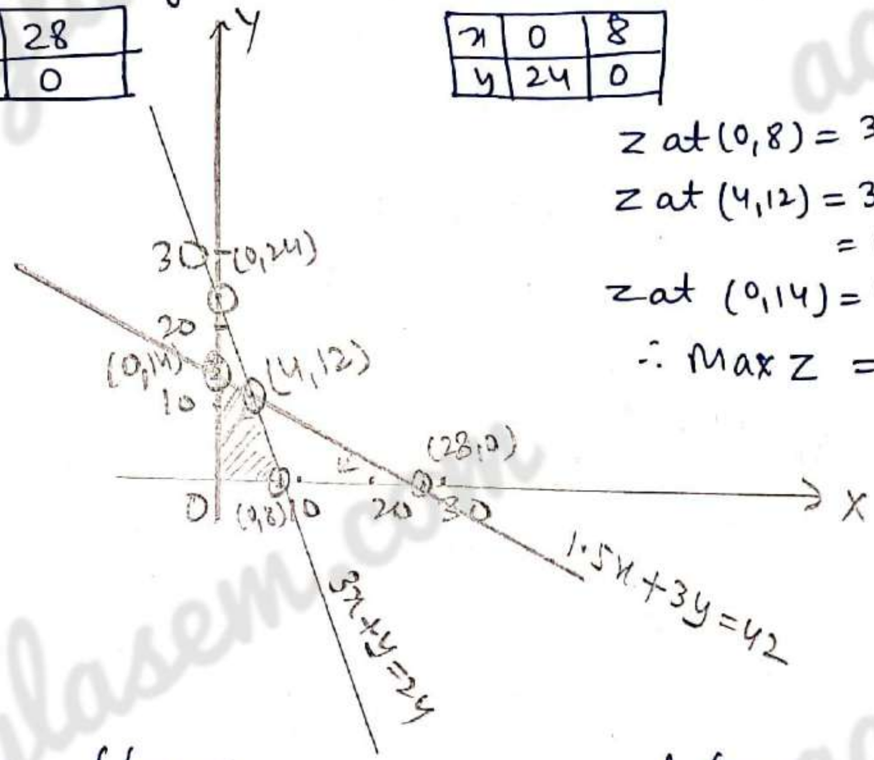
$$\text{Max } Z = 30x + 20y$$

Subject to constraints:

$$1.5x + 3y \le 42 \;;\; 3x + y \le 24,\; x \ge 0,\; y \ge 0$$

x	0	28
y	14	0

x	0	8
y	24	0



Z at $(0,8) = 30\times 0 + 20\times 8 = 160$

Z at $(4,12) = 30\times 4 + 20\times 12$

$= 120 + 240 = 360$

Z at $(0,14) = 30\times 0 + 20\times 14 = 280$

$\therefore$ Max $Z = 360$ at $(4,12)$

Ans

Q.29

$$I = \int\left(\sqrt{\cot x} + \frac{1}{\sqrt{\cot x}}\right)dx = \int\left(\sqrt{\tan x} + \frac{1}{\sqrt{\tan x}}\right)dx$$

$$= \int\left(\frac{\tan x + 1}{\sqrt{\tan x}}\right)dx$$

Putting $\tan x = t^2$

$\sec^2 x\, dx = 2t\, dt$

$dx = \frac{2t\, dt}{1+\tan^2 x} = \frac{2t}{1+t^4}dt$

$$= \int \frac{t^2+1}{\cancel{t}} \times \frac{2\cancel{t}}{1+t^4}dt$$

$$= 2\int \frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}}dt = 2\int \frac{1+\frac{1}{t^2}}{\left(t-\frac{1}{t}\right)^2+2}dt$$

Now $t - \frac{1}{t} = u$

$\left(1+\frac{1}{t^2}\right)dt = du$

$$= 2\int \frac{du}{u^2+(\sqrt{2})^2} = 2\cdot\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + C$$

$$= \sqrt{2}\tan^{-1}\left(\frac{t^2-1}{\sqrt{2}\,t}\right) + C = \sqrt{2}\tan^{-1}\left[\frac{\tan x - 1}{\sqrt{2\tan x}}\right] + C$$

Ans

30 $R = \{(a,b) : a$ is divisor of $b\}$

(i) Reflexive: Since every natural number divisor of itself

$\therefore \forall\, a \in N \Rightarrow (a,a) \in R \quad \therefore R$ is reflexive.

