STD 10. MATHEMATICS .-)

TIME: 3 hrs. MARKS: 100

Note:-(1) There are FIVE QUESTIONS in all, ALL are compulsory.
(2) Draw figures wherever necessary.
(3) Figures to the right indicate total marks of the questions

1A. State Pythagoras theorem and prove its converse. [4]

B. Answer any TWO of the following:- [6]

(1) Factorise: $(a+b)^2(a-b)+(b+c)^2(b-c)+(c+a)^2(c-a)$

(2) Factorise: $2ab(a+2b)+2bc(b+2c)+2ca(c+2a)+9abc$.

(3) Simplify: $\frac{x^3+27}{x^2-2x-15} \div \left[\frac{x^2-9}{x^3-27} \div \frac{x^2-25}{x^4+9x^2+81}\right]$

C. Factorise any TWO of the following:- [4]

(1) $x^3-19x-30$

(2) $9x^2(3x-1)-16y^2(4y-1)$

(3) $(x^2-6x-9)(x^2-6x+5)-576$

D. Solve any ONE of the following:- [2]

(1) Simplify: $\frac{1}{x+4}-\frac{1}{4-x}+\frac{8}{x^2-16}$

(2) If $a^3+b^3 : a^3-b^3 = 91:37$, find $a:b$.

E. Fill in the blanks, by choosing the correct answer from the brackets, so as to make them true: [4]

(1) The range of the function, $f:\{4,16,20,40\} \to N$; $f(x) =$ (g.c.f of x and 8) is _____.
$[\{1,2,4,8\}\ ;\ \{4n/n \in N\}\ ;\ \{4,8\}]$

(2) If $f: R^- \to R$, $f(x) = \frac{1}{2}\log_9 x$, then $f(3) =$ _____.
$[1, \frac{1}{2}, \frac{1}{4}]$

(3) Memory unit is a part of _____. [OPD, IPD, CPU]

(4) In a flow chart, _____ denotes the start.
[⬭, ▭, ◇]

2 A. Prove that chords equidistant from the centre of a circle are congruent. [4]

B. Solve any TWO of the following :- [6]

(1) If $a+b=1$, $c+d=1$, $a-b=\frac{d}{c}$, prove that $c-d=\frac{b}{a}$.

(2) The expense in a hostel is partly constant and partly varies as the number of hostelites. When the number of hostelites is 120, the expense is Rs 20,000 and when the hostelites are 100, the expense is Rs 17,000. Find the number of hostelites when the expense is Rs 18,800.

(3) If $\frac{a+b}{c}=\frac{b+c}{a}=\frac{c+a}{b}$ find the possible values of each ratio.

C. Solve any TWO of the following :- [4]

(1) Some observations are $\frac{x}{4}, \frac{x}{2}, \frac{x}{8}, x$ and $2x$. If median $= 12$, find the mean of the data.

(2) Observations of an ungrouped data in ascending order are 12, 16, m, 24, 34 and 36. If the median is 21, find the value of m.

(3) For a data $\bar{x}-M=2$ and $\bar{x}+M=20$ find the mode.

D. Solve any ONE :- [2]

(1) If $A=\{30°, 45°, 60°\}$; $B=R$, find the range of the function for $f: A \to B$, $f(x)=\sqrt{1-\cos^2 x}$.

(2) For $f(x)=x^2-2x+1$, find $\frac{f(1)+f(2)}{f(3)}$.

E. Fill in the blanks by choosing the correct answer from the brackets :- [4]

(1) If $\frac{ax-by}{bx-ay}=1$; $a-b\neq 0$ then $x=$ ____.

2.E (2) If $x = 4$, then $9x^2 - 3x + 1 - \frac{27x^3}{3x+1} =$ _____

$[\frac{1}{13}; \frac{1}{31}; \frac{27}{13}]$

(3) If $\sin\theta = \frac{1}{3}$, then $\frac{3\operatorname{Cosec}\theta}{1+\operatorname{Cot}^2\theta} =$ ..... $[1, 3, \frac{10}{9}]$

(4) If $xy - 2 + \frac{1}{xy} = 0$, $\therefore x \alpha$ ..... $[y, \frac{1}{y}, \frac{1}{\sqrt{y}}]$

3A. Prove that the tangent of a circle is perpendicular to the radius drawn through the point of contact. [4]

B. Solve any TWO of the following:- [6]

(1) When the angle of elevation of the sun increases from 30° to 60°, then the length of the shadow of the tower reduces by 30 metres. Find the height of the tower. ($\sqrt{3} = 1.7$).

(2) The radius of a circle is 28 cms and the measure of the angle subtended at the centre by an arc ~~chord~~ is $\frac{90}{11}$. Find the area of minor sector and major sector.

(3) A gold biscuit is 12 cm long, 5.5 cm broad and 4.5 cm thick. It is melted and a wire of 0.6 cm diameter is drawn from it. Find the length of the wire.

C. Solve any TWO of the following:- [4]

(1) Prove that $(\cos\theta + \sin\theta)^2 - (\cos\theta - \sin\theta)^2 = \frac{4}{\operatorname{Cosec}\theta \cdot \sec\theta}$

(2) Evaluate: $2\sin^2 60 + \sec^2 60 - 2\cos^4 45 - 3\operatorname{Cosec}^2 60$

(3) Prove that:- $\frac{\tan^2\theta + \operatorname{Cot}^2\theta}{\tan^2\theta - \operatorname{Cot}^2\theta} = \frac{\sin^4\theta + \cos^4\theta}{\sin^2\theta - \cos^2\theta}$.

D. Solve any ONE :- [2]

(1) If $x - y$ varies directly as x and $x + y$ varies inversely as $\frac{x}{y}$; $x \neq 0$, $y \neq 0$,

prove that $x^2 - y^2$ is a constant.

(2) If $\tan\theta - \cot\theta = \operatorname{cosec}\theta$, $0° < \theta < 90°$, find θ.

E. Answer the following:- [4]

(1) If α and β are the roots of the equation $3x^2 - 16x + 5 = 0$ then find the value of $\alpha^2\beta + \alpha\beta^2$.

(2) Define:- adjacent line segment

(3) Define: transversal.

(4) If $\frac{x^2+x}{3} - 2 = 0$ and $x \in N$, then find the value of x.

Q4A In ΔPQR, $\angle Q$ is a right angle, $N \in \overline{PQ}$, $M \in \overline{QR}$ then prove that $PM^2 + RN^2 = PR^2 + MN^2$. [4]

B. Solve any TWO of the following:- [6]

(1) Find the missing frequencies, if a dice is thrown 50 times, in the following data:-

Number	1	2	3	4	5	6	Mean is
Frequency	8	-	9	-	10	4	3.42.

(2) Frequency distribution of salary of workers in a factory is given below.

Salary (Rs)	500-599	600-699	700-799	800-899	900-999
Frequency	31	38	48	50	33

Find the median of the above data

(3) Simplify: $\left(\frac{a^2-b^2}{a^3-b^3}\right)\left(a + \frac{b^2}{a+b}\right) - \left(\frac{a^2-b^2}{a^3+b^3}\right)\left(b - \frac{a^2}{a-b}\right)$

C Answer any TWO of the following:- [4]

(1) $\overleftrightarrow{QE}$ touches $\odot(O, r)$ at A. $\overline{AB}$ is a diameter. A secant $\overleftrightarrow{BC}$ intersects the circle in D and the tangent in C. If $BD = 16$; $CD = 9$, find 'r'.

(2) In a circle $\overline{AB}$ is a diameter, $\overline{AC}$ is a chord and $\overleftrightarrow{CD}$ is a tangent such that $A - B - D$. If $m\angle BAC = 32$, find $m\angle BDC$

(3) If length of minor $\overset{\frown}{AB}$ of $\odot(P, 27)$ is 3π, find $m\angle APB$.

4D. Answer any ONE of the following:- [2]

(1) Solve in R : $\frac{x}{x-1} + \frac{x-1}{x} = \frac{25}{12}$ $[x \neq 0, 1]$

(2) If α and β are the roots of the equation : $x^2 - p(x+1) - c = 0$ then find $(\alpha+1)(\beta+1)$

E. Answer the following :- [4]

(1) Define : Concentric circles

(2) Define : Angle in a segment.

(3) The curved surface area and volume of a sphere are equal. Find the radius.

