Time 3 hrs **Maths** **MM 100**

Q1 **A. State the Pythagoras Theorem and prove its converse** **(4)**

Q1 **B. Obtain the solution of any two of the following** **(6)**

1. **Factorize : $4a^2b + 4b^2c + 4c^2a + 2ab^2 + 2bc^2 + 2ca^2 + 9abc$**
2. **Factorize :-**

$$\frac{x}{4x^2 + 6x + 2} + \frac{x}{4x^2 + 10x + 6} + \frac{-2x}{4x^2 + 8x + 3}$$

3. **Factorise: $a(b-c)^3 + b(c-a)^3 + c(a-b)^3$**

Q1 **C. Factorise (any two)** **(4)**

1. $(x^2 - 7x + 12)^2 + 16x(x-3)^2$
2. $(9x^2 - y^2)^2 - 12xy - 1$
3. $x^3 + x^2 - 3xy + 9y^2 + 27y^3$

Q1 **D. Solve any one of the following** **(2)**

1. **Simplify: $x^{-2} + x^{-3} + x^{-4} - x^{-5} \div x^{-5}$**
2. **If $x : y : z = 3 : 4 : 5$, then find $\frac{x+y}{y+z}$**

Q1 **E. Fill in the blanks by selecting the proper alternatives from those given in brackets** **(4)**

1. **$f : N \to R$, $f(x) = \pi x^2$ then R_f= _____ (N, R, $\{\pi n^2 / n \in N\}$)**
2. **$f : R^- \cup \{0\} \to R$, $f(x) = -x$ is a _____ (line, ray, line segment)**
3. **Loop is used in a flow chart to indicate ____ steps in short. (Pictorial, repeated, alphanumerical)**
4. **There are mainly _____ parts of a Computer. (two, three, four)**

Q2 **A. Prove that the perpendicular drawn through the centre of a circle on a chord bisects the chord.** **(4)**

Q2 B. Solve any two 6

1. If $\frac{x+y}{3a-b} = \frac{y+z}{3b-c} = \frac{z+x}{3c-a}$ prove that
$(x+y+z)(a^2+b^2+c^2) = (a+b+c)(ax+by+cz)$
2. If b is the geometric mean of a and c
then prove that $\frac{a^2+b^2}{b^2+c^2} = \frac{a^2+ab}{b^2+bc}$
3. The height of the cylinder varies directly as the volume and inversely as the square of the radius of it. When the volume of a cylinder is 125.6 cc its radius is 2cm and the height is 10cm. Find the radius of the cylinder whose volume is 1413 c.c and the height is 8 cm.

Q2 C. Solve (any 2)

1. The mean of 25 observations is 19.5. By an error two observations are registered as 40 and 47.5 instead of -40 and 37.5. find the correct mean
2. For the data $3\bar{x} = 2M = 60$ find the mode.
3. Find the median of the observations of some data $\log_{10}10$, $\log_{10}1$, $\log_{10}100$, $\log_{10}0.1$, and $\log_{10}0.01$

Q2 D. Solve (any 2) (2)

1. A = {1, 3, 5, 7} and $f : A \rightarrow N \cup \{0\}$, $f(x) = (-1)^x + x$ then find the range of the function.
2. Express (-6, 2] in the set notation.

Q2 E. Fill in the blanks by selecting the proper alternatives from those given in the brackets. (4)

1. If $x - y = 3$ and $xy = 2$ $\therefore$ $x^3 - y^3 =$ _____ (15, 20, 45)
2. $\frac{x}{2} = \frac{y}{2} = \frac{z}{3} = \frac{3x-5y+2z}{m}$ $\therefore$ m = _____
3. If $x \alpha \frac{1}{y^2}$, and $y \alpha \frac{1}{\sqrt{z}}$, then $z \alpha$ _____ $(x, \frac{1}{x}, \sqrt{x})$
3. $\sin^2(2x+45) + \cos^2(3x+20) = 1$. then x = ____ (15, 25, 65)

Q3 A. Prove that an angle inscribed in a semi circle is a right angle. (4)

B. Solve (any 2) **(6)**

1. A toy is in the form of a cone placed on hemisphere of radius 3.5 cm. The total height of the toy is 15.5 cm. Find the total external surface area.
2. A metallic sphere with 20cm diameter is melted and recasted into spherical beads with 0.25 cm radius. Find the number of beads formed.
3. A cone is 12m high. If radius of its base 5m, find its total surface and curved surface area.