(ii) Symmetric: $(2,4) \in R$ but $(4,2) \notin R \quad \therefore R$ is not symmetric

(iii) Transitive: Let $(a,b) \in R,\ (b,c) \in R \quad \forall\, a,b,c \in N$

$\Rightarrow b = ma \ , \ c = bn$

$c = n(ma) = mna$

$\Rightarrow a$ is divisor of c

$\Rightarrow (a,c) \in R \quad \therefore R$ is Transitive.

OR

30 LHS $= \tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9}$

$= \tan^{-1}\left(\frac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}\cdot\frac{2}{9}}\right) = \tan^{-1}\left(\frac{9+8}{36-2}\right) = \tan^{-1}\left(\frac{17}{34}\right) = \tan^{-1}\frac{1}{2}$

$= \frac{1}{2} \times 2\tan^{-1}\frac{1}{2}$

$= \frac{1}{2} \sin^{-1}\left[\frac{2\times\frac{1}{2}}{1+\frac{1}{4}}\right]$

$= \frac{1}{2}\sin^{-1}\left[\frac{4}{5}\right] =$ RHS

31. Let equation of the required plane be

$(x+2y-3) + k(2x-y+z-1) = 0$ —— (1)

it passes through $(0,0,0)$

$\Rightarrow (0+0-3) + k(0-0+0-1) = 0 \Rightarrow -3-k = 0$

$k = -3$ Putting in (1)

$\therefore$ equation of the plane is

$(x+2y-3) - 3(2x-y+z-1) = 0$

$\Rightarrow x+2y-3-6x+3y-3z+3 = 0$

$\Rightarrow -5x+5y-3z = 0$ OR $\boxed{5x-5y+3z=0}$

Vector form $\vec{r}\cdot(5\hat{i}-5\hat{j}+3\hat{k}) = 0$

32. $\tan^{-1}\left(\frac{y}{x}\right) = \log\sqrt{x^2+y^2} = \frac{1}{2}\log(x^2+y^2)$

Differentiating w.r.t. x

$$\frac{1}{1+\frac{y^2}{x^2}}\left[\frac{x\frac{dy}{dx}-y}{x^2}\right] = \frac{1}{2(x^2+y^2)}\cdot\left(2x+2y\frac{dy}{dx}\right)$$

$$\Rightarrow \frac{\cancel{x^2}}{\cancel{x^2+y^2}}\left[\frac{x\frac{dy}{dx}-y}{\cancel{x^2}}\right] = \frac{1}{\cancel{2}\cancel{(x^2+y^2)}}\cdot\cancel{2}\left(x+y\frac{dy}{dx}\right)$$

$$\Rightarrow x\frac{dy}{dx}-y = x+y\frac{dy}{dx}$$

$$\Rightarrow x\frac{dy}{dx}-y\frac{dy}{dx} = x+y \Rightarrow \frac{dy}{dx} = \frac{x+y}{x-y} \quad \text{HP}$$

OR

32. $y = e^{a\cos^{-1}x}$

$$\log y = a\cos^{-1}x \Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{-a}{\sqrt{1-x^2}}$$

$$\Rightarrow \sqrt{1-x^2}\,\frac{dy}{dx} = -ay \quad \text{——— (1)}$$

Again differentiating w.r.t x

$$\Rightarrow \sqrt{1-x^2}\cdot\frac{d^2y}{dx^2}+\frac{dy}{dx}\cdot\frac{1}{\cancel{2}\sqrt{1-x^2}}\cdot(-\cancel{2}x) = -a\frac{dy}{dx}$$

$$\Rightarrow \frac{(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}}{\sqrt{1-x^2}} = -a\frac{dy}{dx}$$