(4) Total surface area of a cube = _____

5A $\overline{MN}$ is given. Construct a right angled $\triangle ABC$, such that length of hypotenuse $\overline{AC}$ is 5MN and $AB = \frac{5}{2}MN$. Write the steps. [4]

B. Answer any TWO of the following [6]

(1) If $x^2y^2 + 4\alpha xy$, then prove that $x \propto \frac{1}{y}$ $(xy \neq \pm 2)$.

(2) Due to some Technical fault, the speed of an electric train is reduced by 10 km.p.h. The train takes 48 minutes more to cover a distance of 240 km. Find its usual speed.

(3) Find the roots of the quadratic equation : $ax^2 + bx + c = 0$, then find the difference of the roots.

C. Answer any TWO of the following :- [4]

(1) In $\triangle ABC$, G is the centroid ; AB = 7, AC = 11, AG = 4. Find BC.

(2) In $\triangle XYZ$, $\overrightarrow{YP}$ bisects $\angle Y$ and intersects $\overline{XZ}$ in P. If XZ = 7.1 then find XP.

(If $YZ = 1\frac{1}{2} XY$,

(3) $\triangle ABC \sim \triangle PQR$, $ABC = 54$ units. If $BC = 3$ units and $QR = 4$ units then find area of $\triangle PQR$.

D. Answer any one of the following:- [2]

(1) In $\triangle ABC$, $m\angle B = 90$, $\overline{BM} \perp \overline{AC}$, $M \in \overline{AC}$. If $BM = \sqrt{12}$ and $AC = 8$, then find the possible values of AM.

(2) A wire of radius 4mm is drawn by melting a sphere of radius 6cm. Find the length of the wire.

~~E Answer the following:- [4]~~

~~(1) Find the~~

E :- In a circle with centre O, the tangents at A and B intersect each other at P. Prove that $\square AOBP$ is a cyclic quadrilateral [4]

OR

E Answer the following:- [4]

(1) $\angle ABC$ is inscribed in a semicircle with radius r. If $\triangle ABC$ is an isosceles $\triangle$ with $AB = 8$ find 'r'.

(2) The diameter of $\odot(P, r)$ is 13. If the distance of chord $\overline{AB}$ from its centre is 2.5. then find AB.

(3) $\overline{AB}$ and $\overline{CD}$ are distinct chords of $\odot(P, r)$. M and N are the feet of perpendiculars from P on $\overline{AB}$ and $\overline{CD}$ respectively. If $AB = 7.2$, $PM = 4.8$, $PN = 3.6$, find CD.

(4) Tangents drawn from A, lying in the exterior of $\odot(P, 8)$ touch the circle at B and C. If $AP = 17$, find AC.

1. A. Pythagoras theorem :-

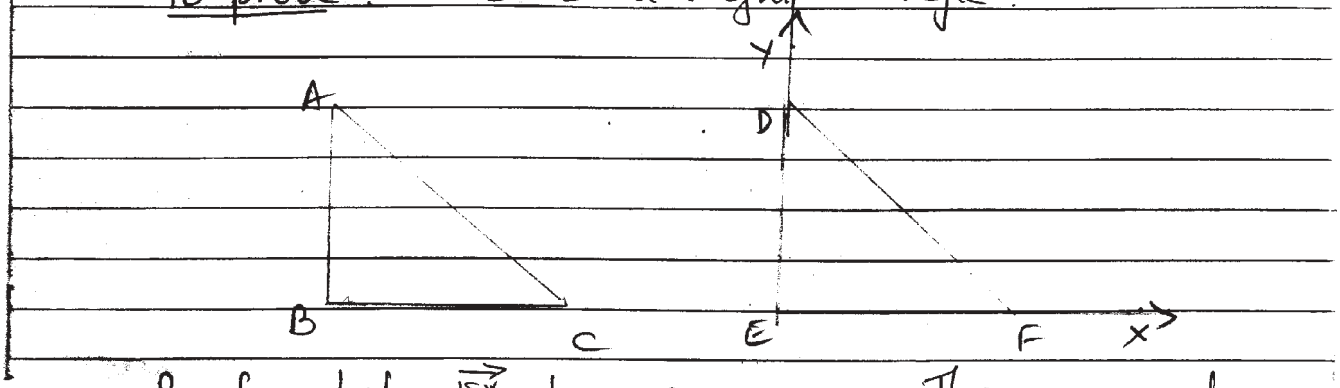
In a right angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the remaining two sides.

Converse of Pythagoras theorem.

In $\triangle ABC$, if $AC^2 = AB^2 + BC^2$, prove that $\angle B$ is a right angle.

Given :- In $\triangle ABC$, $AC^2 = AB^2 + BC^2$

To prove : $\angle B$ is a right angle.



Proof :- Let $\overrightarrow{EX}$ be any ray. Then we get $\overrightarrow{EY}$ perpendicular to it.

According to point plotting theorem, we get point F on $\overrightarrow{EX}$ such that $EF = BC$ and D on $\overrightarrow{EY}$ such that $ED = AB$.

In $\triangle DEF$, $\angle E$ being a right angle, by Pythagoras theorem,

$DF^2 = DE^2 + EF^2$

$= AB^2 + BC^2$ [$\because DE = AB$ & $EF = BC$]

$= AC^2$ [given]

$\therefore DF = AC$.

In $\triangle ABC$ and $\triangle DEF$, $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$ and $\overline{AC} \cong \overline{DF}$,

$\therefore \triangle ABC \leftrightarrow DEF$ is congruence [SSS Theorem]

$\therefore m\angle B = m\angle E$

But $m\angle E = 90$ $\therefore m\angle B = 90$

ie $\angle B$ is a right angle.

1 B.(i) $(a+b)^2(a-b) + (b+c)(b-c) + (c+a)(c-a)$

$= (a+b)(a^2-b^2) + (b+c)(b^2-c^2) + (c+a)(c^2-a^2)$

$= a^3 - ab^2 + a^2b - b^3 + b^3 - bc^2 + b^2c - c^3 + c^3 - a^2c + ac^2 - a^3$

$= a^2b - a^2c - ab^2 + ac^2 + b^2c - bc^2$

$= a^2(b-c) - a(b^2-c^2) + bc(b-c)$

$= a^2(b-c) - a(b-c)(b+c) + bc(b-c)$

$= (b-c)[a^2 - ab - ac + bc]$

$= (b-c)[a(a-b) - c(a-b)]$

$= (b-c)(a-b)(a-c)$

$= -(a-b)(b-c)(c-a)$ Ans

(ii) $2ab(a+2b) + 2bc(b+2c) + 2ca(c+2a) + 9abc$

$= 2a^2b + 4ab^2 + 2b^2c + 4bc^2 + 2ac^2 + 4a^2c + 9abc$

$= 2a^2b + 4a^2c + 4ab^2 + 9abc + 2ac^2 + 2b^2c + 4bc^2$

$= 2a^2(b+2c) + a(4b^2 + 9bc + 2c^2) + 2bc(b+2c)$

$= 2a^2(b+2c) + a(b+2c)(4b+c) + 2bc(b+2c)$

$= (b+2c)[2a^2 + 4ab + ac + 2bc]$

$= (b+2c)[2a+c)(a+2b)$

$= (a+2b)(b+2c)(c+2a)$ Ans

(3) $\dfrac{x^3+27}{x^2-2x-15} \div \left[\dfrac{x^2-9}{x^3-27} \div \dfrac{x^2-25}{x^4+9x^2+81}\right]$

$= \dfrac{(x+3)(x^2-3x+9)}{(x-5)(x+3)} \div \left[\dfrac{(x-3)(x+3)}{(x-3)(x^2+3x+9)} \times \dfrac{(x^2+3x+9)(x^2-3x+9)}{(x-5)(x+5)}\right]$

$= \dfrac{x^2-3x+9}{x-5} \times \dfrac{(x-5)(x+5)}{(x+3)(x^2-3x+9)}$

$= \boxed{\dfrac{x+5}{x+3}}$ Ans

1 C(i) $x^3 - 19x - 30$

$p(-2) = (-2)^3 - 19(-2) - 30 = 0$

$\therefore (x+2)$ is a factor

$\therefore p(x) = x^3 + 2x^2 - 2x^2 - 4x - 15x - 30$

$= x^2(x+2) - 2x(x+2) - 15(x+2)$

$= (x+2)(x^2 - 2x - 15)$

$= (x+2)[x^2 - 5x + 3x - 15]$

$= \underline{(x+2)(x-5)(x+3)}$ Ans.

