SHORT ANSWER TYPE QUESTIONS (1)

Prove that :

20. $\sec^4 \theta - \sec^2 \theta = \tan^4 \theta + \tan^2 \theta$

21. $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \tan\theta + \text{Sec}\,\theta$

22. If $x = p \sec\theta + q \tan\theta$ & $y = p \tan\theta + q \sec\theta$ then prove that $x^2 - y^2 = p^2 - q^2$

23. If $7 \sin^2 \theta + 3 \cos^2 \theta = 4$ then show that $\tan\theta = \frac{1}{\sqrt{3}}$

24. If $\sin(A - B) = \frac{1}{2}$, $\cos(A + B) = \frac{1}{2}$ then find the value of A and B.

25. Find the value of $\frac{\cos^2 20° + \cos^2 70°}{\sin^2 59° + \sin^2 31°}$.

26. If $3 \cot A = 4$, find the value of $\frac{\text{cosec}^2 A + 1}{\text{cosec}^2 A - 1}$.

27. If $\tan(3x - 15°) = 1$ then find the value of x.

28. If A, B, C are interior angles of ΔABC, the prove that $\cos ec\left(\frac{A+B}{2}\right) = \sec\left(\frac{C}{2}\right)$.

(CBSE 2011)

29. In ΔABC, right angled at B, AB = 5 cm and $\angle ACB = 30°$. Find BC and AC.

30. Show that : $\frac{1 - \sin 60°}{\cos 60°} = 2 - \sqrt{3}$. (CBSE, 2014)

31. Find the value of θ, if $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$, $\theta \le 90°$. (CBSE, 2014)

SHORT ANSWER TYPE QUESTIONS

Prove that :

32. $\frac{\tan A + \text{Sec } A - 1}{\tan A - \text{Sec } A + 1} = \frac{1 + \text{Sin } A}{\text{Cos } A}$

33. $\frac{1}{\sec x - \tan x} - \frac{1}{\cos x} = \frac{1}{\cos x} - \frac{1}{\sec x + \tan x}$

34. $\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \tan \theta + \cot \theta = \sec \theta \, \text{cosec} \, \theta + 1$ **(CBSE 2019)**

35. $(\sin \theta + \text{cosec } \theta)^2 + (\cos \theta + \sec \theta)^2 = 7 + \tan^2 \theta + \cot^2 \theta$

36. $\sec A (1 - \sin A) (\sec A + \tan A) = 1$

37. If $\sec \theta = x + \frac{1}{4x}$, prove that $\sec \theta + \tan \theta = 2x$ or $\frac{1}{2x}$

38. If $\sin \theta + \sin^2 \theta = 1$, prove that $\cos^2 \theta + \cos^4 \theta = 1$

39. Without using trigonometric table, the value of
$\cot \theta \tan (90° - \theta) - \sec (90° - \theta) \text{ cosec } \theta + \sin^2 65° + \sin^2 25° + \sqrt{3} \tan 5° \tan 85°$.

40. **Prove that :** $\frac{\cot (90° - \theta)}{\tan \theta} + \frac{\text{cosec} (90° - \theta) \sin \theta}{\tan (90° - \theta)} = \sec^2 \theta$

41. **Find the value of :**

$\frac{\cos^2 20° + \cos^2 70°}{\sec^2 50° - \cot^2 40}$ + 2 Cosec^2 58° – 2 Cot 58° tan 32° – 4 tan 13° tan 37° tan 77° tan 45° tan 53°.

42. If A, B, C are the angles of ΔABC then prove that $\text{cosec}^2 \left(\frac{B + C}{2}\right) - \tan^2 \frac{A}{2} = 1$

43. Find the value of $\sec^2 10° - \cot^2 80° + \frac{\sin 15° \cos 75° + \cos 15° \sin 75°}{\cos \theta \sin (90° - \theta) + \sin \theta \cos (90° - \theta)}$.

44. If $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$, then show that $\cos \theta - \sin \theta = \sqrt{2} \sin \theta$.

45. Evaulate : $\frac{\tan^2 60° + 4 \cos^2 45° + 3 \sec^2 30° + 5 \cos^2 90°}{\text{cosec } 30° + \sec 60° - \cot^2 30°}$

46. If $a \cos \theta + b \sin \theta = m$ and $a \sin \theta - b \cos \theta = n$ (CBSE, 2001 C)
Prove that : $a^2 + b^2 = m^2 + n^2$

LONG ANSWER TYPE QUESTIONS

Prove That:

47. $\left(1+\frac{1}{\tan^2\theta}\right)\left(1+\frac{1}{\cot^2\theta}\right)=\frac{1}{\sin^2\theta-\sin^4\theta}$