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx} = -a\sqrt{1-x^2}\,\frac{dy}{dx}$$

$$= -a \times -ay \quad \text{From (1)}$$

$$= a^2y$$

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx} - a^2y = 0 \quad \text{HP.}$$

SECTION-D

33. $V = \pi r^2 h$

$\Rightarrow 125\cancel{\pi} = \cancel{\pi} r^2 h$

$\Rightarrow h = \dfrac{125}{r^2}$ —(1)

$A = 2\pi r h + \pi r^2$

$= 2\pi \cancel{r}\left(\dfrac{125}{r^{\cancel{2}}}\right) + \pi r^2$ from (1)

$A = \dfrac{250\pi}{r} + \pi r^2$

$\Rightarrow \dfrac{dA}{dr} = -\dfrac{250\pi}{r^2} + 2\pi r$

$\dfrac{dA}{dr} = 0 \Rightarrow -\dfrac{250\pi}{r^2} + 2\pi r = 0$

$\Rightarrow \cancel{2}\cancel{\pi} r = \dfrac{\overset{125}{\cancel{250}}\cancel{\pi}}{r^2}$

$r^3 = 125 \Rightarrow \boxed{r = 5 \text{ cm}}$

$\therefore h = \dfrac{125}{r^2} = \dfrac{125}{25} = 5 \text{ cm}$ $\boxed{h = 5 \text{ cm}}$

$\dfrac{d^2A}{dr^2} = +\dfrac{500\pi}{r^3} + 2\pi > 0$

$\left.\dfrac{d^2A}{dr^2}\right|_{r=5} = +\dfrac{500\pi}{125} + 2\pi = \dfrac{+500\pi + 250\pi}{125} = \dfrac{750\pi}{125} > 0$