(2) $9x^2(3x-1) - 16y^2(4y-1)$

$= 27x^3 - 9x^2 - 64y^3 + 16y^2$

$= 27x^3 - 64y^3 - 9x^2 + 16y^2$

$= (3x-4y)(9x^2 + 12xy + 16y^2) - (3x-4y)(3x+4y)$

$= \underline{(3x-4y)(9x^2 + 12xy + 16y^2 - 3x - 4y)}$ Ans.

(3) $(x^2-6x-9)(x^2-6x+5) - 576$.

Take $x^2 - 6x = a$

$\therefore p(x) = (a-9)(a+5) - 576$

$= a^2 - 4a - 45 - 576$

$= a^2 - 4a - 621$

$= a^2 - 4a + 4 - 625$

$= (a-2)^2 - (25)^2$

$= (a-2-25)(a-2+25)$

$= (a-27)(a+23)$

$= (x^2-6x-27)(x^2-6x+23)$ [putting $a = x^2-6x$]

$= [x^2 - 9x + 3x - 27](x^2-6x+23)$

$= \underline{(x-9)(x+3)(x^2-6x+23)}$ Ans.

1D(ii) $\frac{1}{x+4} - \frac{1}{4-x} + \frac{8}{x^2-16}$

$= \frac{1}{x+4} + \frac{1}{x-4} + \frac{8}{(x-4)(x+4)}$

$= \frac{x-4+x+4+8}{(x-4)(x+4)}$

$= \frac{2(x+4)}{(x-4)(x+4)}$

$= \boxed{\frac{2}{x-4}}$ Ans.

1. D (2) $a^3+b^3 : a^3-b^3 = 91 : 37$. $\therefore a : b = ?$

$$\frac{a^3+b^3}{a^3-b^3} = \frac{91}{37}$$

$$\therefore \frac{a^3+b^3+a^3-b^3}{a^3+b^3-a^3+b^3} = \frac{91+37}{91-37}$$ [Componendo & dividendo]

$$\therefore \frac{2a^3}{2b^3} = \frac{128}{54}$$

$$\therefore \frac{a^3}{b^3} = \frac{64}{27}$$

$$\therefore \frac{a}{b} = \frac{4}{3} \quad \therefore \boxed{a : b = 4 : 3}$$

1 E (1) The range of the function, $f : \{4, 16, 20, 40\} \to N$; $f(x) =$ (g.c.f of x and 8) is $\{4, 8\}$.

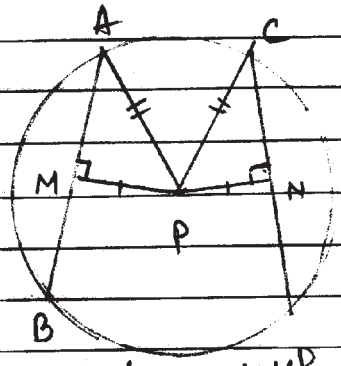
(2) If $R^- \to R$, $f(x) = \frac{1}{2}\log_9 x$, then $f(3) = -\frac{1}{4}$

(3) Memory unit is a part of C.P.U.

(4) In a flow chart, ⬭ denotes the start.

B.2 A. Given:- AB and CD are chords of circle with centre P. $\overline{PM} \perp \overline{AB}$ & $\overline{PN} \perp \overline{CD}$. M and N are the feet of the prependiculars from P on $\overline{AB}$ and $\overline{CD}$ respectively. Also PM = PN.

To prove: $\overline{AB} \cong \overline{CD}$.



Proof: $\overline{PM} \perp \overline{AB}$ and M is the foot of the perpendicular from P on $\overline{AB}$ (given)
∴ M is the midpoint of $\overline{AB}$
∴ AB = 2AM ------ (i)

$\overline{PN} \perp \overline{CD}$ and N is the foot of the perpendicular from P on $\overline{CD}$ (given)
∴ N is the midpoint of $\overline{CD}$
∴ CD = 2CN ------ (ii)

In △AMP and △CNP,
$\overline{AP} \cong \overline{CP}$ (radii of same circle)
∠AMP ≅ ∠CNP (right angles)
$\overline{PM} \cong \overline{PN}$ (given)
∴ AMP ⟷ CNP is congruence (RHS Theorem)
∴ $\overline{AM} \cong \overline{CN}$
∴ AM = CN ie 2AM = 2CN.
∴ AB = CD from (i) and (ii)
∴ $\overline{AB} \cong \overline{CD}$.

B(1) $a+b=1$; $c+d=1$; $a-b=\frac{d}{c}$ prove that $c-d=\frac{b}{a}$

$\frac{a-b}{1}=\frac{d}{c}$

∴ $\frac{a-b}{a+b}=\frac{d}{c}$ [∵ $a+b=1$]

∴ $\frac{a-b+a+b}{a-b-a-b}=\frac{d+c}{d-c}$ [componendo & dividendo

$\therefore \frac{2a}{-2b} = \frac{1}{d-c}$ [$\because c+d=1$]

$\therefore \frac{a}{-b} = \frac{1}{d-c}$

$\therefore \frac{a}{b} = \frac{1}{c-d}$

$\therefore \frac{b}{a} = c-d$ [Invertendo]

ie $c-d = \frac{b}{a}$.

(2) The expense in a hostel is partly constant and partly varies as the number of hostelites. When the number of hostelites is 120, expense is Rs 20,000. When the number of hostelites is 100, expense is Rs 17,000,

x = number of hostelites

y = ~~cost of~~ expense.

$\therefore y = a+bx$ (a & b are constants, $a \neq 0, b \neq 0$)

→ y = Rs 20,000, when $x = 120$.

$\therefore 20000 = a+120b$ ---- (1)

→ y = Rs 17000, when $x = 100$

$\therefore 17000 = a+100b$ ---- (2)

$20000 = a+120b$

$+3000 = +20b$

$\boxed{b = 150}$

putting the value of 'b' in (2)

$\therefore 17000 = a+15000$

$\therefore \boxed{a = 2000}$

$\therefore y = 2000+150x$.

→ when y = Rs 18,800, x = ?

$\therefore 18800 = 2000+150x$

$\therefore 150x = 16800$

$\therefore x = \frac{16800}{150} = 112$

∴ When the expense is Rs 18,800, the number of hostelites is 112.

(3) $\frac{a+b}{c} = \frac{b+c}{a} = \frac{c+a}{b}$ ∴ ER = ? If $a+b+c \neq 0$

∴ Each ratio $= \frac{a+b+b+c+c+a}{c+a+b}$

$= \frac{2(a+b+c)}{a+b+c}$

$= 2$ ---------- (1)

If $a+b+c = 0$, then $c = -a-b$

∴ $\frac{a+b}{c} = \frac{a+b}{-a-b} = \frac{a+b}{-(a+b)} = (-1)$ ---- (2)

∴ E.R = (-1) or 2

(1) Observations in ascending order are $\frac{x}{8}, \frac{x}{4}, \frac{x}{2}, x$ and $2x$

∴ n = 5 (odd) M = 12 $\bar{x}$ = ?

M = Value of $\left(\frac{n+1}{2}\right)^{th}$ observation

= Value of 3rd observation

∴ $M = 12 = \frac{x}{2}$

∴ $x = 24$

∴ Observations are 3, 6, 12, 24, 48

∴ $\bar{x} = \frac{\Sigma x_i}{n} = \frac{3+6+12+24+48}{5}$

$= \frac{93}{5} = 18.6$

$\bar{x} = 18.6$

(2) Observations in ascending order are :
12, 16, m, 24, 34 and 36
$n = 6$ (even) , $M = 21$, $m = ?$

$$M = \frac{\text{Value of } \left(\frac{n}{2}\right)^{th} \text{ obs} + \left(\frac{n}{2}+1\right)^{th} \text{ obs}}{2}$$

$$\therefore 21 = \frac{3^{rd} \text{ observation} + 4^{th} \text{ obs}}{2}$$

$$\therefore 21 = \frac{m+24}{2}$$

$$\therefore 42 = m + 24 \quad \therefore \boxed{m = 18}$$

(3) $\bar{X} - M = 2$, $Z = ?$
$\bar{X} + M = 20$
$\therefore 2\bar{X} = 22$
$\therefore \bar{X} = 11$ $\therefore M = 9$.

