X - RAYS
CHAPTER 44

1. $\lambda = 0.1$ nm

   a) Energy = $\frac{hc}{\lambda} = \frac{1242 \text{ ev.nm}}{0.1 \text{ nm}}$

   = 12420 ev = 12.42 Kev = 12.4 kev.

   b) Frequency = $\frac{C}{\lambda} = \frac{3\times10^8}{0.1\times10^{-9}} = \frac{3\times10^8}{10^{-10}} = 3\times10^{18}$ Hz

   c) Momentum = E/C = $\frac{12.4\times10^3\times1.6\times10^{-19}}{3\times10^8} = 6.613 \times 10^{-24}$ kg-m/s = 6.62×10^{-24} kg-m/s.

2. Distance = 3 km = 3×10^3 m

   C = 3×10^8 m/s

   $t = \frac{\text{Dist}}{\text{Speed}} = \frac{3\times10^3}{3\times10^8} = 10^{-5}$ sec.

   $\Rightarrow 10 \times 10^{-8}$ sec = 10 μs in both case.

3. V = 30 KV

   $\lambda = \frac{hc}{E} = \frac{hc}{eV} = \frac{1242 \text{ ev} - \text{nm}}{e\times30\times10^3} = 414 \times 10^{-4}$ nm = 41.4 Pm.

4. $\lambda = 0.10$ nm = 10^{-10} m ; $h = 6.63 \times 10^{-34}$ J-s

   C = 3×10^8 m/s ; $e = 1.6 \times 10^{-19}$ C

   $\lambda_{min} = \frac{hc}{eV}$ or $V = \frac{hc}{e\lambda}$

   $= \frac{6.63\times10^{-34}\times3\times10^8}{1.6\times10^{-19}\times10^{-10}} = 12.43 \times 10^3$ V = 12.4 KV.

   Max. Energy = $\frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^8}{10^{-10}} = 19.89 \times 10^{-18} = 1.989 \times 10^{-15} = 2 \times 10^{-15}$ J.

5. $\lambda = 80$ pm, $E = \frac{hc}{\lambda} = \frac{1242}{80\times10^{-3}} = 15.525 \times 10^3$ eV = 15.5 KeV

6. We know $\lambda = \frac{hc}{V}$

   Now $\lambda = \frac{hc}{1.01V} = \frac{\lambda}{1.01}$

   $\lambda - \lambda' = \frac{0.01}{1.01}\lambda$.

   % change of wave length = $\frac{0.01\times\lambda}{1.01\times\lambda}\times100 = \frac{1}{1.01} = 0.9900 = 1\%$.

7. d = 1.5 m, $\lambda = 30$ pm = 30×10^{-3} nm

   $E = \frac{hc}{\lambda} = \frac{1242}{30\times10^{-3}} = 41.4 \times 10^3$ eV

   Electric field = $\frac{V}{d} = \frac{41.4\times10^3}{1.5} = 27.6 \times 10^3$ V/m = 27.6 KV/m.

8. Given $\lambda' = \lambda - 26$ pm, $V' = 1.5$ V

   Now, $\lambda = \frac{hc}{ev}$, $\lambda' = \frac{hc}{ev'}$

   or $\lambda V = \lambda' V'$

   $\Rightarrow \lambda V = (\lambda - 26 \times 10^{-12}) \times 1.5$ V

   $\Rightarrow \lambda = 1.5\lambda - 1.5 \times 26 \times 10^{-12}$

   $\Rightarrow \lambda = \frac{39\times10^{-12}}{0.5} = 78 \times 10^{-12}$ m

   $V = \frac{hc}{} = \frac{6.63\times3\times10^{-34}\times10^8}{}$ $= 0.15937 \times 10^5 = 15.93 \times 10^3$ V = 15.93 KV.