48. $2(\sin^6\theta+\cos^6\theta)-3(\sin^4\theta+\cos^4\theta)+1=0$

49. $(1+\cot A+\tan A)(\sin A-\cos A)=\sin A\tan A-\cot A\cos A$

50. If $\sin\theta+\cos\theta=m$ and $\sec\theta+\operatorname{cosec}\theta=n$ then show that $n(m^2-1)=2m$

51. find the value of :

$$\frac{\cot(90°-\theta)\tan\theta-\operatorname{cosec}(90°-\theta)\sec\theta}{\sin 12°\,\operatorname{Cos}15°\sec 78°\operatorname{cosec}75°}+\frac{\sin^2(50°+\theta)+\sin^2(40°-\theta)}{\tan 15°\tan 37°\tan 53°\tan 75°}$$

52. Prove that :

$$\frac{1}{\operatorname{cosec}\theta+\cot\theta}-\frac{1}{\sin\theta}=\frac{1}{\sin\theta}-\frac{1}{\operatorname{cosec}\theta-\cot\theta}$$

53. If $\frac{\cos\alpha}{\cos\beta}=m$ and $\frac{\cos\alpha}{\sin\beta}=n$, then prove that $(m^2+n^2)\operatorname{Cos}^2\beta=n^2$

54. Prove that :

$$\sec^2\theta-\frac{\sin^2\theta-2\sin^4\theta}{2\cos^4\theta-\cos^2\theta}=1$$

55. $\cot\theta\tan(90°-\theta)-\sec(90°-\theta)\operatorname{cosec}\theta+\sqrt{3}\tan 12°\tan 60°\tan 78°$ find its value.

56. Find the value of —

$$\frac{\sec(90°-\theta)\operatorname{cosec}\theta-\tan(90°-\theta)\cot\theta+\cos^2 25°+\cos^2 65°}{3\tan 27°\tan 63°}$$

57. If $\sin\theta+\cos\theta=\sqrt{3}$, then prove that $\tan\theta+\cot\theta=1$ **(CBSE 2020)**

58. Prove $\frac{\cot A-\cos A}{\cot A+\cos A}=\sec^2A+\tan^2A-2\sec A\tan A$ **(CBSE 2020 Basic)**

59. Prove $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$ **(CBSE 2020 Basic)**

60. If $\cos(A + B) = \sin(A - B) = \frac{1}{2}$, $0 < A + B < 90°$ and $A > B$ then find the value of A and B. **(CBSE 2020 Basic)**

61. If $\tan\theta + \sin\theta = m$, $\tan\theta - \sin\theta = n$, then prove that $m^2 - n^2 = 4\sqrt{mn}$. **(CBSE 2020 Standard)**

62. Find $\frac{\sec^2(90° - \theta) - \cot^2\theta}{2(\sin^2 25° + \sin^2 65°)} + \frac{2\cos^2 60° \tan^2 28° \tan^2 62°}{3(\sec^2 43° - \cot^2 47°)}$

(CBSE 2020 Standard)

63. Prove $\frac{1 + \sec\theta - \tan\theta}{1 + \sec\theta + \tan\theta} = \frac{1 - \sin\theta}{\cos\theta}$ **(CBSE 2020 Standard)**

64. Evaluate $\left(\frac{3\sin 43°}{\cos 47°}\right)^2 - \frac{\cos 37° \operatorname{cosec} 53°}{\tan 5° \tan 25° \tan 45° \tan 65° \tan 85°}$

(CBSE 2019)

65. Prove that $\frac{\sin\theta}{\cot\theta + \operatorname{cosec}\theta} = 2 + \frac{\sin\theta}{\cot\theta - \operatorname{cosec}\theta}$ **(CBSE 2019)**

66. If $4\tan\theta = 3$ then find the value of $\frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1}$ **(CBSE 2018)**

67. Prove $\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$ **(CBSE 2018)**

ANSWERS AND HINTS

1. 45° **2.** 30°
3. 48° **4.** 7
5. $\frac{1}{3}$ **6.** a^2
7. 0 **8.** 9

9. $\sqrt{\frac{1+\cot^2\theta}{\cot\theta}}$ **10.** 0°

11. 40° **12.** $\tan^2\theta$

13. $\frac{1}{2}$ **14.** sec 42° + cot 2°

15. (c) **16.** (d)

17. (a) **18.** (a)

19. (a)