$\therefore Z = 3M - 2\bar{X}$
$= 3(9) - 2(11)$
$= 27 - 22$
$\therefore Z = 5$ $\boxed{Z = 5}$

D(1) $A = \{30°, 45°, 60°\}$, $B = R$, $f : A \to B$,
$f(x) = \sqrt{1 - \cos^2 x}$; $R_f = ?$

$f(x) = \sqrt{1 - \cos^2 x} = \sqrt{\sin^2 x} = \sin x$

$\therefore f(30°) = \sin 30° = \frac{1}{2}$

$f(45°) = \sin 45° = \frac{1}{\sqrt{2}}$

$f(60°) = \sin 60° = \frac{\sqrt{3}}{2}$

$\therefore \boxed{R_f = \left\{\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}\right\}}$ Ans.

2.D (2) $f(x) = x^2 - 2x + 1$ $\quad \frac{f(1)+f(2)}{f(3)} = ?$

$\therefore f(1) = (1)^2 - 2(1) + 1$

$= 0$

$f(2) = 2^2 - 2(2) + 1$

$= 1$

$f(3) = 3^2 - 2(3) + 1$

$= 4$

$\therefore \frac{f(1)+f(2)}{f(3)} = \frac{0+1}{4} = \boxed{\frac{1}{4}}$

E(1) If $\frac{ax-by}{bx-ay} = 1$; $a-b \neq 0$ then $\frac{x}{y} = (-1)$.

(2) If $x = 4$, then $9x^2 - 3x + 1 - \frac{27x^3}{3x+1} = \frac{1}{13}$.

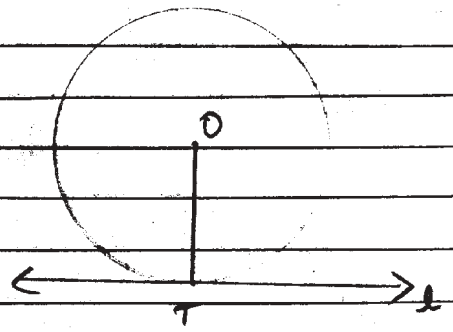
(3) If $\sin\theta = \frac{1}{3}$ then $\frac{3\,\mathrm{Cosec}\,\theta}{1+\cot^2\theta} = 1$.

(4) If $xy - 2 + \frac{1}{xy} = 0$ $\therefore x \propto \frac{1}{y}$.

.3 A. Given:- line l is a tangent of a circle with centre O. T is the point of contact. $\overline{OT}$ is the radius drawn through the point of contact.

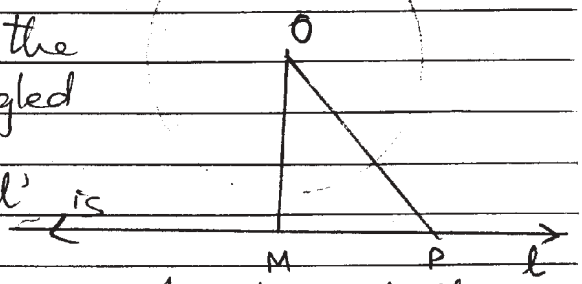
To prove: $\overline{OT} \perp l$

Proof : 'l' intersects the circle in only one point T, because it is a tangent of the circle. There are three alternatives for M the foot of the perpendicular from the centre 'O' of the circle to the line 'l'



(i) If M is in the exterior of the circle; then $OM > r$. For any other point P on line l, $OP > OM > r$, as $\overline{OP}$ is the hypotenuse of right angled ΔOMP.

∴ Each point of line 'l' is in the exterior of the circle. Thus the line does not intersect the circle. ∴ this alternative is not possible.



(ii) If M is in the interior of the circle. Suppose $OM = p$, $p < r$

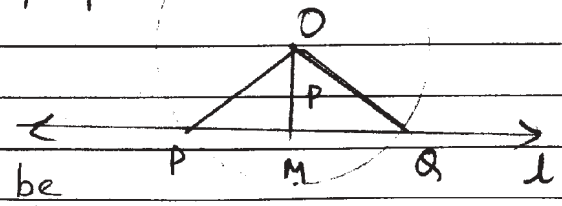
∴ $r^2 - p^2$ is a positive real number.

∴ On line 'l', two points P and Q can be obtained at a distance $\sqrt{r^2 - p^2}$ from M

Moreover $OP^2 = OQ^2 = OM^2 + PM^2 = p^2 + r^2 - p^2 = r^2$

∴ $OP = OQ = r$

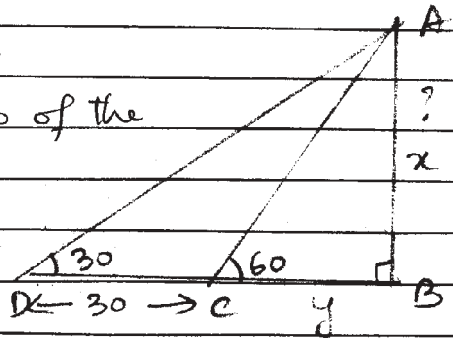
Thus the line l intersects the circle in two



distinct points P and Q.
But line l intersects the circle in only one point, so this alternative is also not possible.

(iii) If M is on the circle.
But Line l intersects the circle in only one point T.
$\therefore T = M$
$\therefore \overline{OM} \perp l$ so $\overline{OT} \perp l$.

B (1) When the angle of elevation of the sun increases from 30° to 60°, the length of the shadow of the tower reduces by 30 metres. ($\sqrt{3} = 1.7$)



AB = tower = x m.
BC = y m } shadow of the
BD = y+30 m } tower
$m\angle ADB = 30°$
$m\angle ACB = 60°$

In $\triangle ABD$, $\tan 30 = \frac{AB}{BD} = \frac{x}{y+30}$

$\therefore \frac{1}{\sqrt{3}} = \frac{x}{y+30}$

$\therefore y+30 = \sqrt{3}x$

$\therefore y = \sqrt{3}x - 30$ ——— (1)

In $\triangle ABC$, $\tan 60 = \frac{AB}{BC} = \frac{x}{y}$

$\therefore \sqrt{3} = \frac{x}{y}$

$\therefore y = \frac{x}{\sqrt{3}}$ ——— (2)

From (1) and (2) $\sqrt{3}x - 30 = \frac{x}{\sqrt{3}}$

$\therefore 3x - 30\sqrt{3} = x$

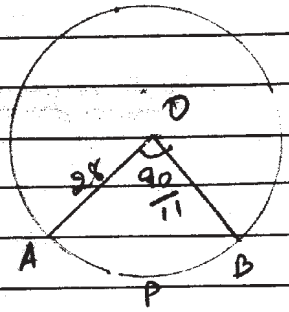
$\therefore 2x = 30\sqrt{3} = 30 \times 1.7$

$\therefore x = \frac{30 \times 1.7}{2} = 15 \times 1.7$

$= 25.5$ metres.

∴ The height of the tower is 25.5 metres.

(2) Radius of a circle is 28 cm. $\theta = \frac{90}{11}$.



→ Area of minor sector $OAPB = \frac{\pi r^2 \theta}{360}$

$= \frac{\cancel{22}}{\cancel{7}} \times \frac{\cancel{28} \times 28 \times \cancel{90}}{\cancel{360}} \times \frac{}{\cancel{11}} = 56$ sq.cm

→ Area of circle $= \pi r^2 = \frac{22}{7} \times \cancel{28} \times 28$

$= 2464$ sq.cm.

→ ∴ Area of major sector = Area of circle − Area of minor sector

$= (2464 - 56)$ sq.cm.

$= 2408$ sq.cm

Area of minor sector = 56 sq.cm
Area of major sector = 2408 sq.cm.

(3) Gold biscuit (cuboid)
$l = 12$ cm, $b = 5.5$ cm, $h = 4.5$ cm.

∴ Volume of gold biscuit $= l \times b \times h$

$= 12 \times \frac{55}{10} \times \frac{45}{10}$ cu.cm

Wire (cylinder)

diameter = 0.6 cm , $\therefore$ r = 0.3 cm. , h = ?