$\therefore$ A is minimum when $r = 5$ cm, $h = 5$ cm

34. $x^2 + y^2 - 8x = 0$

$x^2 - 8x + 16 + y^2 = 16$

$\Rightarrow (x-4)^2 + y^2 = 4^2$

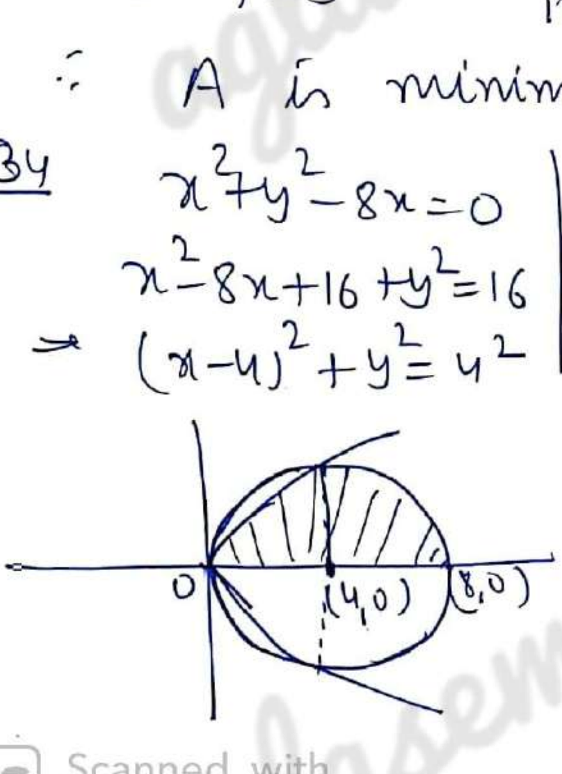
$y^2 = 4x$

$x^2 + 4x - 8x = 0$

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0$ or $x = 4$



Required Area $= \int_0^4 2\sqrt{x}\,dx + \int_4^8 \sqrt{4^2 - (x-4)^2}\,dx$

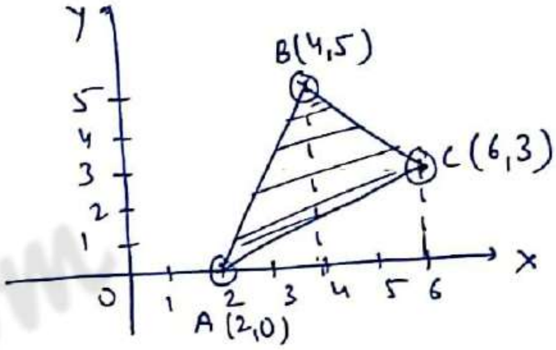
$= 2 \cdot \frac{2}{3}\left[x^{3/2}\right]_0^4 + \left[\frac{x-4}{2}\sqrt{4^2 - (x-4)^2} + \frac{16}{2}\sin^{-1}\frac{x-4}{4}\right]_4^8$

$= \frac{4}{3}\left[2^{2 \times \frac{3}{2}}\right] + \left[(0 + 8\sin^{-1} 1) - (0)\right]$

$= \frac{32}{3} + 8 \cdot \frac{\pi}{2} = \left(\frac{32}{3} + 4\pi\right)$ sq. units

OR

Q.34 Equation of AB



$$\frac{y-0}{5-0} = \frac{x-2}{4-2}$$

$$\frac{y}{5} = \frac{x-2}{2} \Rightarrow 2y = 5x-10$$

$$y = \frac{5x-10}{2}$$

Equation of BC

$$\frac{y-3}{5-3} = \frac{x-6}{4-6}$$

$$\Rightarrow \frac{y-3}{\cancel{2}} = \frac{x-6}{\cancel{-2}_{-1}} = -x+6$$

$$y = -x+6+3$$

$$y = 9-x$$

Equation of AC

$$\frac{y-0}{3-0} = \frac{x-2}{6-2}$$

$$\frac{y}{3} = \frac{x-2}{4} \Rightarrow y = \frac{3x-6}{4}$$

Area of $\Delta ABC = \int_2^4 \frac{5x-10}{2}dx + \int_4^6 (9-x)dx - \int_2^6 \frac{3x-6}{4}dx$

$$= \frac{1}{2}\left(\frac{5x^2}{2} - 10x\right)_2^4 + \left(9x - \frac{x^2}{2}\right)_4^6 - \frac{1}{4}\left(\frac{3x^2}{2} - 6x\right)_2^6$$

$$= \frac{1}{2}\left[\left(\frac{5\times\cancel{16}^{8}}{\cancel{2}} - 40\right) - \left(\frac{5\times\cancel{4}^{2}}{\cancel{2}} - 20\right)\right] + \left[\left(9\times6 - \frac{\cancel{36}^{18}}{\cancel{2}}\right) - \left(9\times4 - \frac{\cancel{16}^{8}}{\cancel{2}}\right)\right]$$

$$- \frac{1}{4}\left[\left(\frac{3\times\cancel{36}^{18}}{\cancel{2}} - 36\right) - \left(\frac{3\times\cancel{4}^{2}}{\cancel{2}} - 6\times2\right)\right]$$

$$= \frac{1}{2}[0+10] + (36-28) - \frac{1}{4}[18+6]$$

$$= 5 + 8 - 6 = 7 \text{ sq. units} \quad \text{Ans}$$

Q.35

$$A = \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix}$$

$A_{11} = +28 \quad A_{12} = +13 \quad A_{13} = +(-19) = -19$

$A_{21} = -2 \quad A_{22} = +10 \quad A_{23} = +5$

$A_{31} = +(-17) = -17 \quad A_{32} = -17 \quad A_{33} = +17$

$$|A| = 5(18+10) + 1(12-25) + 4(-4-15)$$

$$= 140 - 13 - 76$$

$$= 140 - 89$$

$$= 51$$

$$\text{adj}\,A = \begin{pmatrix} 28 & 13 & -19 \\ -2 & 10 & 5 \\ -17 & -17 & 17 \end{pmatrix}^T = \begin{pmatrix} 28 & -2 & -17 \\ 13 & 10 & -17 \\ -19 & 5 & 17 \end{pmatrix}$$