20. LHS = $\sec^2\theta(\sec^2\theta - 1)$

RHS = $\tan^2\theta(\tan^2\theta + 1)$

Use $1 + \tan^2\theta = \sec^2\theta$

21. Relationalise and proceed in LHS

22. Squaring both sides of x and y and subtracting.

23. Divide both sides by $\cos^2\theta$

24. A = 45°, B = 15°

25. 1

26. $\frac{17}{8}$

27. 20°

28. Use (A + B + C = 180°)

29. AC = 10, BC = $5\sqrt{3}$, use Pythagoras theorem

30. Substitute values of sin 60° and cos 60° and solve

31. 60°

Note : 32 to 38 use trigonometric identities and prove (based on Ex. 8.4 of NCERT)

39. $\sqrt{3}$

40. Use $\cot(90 - \theta) = \tan\theta$, $\operatorname{cosec}(90 - \theta) = \sec\theta$, $\tan(90 - \theta) = \cot\theta$

41. – 1

42. Use A + B + C = 180°

43. 2

44. $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

Square both sides and get $1 + 2\cos\theta\sin\theta = 2\cos^2\theta$

$\Rightarrow 2\cos\theta\sin\theta = 2\cos^2\theta - 1$ ...(1)

Now square $(\cos\theta - \sin\theta)^2$ and get

$(\cos\theta - \sin\theta)^2 = 1 - 2\cos\theta\sin\theta$ ...(2)

Substitute (1) in (2)

45. 9.

46. Find m^2 and n^2 and add

Note : Q47 to Q50 Use identities to prove

51. 0

52. Rationalise $\frac{1}{\text{cosec}\,\theta + \cot\theta}$ in LHS and proceed, use $\frac{1}{\sin\theta} = \text{cosec}\,\theta$.

Rationalise $\frac{1}{\text{cosec}\,\theta - \cot\theta}$ on RHS and proceed, use $\frac{1}{\sin\theta} = \text{cosec}\,\theta$.

53. Find m^2 and n^2 and substitute in LHS.

54. Take common $\sin^2\theta$ in Numerator and $\cos^2\theta$ in Denominator of 2nd term on LHS and replace 1 by $\sin^2\theta + \cos^2\theta$.

55. 0

56. $\frac{2}{3}$

57. $(\sin\theta + \cos\theta) = \sqrt{3}$

square both sides and get value of $\frac{1}{\sin\theta \times \cos\theta}$

Change $\tan\theta + \cot\theta$ into $\sin\theta$ and $\cos\theta$ proceed.

58. Change $\cot A = \frac{\cos A}{\sin A}$, take cos A common from Numerator and Denominator, Rationalise remaining term and change into sec A and tan A.

59. LHS = $\frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta-1)}$, write $1 = \sin^2\theta + \cos^2\theta$ and proceed.

60. $\cos(A + B) = \frac{1}{2} = \cos 60°$

$\sin(A - B) = \frac{1}{2} = \sin 30°$

$\Rightarrow \left.\begin{matrix} A+B=60° \\ A-B=30° \end{matrix}\right]$ Solve these equations

$A = 45°, B = 15°$

61. Find m^2 and n^2 substitute in $m^2 - n^2$ and substitute m and n in $4\sqrt{mn}$

62. $\frac{2}{3}$

63. Refer NCERT

64. (Use complementary form), 8

65. Convert cot θ and cosec θ into sin θ and cos θ
and use $\sin^2\theta = 1 - \cos^2\theta$

66. Divide Numerator and Denominator by cos θ, and use $\sec\theta = \sqrt{1+\tan^2\theta}$
or use pythagoras theorem and trigonometeric ratios,

Ans. $\frac{13}{11}$

67. Same as Q 59.

PRACTICE-TEST

Introduction to Trigonometry

Time : 1 Hrs. ***M.M.: 20***

SECTION-A

1. If $\sin\theta = \frac{4}{5}$ what is the value of $\cos\theta$. **1**

2. Write the value of $\sin(45° + \theta) - \cos(45° - \theta)$. **1**

3. If $\cos 9\alpha = \sin\alpha$ and $9\alpha < 90°$, then the value of $\tan 5\alpha$ is **1**

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$ (c) 1 (d) 0

4. If $\sin A + \sin^2 A = 1$, then the value of $(\cos^2 A + \cos^4 A)$ is **1**

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) 3

SECTION-B

5. If $5\tan\theta = 4$ then find the value of $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta}$ **2**

6. Find the value of $\tan 35° \tan 40° \tan 45° \tan 50° \tan 55°$ **2**

7. Prove that $(\sin\alpha + \cos\alpha)(\tan\alpha + \cot\alpha) = \sec\alpha + \operatorname{cosec}\alpha$ **2**

SECTION-C

8. Prove that $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$ **3**

9. Prove that $\frac{\cos A}{1-\tan A} - \frac{\sin^2 A}{\cos A - \sin A} = \sin A + \cos A$ **3**

SECTION-D

10. Prove that $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{\cos\theta}{1-\sin\theta}$. **4**