$e\lambda \quad 1.6\times10^{-19}\times78\times10^{-12}$

9. $V = 32\ KV = 32\times10^{3}\ V$

When accelerated through 32 KV

$E = 32\times10^{3}\ eV$

$\lambda = \frac{hc}{E} = \frac{1242}{32\times10^{3}} = 38.8\times10^{-3}\ nm = 38.8\ pm.$

10. $\lambda = \frac{hc}{eV}$; $V = 40\ kV$, $f = 9.7\times10^{18}\ Hz$

or, $\frac{h}{c} = \frac{h}{eV}$; or, $\frac{i}{f} = \frac{h}{eV}$; or $h = \frac{eV}{f}V-s$

$= \frac{eV}{f}V-s = \frac{40\times10^{3}}{9.7\times10^{18}} = 4.12\times10^{-15}\ eV\text{-}s.$

11. $V = 40\ KV = 40\times10^{3}\ V$

Energy $= 40\times10^{3}\ eV$

Energy utilized $= \frac{70}{100}\times40\times10^{3} = 28\times10^{3}\ eV$

$\lambda = \frac{hc}{E} = \frac{1242-ev\ nm}{28\times10^{3}\ ev} \Rightarrow 44.35\times10^{-3}\ nm = 44.35\ pm.$

For other wavelengths,

E = 70% (left over energy) $= \frac{70}{100}\times(40-28)10^{3} = 84\times10^{2}.$

$\lambda' = \frac{hc}{E} = \frac{1242}{8.4\times10^{3}} = 147.86\times10^{-3}\ nm = 147.86\ pm = 148\ pm.$

For third wavelength,

$E = \frac{70}{100} = (12-8.4)\times10^{3} = 7\times3.6\times10^{2} = 25.2\times10^{2}$

$\lambda' = \frac{hc}{E} = \frac{1242}{25.2\times10^{2}} = 49.2857\times10^{-2}\ nm = 493\ pm.$

12. $K_{\lambda} = 21.3\times10^{-12}\ pm,$ Now, $E_K - E_L = \frac{1242}{21.3\times10^{-3}} = 58.309\ kev$

$E_L = 11.3\ kev,$ $E_K = 58.309 + 11.3 = 69.609\ kev$

Now, Ve = 69.609 KeV, or V = 69.609 KV.

13. $\lambda = 0.36\ nm$

$E = \frac{1242}{0.36} = 3450\ eV\ (E_M - E_K)$

Energy needed to ionize an organ atom = 16 eV

Energy needed to knock out an electron from K-shell

= (3450 + 16) eV = 3466 eV = 3.466 KeV.

14. $\lambda_1 = 887\ pm$

$v = \frac{C}{\lambda} = \frac{3\times10^{8}}{887\times10^{-12}} = 3.382\times10^{7} = 33.82\times10^{16} = 5.815\times10^{8}$

$\lambda_2 = 146\ pm$

$v = \frac{3\times10^{8}}{146\times10^{-12}} = 0.02054\times10^{20} = 2.054\times10^{18} = 1.4331\times10^{9}.$

We know, $\sqrt{v} = a(z-b)$

$\Rightarrow \frac{\sqrt{5.815\times10^{8}} = a(13-b)}{\sqrt{1.4331\times10^{9}} = a(30-b)}$

$\Rightarrow \frac{13-b}{30-b} = \frac{5.815\times10^{-1}}{1.4331} = 0.4057.$

$\Rightarrow 30\times0.4057 - 0.4057\ b = 13 - b$

$\Rightarrow 12.171 - 0.4.57\ b + b = 13$

$\Rightarrow b = \frac{0.829}{0.5943} = 1.39491$

0.5943

$\Rightarrow a = \frac{5.815\times10^{8}}{11.33} = 0.51323\times10^{8} = 5\times10^{7}.$

For 'Fe',

$\sqrt{v} = 5\times10^{7}(26-1.39) = 5\times24.61\times10^{7} = 123.05\times10^{7}$

$c/\lambda = 15141.3\times10^{14}$

$= \lambda = \frac{3\times10^{8}}{15141.3\times10^{14}} = 0.000198\times10^{-6}\ m = 198\times10^{-12} = 198\ pm.$

15. E = 3.69 kev = 3690 eV

$\lambda = \frac{hc}{E} = \frac{1242}{3690} = 0.33658\ nm$

$\sqrt{c/\lambda} = a(z-b);\quad a = 5\times10^{7}\ \sqrt{Hz}$, b = 1.37 (from previous problem)

$\sqrt{\frac{3\times10^{8}}{0.34\times10^{-9}}} = 5\times10^{7}(Z-1.37) \Rightarrow \sqrt{8.82\times10^{17}} = 5\times10^{7}(Z-1.37)$

$\Rightarrow 9.39\times10^{8} = 5\times10^{7}(Z-1.37) \Rightarrow 93.9/5 = Z-1.37$

$\Rightarrow Z = 20.15 = 20$

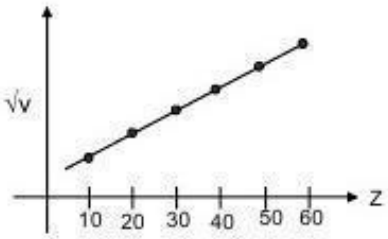
$\therefore$ The element is calcium.