$$A^{-1} = \frac{\text{adj}\,A}{|A|} = \frac{1}{51}\begin{bmatrix} 28 & -2 & -17 \\ 13 & 10 & -17 \\ -19 & 5 & 17 \end{bmatrix}$$

$\longrightarrow$

Q35 (continued)

The system can be expressed as

$AX = B$ where $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $B = \begin{pmatrix} 5 \\ 2 \\ -1 \end{pmatrix}$

$\Rightarrow X = A^{-1} B$

$= \frac{1}{51} \begin{bmatrix} 28 & -2 & -17 \\ 13 & 10 & -17 \\ -19 & 5 & 17 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{51} \begin{bmatrix} 140 - 4 + 17 \\ 65 + 20 + 17 \\ -95 + 10 - 17 \end{bmatrix} = \frac{1}{51} \begin{bmatrix} 153 \\ 102 \\ -102 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -2 \end{bmatrix}$

$\Rightarrow x = 3,\ y = 2,\ z = -2$

OR

$\Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix}$

$= \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$

$= (1+xyz) \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$

$R_1 \to R_1 - R_2,\ R_2 \to R_2 - R_3$

$= (1+xyz) \begin{vmatrix} 0 & x-y & x^2-y^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix}$

$= (1+xyz)(x-y)(y-z) \begin{vmatrix} 0 & 1 & x+y \\ 0 & 1 & y+z \\ 1 & z & z^2 \end{vmatrix}$

$= (1+xyz)(x-y)(y-z)[1(\cancel{y}+z-x-\cancel{y})]$

Expanding along C_1

$\Delta = (1+xyz)(x-y)(y-z)(z-x)$

It is given $\Delta = 0$

$\Rightarrow (1+xyz)(x-y)(y-z)(z-x) = 0$

$\Rightarrow 1 + xyz = 0$ HP $(\because x-y \neq 0,\ y-z \neq 0,\ z-x \neq 0$ x, y, z are different$)$

Q-36. $\Sigma P_i = 1$

$\Rightarrow \quad 4c^2 + 3c^2 + 2c^2 + c^2 + c + 2c = 1$

$\Rightarrow \quad 10c^2 + 3c - 1 = 0$

$\Rightarrow \quad 10c^2 + 5c - 2c - 1 = 0$

$\Rightarrow \quad 5c(2c+1) - 1(2c+1) = 0$

$\Rightarrow \quad (2c+1)(5c-1) = 0$

$\Rightarrow \quad 2c+1 = 0$ or $5c-1 = 0$

$\Rightarrow \quad c = -\frac{1}{2}$ or $c = \frac{1}{5}$

But $c \neq -\frac{1}{2}$ $\therefore$ $\boxed{c = \frac{1}{5}}$

Mean $= \Sigma P_i x_i$

$= 0 + 1 \times 3c^2 + 2 \times 2c^2 + 3c^2 + 4c + 10c$

$= 10c^2 + 14c$

$= \cancel{10}^{2} \times \frac{1}{\cancel{25}_5} + 14 \times \frac{1}{5}$

$= \frac{2+14}{5} = \frac{16}{5}$

Variance $= \Sigma P_i x_i^2 - (\Sigma P_i x_i)^2$

$= 14 - \left(\frac{16}{5}\right)^2$

$= 14 - \frac{256}{25}$

$= \frac{350-256}{25}$

$= \frac{94}{25}$ Ans