Volume of the wire = $\pi r^2 h$

$= \frac{22}{7} \times \frac{3}{10} \times \frac{3}{10} \times h$

$\therefore h = \frac{\cancel{12}^{6} \times \cancel{55}^{5}}{\cancel{10}} \times \frac{\cancel{45}^{5}}{\cancel{10}} \times \frac{7 \times 10 \times 10}{\cancel{22} \times \cancel{3} \times \cancel{3}}$

$= 1050$ cm.

Length of the wire = 1050 cm or 10.50m.

C(1) Prove that $(\cos\theta + \sin\theta)^2 - (\cos\theta - \sin\theta)^2 = \frac{4}{\operatorname{Cosec}\theta \cdot \sec\theta}$

LHS = $(\cos\theta + \sin\theta)^2 - (\cos\theta - \sin\theta)^2$

$= (\cos\theta + \sin\theta - \cos\theta + \sin\theta)(\cos\theta + \sin\theta + \cos\theta - \sin\theta)$

$= (2\sin\theta)(2\cos\theta)$

$= 4 \sin\theta \cdot \cos\theta$

$= \frac{4}{\operatorname{Cosec}\theta \cdot \sec\theta}$

$=$ RHS.

(2) $2\sin^2 60 + \sec^2 60 - 2\cos^4 45 - 3\operatorname{Cosec}^2 60$

$= 2\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{2}{1}\right)^2 - 2\left(\frac{1}{\sqrt{2}}\right)^4 - 3\left(\frac{2}{\sqrt{3}}\right)^2$

$= \cancel{2}\left(\frac{3}{\cancel{4}}\right)_{2} + 4 - \cancel{2}\left(\frac{1}{\cancel{4}}\right)_{2} - \cancel{3}\left(\frac{4}{\cancel{3}}\right)$

$= \frac{3}{2} + 4 - \frac{1}{2} - 4$

$= \frac{2}{2} = \boxed{1}$.

(3) Prove that $\frac{\tan^2\theta + \cot^2\theta}{\tan^2\theta - \cot^2\theta} = \frac{\sin^4\theta + \cot^4\theta}{\sin^2\theta - \cot^2\theta}$

$$\text{LHS} = \frac{\tan\theta + \cot^2\theta}{\tan^2\theta - \cot^2\theta}$$

$$= \frac{\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\sin^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta} - \frac{\cos^2\theta}{\sin^2\theta}}$$

$$= \frac{\sin^4\theta + \cos^4\theta}{\sin^2\theta \cdot \cos^2\theta} \times \frac{\sin^2\theta \cdot \cos^2\theta}{\sin^4\theta - \cos^4\theta}$$

$$= \frac{\sin^4\theta + \cos^4\theta}{(\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta)}$$

$$= \frac{\sin^4\theta + \cos^4\theta}{\sin^2\theta - \cos^2\theta} \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \text{RHS}$$

3 D (1) $x - y \propto \frac{x}{y}$; $x + y \propto \frac{1}{x/y}$ $[x \neq 0, y \neq 0]$

$\therefore x - y = k\left(\frac{x}{y}\right)$; $x + y = m\left(\frac{y}{x}\right)$ [k, m are constants, $k, m \neq 0$]

$\therefore (x-y)(x+y) = k\left(\frac{x}{y}\right) m\left(\frac{y}{x}\right)$

$\therefore x^2 - y^2 = km$

ie $x^2 - y^2 =$ constant.

(2) $\tan\theta - \cot\theta = \operatorname{cosec}\theta$, $0^\circ < \theta < 90^\circ$ $\therefore \theta = ?$

Squaring both sides:-

$\tan^2\theta - 2 + \cot^2\theta = \operatorname{cosec}^2\theta$

$\therefore \tan^2\theta = \operatorname{cosec}^2 - \cot^2\theta + 2$

$= 1 + 2$ $[\because \operatorname{cosec}^2\theta - \cot^2\theta = 1]$

$\therefore \tan^2\theta = 3$

$\therefore \tan\theta = \sqrt{3}$ $\therefore \boxed{\theta = 60^\circ}$

Q.5.(1) α and β are the roots of the equation $3x^2 - 16x + 5 = 0$ $\therefore \alpha^2\beta + \alpha\beta^2 = ?$

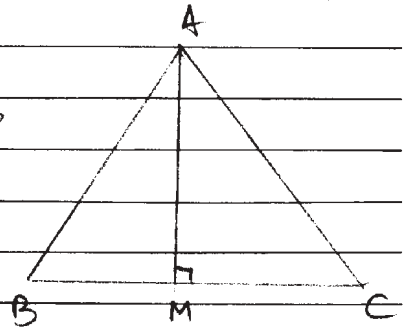
$a = 3, b = -16, c = 5$

$\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta)$

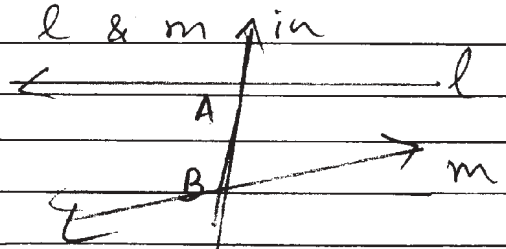
$\rightarrow \alpha + \beta = \frac{-b}{a} = \frac{16}{3}$ $\alpha\beta = \frac{c}{a} = \frac{5}{3}$

$\therefore \alpha^3\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta) = \left(\frac{16}{3}\right)\left(\frac{5}{3}\right) = \boxed{\frac{80}{9}}$

(2) Adjacent line segment.
If $\overline{AM}$ is an altitude of ΔABC and M is the foot of the perpendicular, $\angle B$ and $\angle C$ are acute angles, then B-M-C $\overline{BM}$ is called the ~~adjacent~~ adjacent line segment of $\overline{AB}$ and $\overline{CM}$ is the adjacent line segment of $\overline{BC}$.



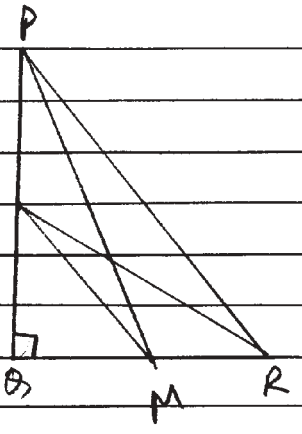
(3) Transversal:- If a line 't' intersects two coplanar lines l & m in distinct points then 't' is the transversal of l and m



(4) $\frac{x^2 + x}{3} - 2 = 0$, $x \in N$ $\therefore x = ?$

$\therefore x^2 + x - 6 = 0$
$\therefore (x + 3)(x - 2) = 0$
$\therefore x = -3$ or $x = 2$
But $x \in N$ $\therefore \boxed{x = 2}$

4A. Given:- In $\triangle PQR$, $\angle Q$ is a right angle.
$N \in \overline{PQ}$, $M \in \overline{QR}$.
To prove: $PM^2 + RN^2 = PR^2 + MN^2$.



Proof:- In $\triangle PQR$, $\angle Q$ is a right angle (given)

$\therefore$ $\triangle PQR$, $\triangle PQM$, $\triangle NQR$ and $\triangle NQM$ are right angled triangles.

$\therefore$ By Pythagoras theorem

In $\triangle PQR$, $PQ^2 + QR^2 = PR^2$ ---(1)

$\triangle PQM$, $PQ^2 + QM^2 = PM^2$ ---(2)

$\triangle NQR$, $NQ^2 + QR^2 = RN^2$ ---(3)

$\triangle NQM$, $NQ^2 + QM^2 = NM^2$ ---(4)

Adding (2) and (3) we get

$PM^2 + RN^2 = PQ^2 + QM^2 + NQ^2 + QR^2$

$= (PQ^2 + QR^2) + (QM^2 + NQ^2)$

$= PR^2 + MN^2$ from (1) & (4)

$\therefore$ $PM^2 + RN^2 = PR^2 + MN^2$.

B(1)

Number(x_i)	Frequencies(f_i)	$f_i x_i$
1	8	8
2	x	$2x$
3	9	27
4	y	$4y$
5	10	50
6	4	24
	$n = 50 - 31 + x + y$	$\Sigma f_i x_i = 109 + 2x + 4y$

$\bar{X} = 3.42$.