16. K_B radiation is when the e jumps from
n = 3 to n = 1 (here n is principal quantum no)

$\Delta E = h\nu = Rhc(z-h)^{2}\left(\frac{1}{2^{2}}-\frac{1}{3^{2}}\right)$

$\Rightarrow \sqrt{v} = \sqrt{\frac{9RC}{8}}(z-h)$

$\therefore \sqrt{v} \propto z$



Second method :

We can directly get value of v by `

hv = Energy

$\Rightarrow v = \frac{Energy(in\ kev)}{h}$

This we have to find out $\sqrt{v}$ and draw the same graph as above.

17. b = 1

For $\propto$ a (57)

$\sqrt{v} = a(Z-b)$

$\Rightarrow \sqrt{v} = a(57-1) = a\times56$ ...(1)

For Cu(29)

$\sqrt{1.88\times10^{78}} = a(29-1) = 28\ a$ ...(2)

dividing (1) and (2)

$\sqrt{\frac{v}{1.88\times10^{18}}} = \frac{a\times56}{a\times28} = 2.$

$\Rightarrow v = 1.88\times10^{18}(2)^{2} = 4\times1.88\times10^{18} = 7.52\times10^{8}$ Hz.

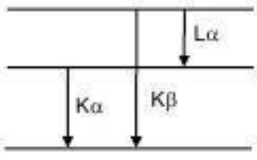
18. $K_\alpha = E_K - E_L$ ...(1) $\lambda K_\alpha = 0.71$ A°

$K_\beta = E_K - E_M$ ...(2) $\lambda K_\beta = 0.63$ A°

$L_\alpha = E_L - E_M$ ...(3)

Subtracting (2) from (1)

$K_\alpha - K_\beta = E_M - E_L = -L_\alpha$

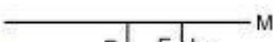


or, $L_\alpha = K_\beta - K_\alpha = \frac{3\times10^{8}}{0.63\times10^{-10}} - \frac{3\times10^{8}}{0.71\times10^{-10}}$

$= 4.761\times10^{18} - 4.225\times10^{18} = 0.536\times10^{18}$ Hz.

Again $\lambda = \frac{3\times10^{8}}{0.536\times10^{18}} = 5.6\times10^{-10} = 5.6$ A°.

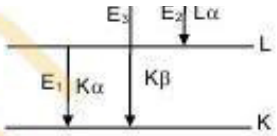
19. $E_1 = \frac{1242}{21.3\times10^{-3}} = 58.309\times10^{3}$ ev



$E_2 = \frac{1242}{141\times10^{-3}} = 8.8085 \times 10^3$ ev

$E_3 = E_1 + E_2 \Rightarrow (58.309 + 8.809)$ ev $= 67.118 \times 10^3$ ev

$\lambda = \frac{hc}{E_3} = \frac{1242}{67.118\times10^3} = 18.5 \times 10^{-3}$ nm $= 18.5$ pm.



20. $E_K = 25.31$ KeV, $E_L = 3.56$ KeV, $E_M = 0.530$ KeV

$K_\alpha = E_K - K_L = hv$

$\Rightarrow v = \frac{E_K - E_L}{h} = \frac{25.31-3.56}{4.14\times10^{-15}}\times10^3 = 5.25 \times 10^{15}$ Hz

$K_\beta = E_K - K_M = hv$

$\Rightarrow v = \frac{E_K - E_M}{h} = \frac{25.31-0.53}{4.14\times10^{-15}}\times10^3 = 5.985 \times 10^{18}$ Hz.

21. Let for, k series emission the potential required = v

$\therefore$ Energy of electrons = ev

This amount of energy ev = energy of L shell

The maximum potential difference that can be applied without emitting any electron is 11.3 ev.

22. V = 40 KV, i = 10 mA

1% of T_{KE} (Total Kinetic Energy) = X ray

$i = ne$ or $n = \frac{10^{-2}}{1.6\times10^{-19}} = 0.625 \times 10^{17}$ no.of electrons.