$\bar{X} = \frac{\Sigma f x_i}{n}$

$31 + x + y = 50$

$\therefore x + y = 19$ ---(1)

$\therefore \quad 3.42 = \frac{109 + 2x + 4y}{50}$

$\therefore \quad 171.00 = 109 + 2x + 4y$

$\therefore \quad 2x + 4y = 62 \quad \text{——— (2)}$

$x + y = 19 \quad \text{——— (1)} \times 2$

$\therefore \quad 2x + 2y = 38$

$2x + 4y = 62$

$-2y = -24$

$\therefore \quad y = 12 \qquad \therefore \quad x = 7$

Missing frequencies are 7 and 12.

(2)

Salary in Rs.	Lower boundary points	Freq. (f_i)	C.F.
500 – 599	499.5	31	31
600 – 699	599.5	38	69 F
700 – 799	699.5 = L	48 f	117
800 – 899	799.5	50	167
900 – 999	899.5	33	200
		n = 200	

$M = L + \left(\frac{\frac{n}{2} - F}{f}\right) \times C$

$L = 699.5, \quad \frac{n}{2} = 100$

$F = 69$

$f = 48; \quad C = 100.$

$\therefore \quad M = 699.5 + \left(\frac{100 - 69}{48}\right) 100$

$= 699.5 + \frac{3100}{48}$

$= 699.5 + 64.58$

$= 764.08$

$\boxed{M = 764.08}$

4 B (3) $\left(\frac{a^2-b^2}{a^3-b^3}\right)\left(a+\frac{b^2}{a+b}\right)-\left(\frac{a^2-b^2}{a^3+b^3}\right)\left(b-\frac{a^2}{a-b}\right)$

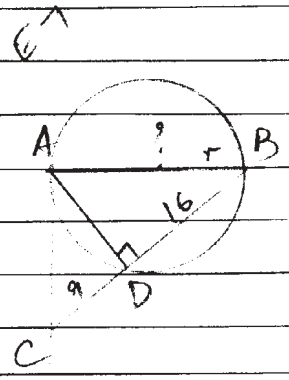
$= \frac{(a-b)(a+b)}{(a-b)(a^2+ab+b^2)} \cdot \frac{(a^2+ab+b^2)}{(a+b)} - \frac{(a+b)(a+b)}{(a+b)(a^2-ab+b^2)} \cdot \frac{(ab-b^2-a^2)}{(a-b)}$

$= 1 - \frac{(ab-b^2-a^2)}{(a^2-ab+b^2)}$

$= 1 + \frac{(a^2-ab+b^2)}{(a^2-ab+b^2)}$

$= 1 + 1 = \boxed{2}$

C (1) $\overleftrightarrow{DE}$ touches $\odot(O,r)$ at A, $\overline{AB}$ is a diameter. Secant $\overleftrightarrow{BC}$ intersects $\odot$ in D and tangent in C. BD = 16, CD = 9, r = ?



$\angle ADB$ = right angle (angle in a semicircle)

$\overline{AB} \perp \overline{AC}$ (Theorem)

∴ In right angled $\triangle CAB$, $\overline{AD} \perp \overline{BC}$

∴ $AB^2 = BD \cdot BC$ (Geometric mean Theorem)

$= 16 \times 25$

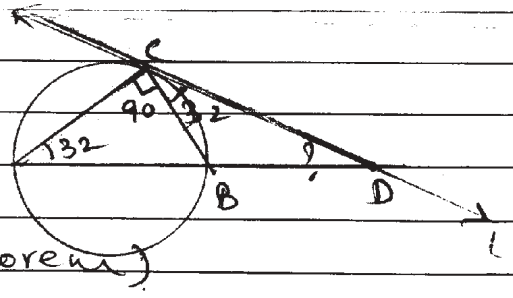
∴ $AB = 4 \times 5 = \boxed{20 = d.}$

∴ $\boxed{\text{radius} = 10}$

(2) $\overline{AB}$ is a diameter in a circle. $\overline{AC}$ is a chord and $\overleftrightarrow{CD}$ is a Tangent. A-B-D

$m\angle BAC = 32$; $m\angle BDC = ?$

$m\angle BAC = 32$ (given)



$\therefore m\angle BCD = 32$
(alternate Segment theorem)

$m\angle ACB = 90$ (angle in a semicircle)

$\therefore$ In $\triangle ACD$, $m\angle BAC + m\angle ACD + m\angle ADC = 180$

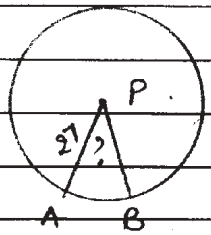
$\therefore 32 + (90+32) + m\angle ADC = 180$

$\therefore m\angle ADC = 180 - 154 = 26$

$\boxed{\text{ie } m\angle BDC = 26}$ [$\because$ A-B-D]

(3) Length of minor $\overset{\frown}{AB}$ of $\odot(P, 27)$ is 3π
$\therefore m\angle APB = ?$

Length of minor arc $= \frac{\pi r\theta}{180}$



$\therefore \frac{\pi r\theta}{180} = 3\pi$

$\therefore \theta = \frac{3\pi \times 180^{20}}{\pi \times 27_{9}}$

$\theta = 20°$ $\therefore \boxed{m\angle APB = 20°}$

Q(1) $\frac{x}{x-1} + \frac{x-1}{x} = \frac{25}{12}$ [$x \neq 0, x \neq 1$]

Take $\frac{x}{x-1} = m$

$\therefore m + \frac{1}{m} = \frac{25}{12}$

$\therefore 12m^2 - 25m + 12 = 0$

$\therefore 12m^2 - 16m - 9m + 12 = 0$

$\therefore (4m-3)(3m-4) = 0$

$\therefore m = \frac{3}{4}$ OR $m = \frac{4}{3}$

12×12
$4 \times 3 \times 4 \times 3$

$\therefore \frac{x}{x-1} = \frac{3}{4}$ | OR | $\frac{x}{x-1} = \frac{4}{3}$

$\therefore 4x = 3x - 3$ | | $\therefore 3x = 4x - 4$

$\therefore \boxed{x = -3}$ | | $\therefore \boxed{x = 4}$

(2) α and β are the roots of the equation $x^2 - p(x+1) - c = 0$

$\therefore x^2 - px - p - c = 0$

$a = 1, \quad b = -p, \quad c = -p-c$

$\therefore (\alpha+1)(\beta+1) = \alpha\beta + (\alpha+\beta) + 1$ ---

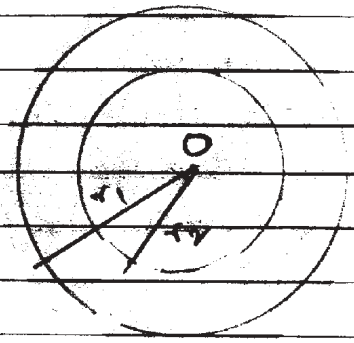
$\alpha\beta = \frac{c}{a} = \frac{-p-c}{1}$

$\alpha + \beta = \frac{-b}{a} = \frac{p}{1}$

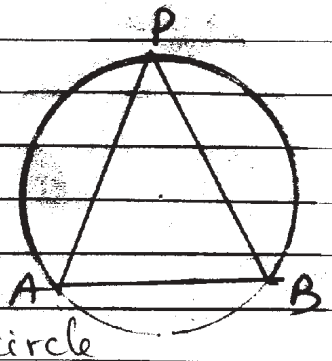
$\therefore (\alpha+1)(\beta+1) = \alpha\beta + (\alpha+\beta) + 1$

$= -p - c + p + 1 = \boxed{1-c}$

Eg) Concentric circles:
Two circles lying in the same plane having same centre but radii of different measures are called concentric circles.



(2) Angle in a segment:
If a segment of a circle is $\overset{\frown}{AB}$ UAB, then any angle in $\overset{\frown}{AB}$ is called an angle in the segment of the circle



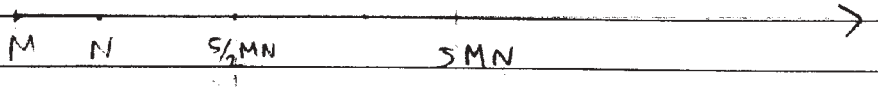
(3) CSA of sphere = Volume of sphere

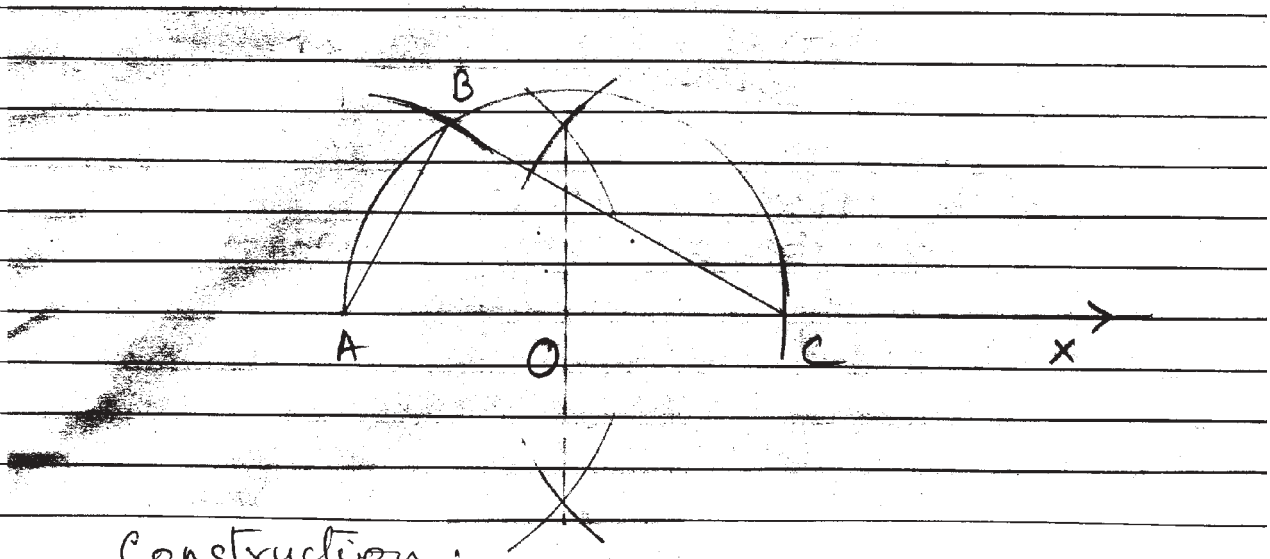
$\therefore 4\pi r^2 = \frac{4}{3}\pi r^3 \quad \therefore \boxed{r = 3}$

(4) Total surface area of a cube = $6l^2$

5 A. Given: MN

To construct: Right angled $\triangle ABC$, such that $AC = 5MN$ and $AB = \frac{5}{2}MN$, $m\angle B = 90°$.





Construction :-

Steps	Support.
(1) Construct $\overrightarrow{AX}$	- Ruler post.
(2) Const. $\odot(A, 5MN)$ to intersect $\overrightarrow{AX}$ in C.	- compass post.
(3) Const. $\perp$ bisector of $\overline{AC}$ and obtain its mid-point O	- Perpendicular bisector const.
(4) Const. $\odot(O, OA)$	- Compass post.
(5) Const. $\odot(A, \frac{5}{2}MN)$ to intersect $\odot(O, OA)$ in B	compass post.
(6) Const $\overline{AB}$, $\overline{BC}$	Ruler post-

Thus $\triangle ABC$ is the required right angled $\triangle$.

∴ B(ii) $x^2y^2+4 \propto xy$, p.t. $x \propto \frac{1}{y}$ $[xy \neq \pm 2]$

$\therefore \frac{x^2y^2+4}{xy} = k \quad [k \neq 0]$

$\therefore \frac{x^2y^2+4}{4xy} = \frac{k}{4}$ [Multiplying by $\frac{1}{4}$]

$\therefore \frac{x^2y^2+4+4xy}{x^2y^2+4-4xy} = \frac{k+4}{k-4}$ [componendo & dividendo]

$\therefore \frac{(xy+2)^2}{(xy-2)^2} = \frac{k+4}{k-4}$

$\therefore \frac{xy+2}{xy-2} = \sqrt{\frac{k+4}{k-4}} = m \quad [m \neq 0]$

$\therefore \frac{xy+2+xy-2}{xy+2-xy+2} = \frac{m+1}{m-1}$ [componendo & dividendo]

$\therefore \frac{2xy}{4} = \frac{m+1}{m-1}$

$\therefore xy = 2\left(\frac{m+1}{m-1}\right)$

$\therefore xy =$ Non-zero constant

$\therefore \boxed{x \propto \frac{1}{y}}$

(2) Due to some technical fault the speed of an electric train is reduced by 10 km.p.h. The train takes 48 minutes to cover a distance of 240 km.

Let the usual speed be x km.p.h.

∴ Time taken $= \left(\frac{240}{x}\right)$ hrs.

Decrease in speed = 10 km.p.h.

$\therefore$ new speed $= (x-10)$ km.p.h

$\therefore$ Time taken $= \left(\frac{240}{x-10}\right)$ hrs.

$\therefore$ By data : $\frac{240}{x-10} - \frac{240}{x} = \frac{48}{60}$

$\therefore \frac{240}{x-10} - \frac{240}{x} = \frac{4}{5}$

$\therefore 1200x - 1200x + 12000 = 4x(x-10)$

$\therefore 4x^2 - 40x + 12000 = 0$

$\therefore x^2 - 10x + 3000 = 0$

$(x-60)(x+50) = 0$

$\therefore x = 60$ or $x = -50$

$\because$ speed cannot be negative, so $x = -50$ is discarded

$\therefore$ **Speed = 60 km.p.h**

(3) $ax^2 + bx + c = 0$.

Multiplying by $4a$

$4a^2x^2 + 4abx + 4ac = 0$

Adding b^2 to both sides

$4a^2x^2 + 4abx + b^2 + 4ac = b^2$

$\therefore (2ax+b)^2 = b^2 - 4ac = \Delta$

$\therefore (2ax+b)^2 = (\sqrt{\Delta})^2$

$\therefore (2ax+b-\sqrt{\Delta})(2ax+b+\sqrt{\Delta}) = 0$

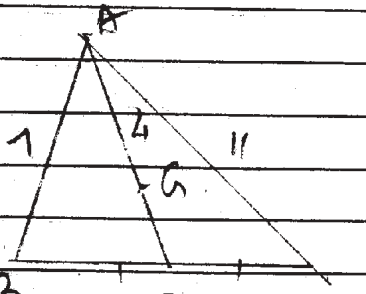
$\therefore x = \frac{-b+\sqrt{\Delta}}{2a}$ or $x = \frac{-b-\sqrt{\Delta}}{2a}$

$\therefore$ **$\alpha = \frac{-b+\sqrt{\Delta}}{2a}$** ; **$\beta = \frac{-b-\sqrt{\Delta}}{2a}$**

$\therefore \alpha - \beta = \frac{-b+\sqrt{\Delta}+b+\sqrt{\Delta}}{2a} = \frac{2\sqrt{\Delta}}{2a} = \frac{\sqrt{\Delta}}{a}$.

$\therefore$ **$\alpha - \beta = \frac{\sqrt{\Delta}}{a}$**

(1) In $\triangle ABC$, G is the centroid, AB = 7, AC = 11, AG = 4, BC = ?



$AG = 4 \therefore AD = \frac{3}{2}(AG)$

$AD = 6$

By Apollonius Theorem

$AB^2 + AC^2 = 2(AD^2 + BD^2)$

$\therefore 49 + 121 = 2(36 + BD^2)$

$\therefore \frac{170}{2} = 36 + BD^2$

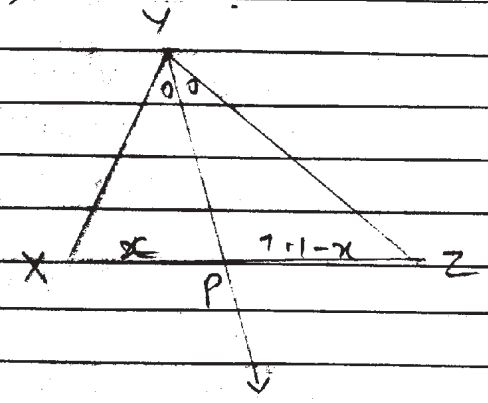
$\therefore 85 - 36 = BD^2$

$49 = BD^2 \quad \therefore BD = 7$

$\therefore \boxed{BC = 14}$

(2) In $\triangle XYZ$, $\overrightarrow{YP}$ bisects $\angle Y$ and intersects $\overline{XZ}$ in P. If XZ = 7.1, XP = ?