KE of one electron = eV $= 1.6 \times 10^{-19} \times 40 \times 10^3 = 6.4 \times 10^{-15}$ J

$T_{KE} = 0.625 \times 6.4 \times 10^{17} \times 10^{-15} = 4 \times 10^2$ J.

a) Power emitted in X-ray $= 4 \times 10^2 \times (-1/100) = 4w$

b) Heat produced in target per second $= 400 - 4 = 396$ J.

23. Heat produced/sec = 200 w

$\Rightarrow \frac{neV}{t} = 200 \Rightarrow (ne/t)V = 200$

$\Rightarrow i = 200/V = 10$ mA.

24. Given : $v = (25 \times 10^{14}\ Hz)(Z-1)^2$

Or $C/\lambda = 25 \times 10^{14}(Z-1)^2$

a) $\frac{3\times10^8}{78.9\times10^{-12}\times25\times10^{14}} = (Z-1)^2$

or, $(Z-1)^2 = 0.001520 \times 10^6 = 1520$

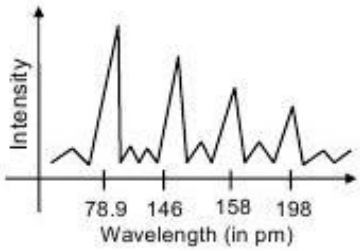
$\Rightarrow Z - 1 = 38.98$ or $Z = 39.98 = 40$. It is (Zr)

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b) $\frac{3\times10^8}{146\times10^{-12}\times25\times10^{14}} = (Z-1)^2$

or, $(Z-1)^2 = 0.0008219 \times 10^6$

$\Rightarrow Z - 1 = 28.669$ or $Z = 29.669 = 30$. It is (Zn).



c) $\frac{3\times10^8}{158\times10^{-12}\times25\times10^{14}} = (Z-1)^2$

or, $(Z-1)^2 = 0.0007594 \times 10^6$

$\Rightarrow Z - 1 = 27.5589$ or $Z = 28.5589 = 29$. It is (Cu).

d) $\frac{3\times10^8}{198\times10^{-12}\times25\times10^{14}} = (Z-1)^2$

or, $(Z-1)^2 = 0.000606 \times 10^6$

$\Rightarrow Z - 1 = 24.6182$ or $Z = 25.6182 = 26$. It is (Fe).

25. Here energy of photon = E

$E = 6.4$ KeV $= 6.4 \times 10^3$ ev

Momentum of Photon = E/C $= \frac{6.4\times10^3}{3\times10^8} = 3.41 \times 10^{-24}$ m/sec.

According to collision theory of momentum of photon = momentum of atom

$\therefore$ Momentum of Atom = P $= 3.41 \times 10^{-24}$ m/sec

$\therefore$ Recoil K.E. of atom = $P^2/2m$

$\Rightarrow \frac{(3.41\times10^{-24})^2 eV}{(2)(9.3\times10^{-26}\times1.6\times10^{-19})} = 3.9$ eV [1 Joule = 1.6×10^{-19} ev]

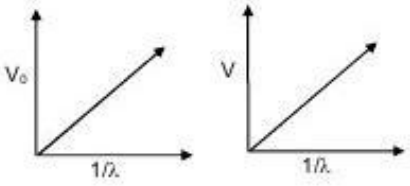
26. $V_0 \rightarrow$ Stopping Potential, $\lambda \rightarrow$ Wavelength, $eV_0 = hv - hv_0$

$eV_0 = hc/\lambda \Rightarrow V_0\lambda = hc/e$

$V \rightarrow$ Potential difference across X-ray tube, $\lambda \rightarrow$ Cut of wavelength

$\lambda = hc / eV$ or $V\lambda = hc / e$

Slopes are same i.e. $V_0\lambda = V\lambda$



$$\frac{hc}{e} = \frac{6.63\times10^{-34}\times3\times10^{8}}{1.6\times10^{-19}} = 1.242 \times 10^{-6}\ Vm$$

27. $\lambda = 10$ pm $= 100 \times 10^{-12}$ m

$D = 40$ cm $= 40 \times 10^{-2}$ m

$\beta = 0.1$ mm $= 0.1 \times 10^{-3}$ m

$$\beta = \frac{\lambda D}{d}$$

$$\Rightarrow d = \frac{\lambda D}{\beta} = \frac{100\times10^{-12}\times40\times10^{-2}}{10^{-3}\times0.1} = 4 \times 10^{-7}\ m.$$