$YZ = 1\frac{1}{2} XY$.



$\therefore \frac{YZ}{XY} = \frac{3}{2}$

$\therefore \frac{YZ}{XY} = \frac{PZ}{XP}$

$\therefore \frac{3}{2} = \frac{7.1 - x}{x}$

$\therefore 3x = 14.2 - 2x$

$\therefore 5x = 14.2$

$\therefore x = 2.84 \quad \therefore \boxed{XP = 2.84}$

(3) $\triangle ABC \sim \triangle PQR$, ABC = 54 units, BC = 3 units, QR = 4 units

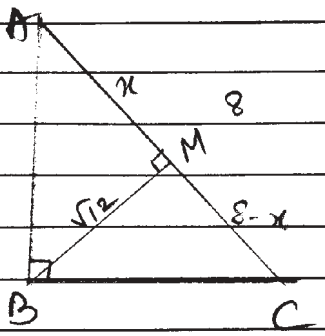
$\therefore \frac{ABC}{PQR} = \frac{BC^2}{QR^2}$ [Theorem]

$\therefore \frac{54}{PQR} = \frac{9}{16}$

$\therefore PQR = \frac{54 \times 16}{9} = 96$

$\therefore$ **Area of $\triangle PQR$ = 96 units**

D(i) In $\triangle ABC$, $m\angle B = 90$, $\overline{BM} \perp \overline{AC}$, $BM = \sqrt{12}$, $AC = 8$. $\therefore AM = ?$



By Geometric mean theorem;

$BM^2 = AM \cdot AC$

$\therefore 12 = x(8-x)$

$\therefore 12 = 8x - x^2$

$\therefore x^2 - 8x + 12 = 0$

$\therefore (x-6)(x-2) = 0$

$\therefore$ **$x = 6$ or $x = 2$** $\quad$ **$\therefore AM = 2$ or 6**

(2) **Wire (Cylinder)**

$r = 4\,mm = \frac{4}{10}\,cm$

$\therefore$ Volume $= \pi r^2 h$

$= \pi \times \frac{4}{10} \times \frac{4}{10} \times h$

$\therefore h = \frac{4\pi \times 2 \times 6 \times 6 \times 100}{\pi \times 4 \times 4}$ (3)

$h = 1800\ cm$

Sphere

$r = 6\,cm.$

Volume $= \frac{4}{3}\pi r^3$

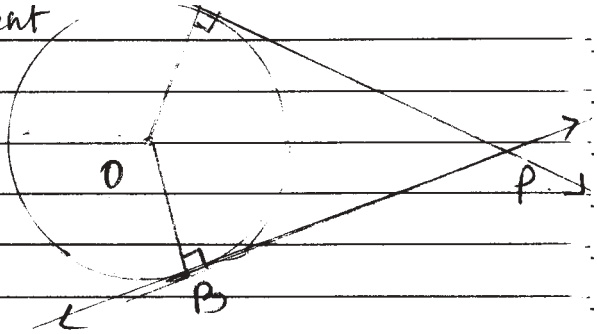
$= \frac{4}{3}\pi \times 6 \times 6 \times 6$ (2)

Length of the wire = 1800 cm.

E Given: In a circle with centre O, the tangents at A and B; intersect each other at P.

To prove: $\square AOBP$ is a cyclic quadrilateral.

• Proof: $\overline{PA}$ is the tangent of circle with centre at A and $\overline{OA}$ is a radius



$\therefore \overline{OA} \perp \overline{AP}$

$\therefore m\angle OAP = 90$

Similarly $m\angle OBP = 90$

$\therefore m\angle OAP + m\angle OBP = 180$ ——— (1)

In $\square AOBP$, $m\angle OAP + m\angle OBP + m\angle AOB + m\angle APB = 360$

$\therefore m\angle AOB + m\angle APB = 360 - 180$ from (1)

$= 180$ ——— (2)

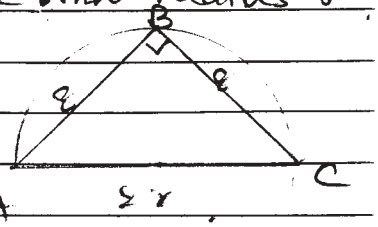
From (1) & (2), we get opposite angles of $\square AOBP$ are supplementary.

$\therefore \square AOBP$ is cyclic quadrilateral.

OR

(1) $\angle ABC$ is inscribed in semicircle with radius 'r'

$\triangle ABC$ is isosceles. $AB = 8$.



$m\angle ABC = 90$ [$\angle$ in a semicircle]

$\therefore AB^2 + BC^2 = AC^2$

$\therefore 64 + 64 = 4r^2$

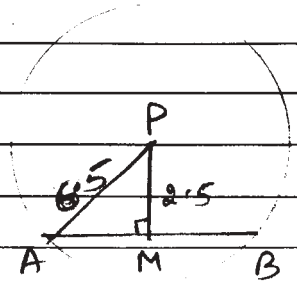
$128 = 4r^2$

$\therefore r^2 = 32$ $\therefore \boxed{r = 4\sqrt{2}}$

(2) Diameter of $\odot(P, r)$ is 13. Distance of chord $\overline{AB}$ from the centre is 2.5.

$\therefore AB = ?$

$\overline{PM} \perp \overline{AB}$



In right angled $\triangle AMP$

$AM^2 + PM^2 = AP^2$

$\therefore AM^2 = AP^2 - PM^2$

$= 6.5^2 - 2.5^2$

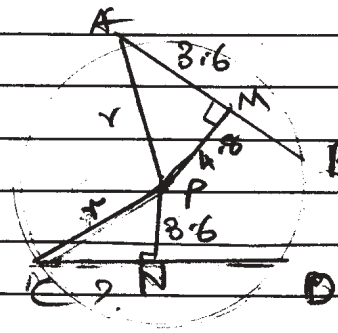
$= (4)(9)$

$\therefore AM = 2 \times 3 = 6$

$\therefore \boxed{AB = 2(AN) = 12}$

5 E(3) AB and CD are distinct chords of ⊙(P, r). $\overline{PM} \perp \overline{AB}$ and $\overline{PN} \perp \overline{CD}$

∴ AB = 7.2, PM = 4.8, PN = 3.6, CD = ?



$\overline{PM} \perp \overline{AB}$

$\therefore AM = \frac{1}{2} AB$

$= \frac{1}{2}(7.2)$

$= 3.6$

In right angled △AMP, $r^2 = PM^2 + AM^2$

$= 4.8^2 + 3.6^2$ ——(1)

In right angled △CNP, $r^2 = CN^2 + PN^2$

$= CN^2 + 3.6^2$ ——(2)

From (1) & (2) : $CN^2 = 4.8^2$

$\therefore CN = 4.8$

$\therefore CD = 2(4.8) = 9.6$

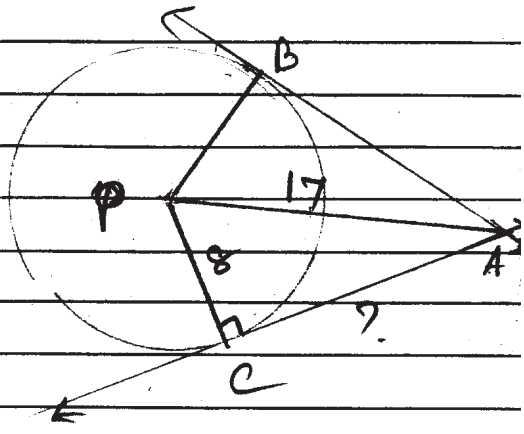
CD = 9.6

(4) Tangents drawn from A, lying in the exterior of ⊙(P, 8) touch the circle at B and C. AP = 17 ∴ AC = ?

$\overline{AC}$ is the tangent

$\overline{PC}$ is the radius

$\therefore \overline{PC} \perp \overline{AC}$



∴ In right angled △ACP

$AC^2 = AP^2 - PC^2$

$= 17^2 - 8^2$

$= (9)(25)$

$\therefore AC = 3 \times 5 = 15$

AC = 15