Q3 **C. Solve (any 2)** **(4)**

1. Prove : $\frac{\text{Sec}\,\theta + 2\,\text{Sin}\,\theta}{\text{Cos}\,\theta + \text{Sin}\,\theta} = 1 + \tan\theta$
2. If $\text{Cos}\,\theta - \text{Sin}\,\theta = \sqrt{2}\,\text{Sin}\,\theta$, then prove that $\text{Cos}\,\theta + \text{Sin}\,\theta = \sqrt{2}\text{Cos}\,\theta$
3. Find the value of $4\tan^2 30 - 5\,\text{Cosec}^2 45 - \frac{1}{3}\,\text{Sin}^4 45$

Q3 **D. Solve (any 1)** **(2)**

1. If $a \propto b$ and $b \propto c$ $(a \neq -c)$ then prove that $a^2 + b^2 + c^2 \propto ab + bc$
2. Prove that $\frac{\text{Cot}\,30 + 1}{\text{Cot}\,30 - 1} = 2\,(1 + \text{Cos}\,\theta)$

Q3 **E. Answer the following** **(4)**

1. If $9x^2 + mx - 25 = 0$ has opposite roots then find m.
2. In quadratic equation $mx^2 + nx + 1 = 0$ then find the sum of roots $\alpha + \beta$.
3. Define : Intercept made by the transversal
4. Define : Adjacent line Segment.

Q4 **A. In Δ ABC, $\angle$B is a right angle and $\overline{AD}$ is the median.** **(4)**
Prove that $AC^2 = AD^2 + 3\,CD^2$

Q4 **B. Solve (any 2)** **(6)**

1. Simplify: $\frac{a^2 + b^2 - a^{-2} - b^{-2}}{a^2b^2 - a^{-2}b^{-2}} + \frac{(a - a^{-1})(b - b^{-1})}{ab + a^{-1}b^{-1}}$

2. Frequency of number of children in twenty five families is given below. If mean of data is 2.08 ,then find y

No of children	0	1	2	Y	4
No of families	4	3	6	11	1

3. Find the mean and median of the following data.

No s	1	2	3	4	5	6
Frequency	14	18	20	24	20	04

Q4 C. Answer the following questions (any two) (4)

1. In ⊙ (P, r) , $\overline{AB}$ is the chord $\overline{PD} \perp \overline{AB}$, $\overline{PD} \cap \overline{AB} = \{M\} \in$ ⊙ (P, r). If AB = 8 , DM = 2. find radius.
2. $\angle$ PQR is angle inscribed in semicircle. PQ = 9. If the corresponding radius is 7.5, then find QR.
3. The radii of two concentric circles are 3.4 and 1.6 respectively. The chord of bigger circle touches the chord of smaller circle. Find the length of the chord.

Q4 D. Solve (any 1) (2)

1. If one root of the equation $x^2 - 5x + 6 = 0$ is also the root of equation $x^2 + 3x + K = 0$ then find the value of K.
2. Find the product of the roots of the equation $2x^2 - 11x - 16 = 0$

Q4 E. Answer the following (4)

1. Define: Major arc of the circle.
2. Define : Tangent of the circle.
3. If the surface area of the sphere is 2464 sq.m the radius of it is __ m.
4. The volume of a cone and sphere of the same radius x are equal. Then the height of the cone is _____.

Q5 A. $\overline{MA}$ is given. Construct ⊙ (0, ⅓ MA) and a regular octagon circumscribed about it. Write steps of construction. (4)

Q5 B. Solve (any 2) (6)

1. Solve : $8\left(x^2 + \frac{1}{x^2}\right) - 14\left(x + \frac{1}{x}\right) - 69 = 0$

2. The speed of the motor boat is 15km/ h in still water. If takes 9 hours for a round trips of 60 kms. Find the speed of water.
3. The cost of printing a book is partly constant and partly varies directly as the number of copies printed. If the printing of 120 copies Rs 2000 and that of 100 copies Rs 1700. Compute the number of copies when the cost is Rs 1880.

Q5 C. Solve (any 2) **(4)**

1. In $\square^{m}ABCD$, $AB^2 + AD^2 = 260$, $BD = 18$, Find AC.
2. In ΔABC, the bisector of $\angle B$ intersects $\overline{AC}$ at D. If BC: BA = 4.5 and CD = 12 then find CA.
3. In ΔABC , $\angle B$ is a right angle $\overline{BM} \perp \overline{AC}$. AM = 10 AC = 18 , find BC

Q5 D. Solve (any 1) **(2)**

1. In ΔABC , AB = AC = 15, BC = 24, $\overline{AD}$ is the median and G is the centroid then find AG.
2. The curved surface area of a planet is 49,89,60,000 kms. Find its diameter.

Q5 E. In a circle, length of the chord $\overline{AB}$ of a circle is twice the length of the chord $\overline{CD}$. The radius of the circle is r. The distance of $\overline{CD}$ from centre is twice the distance of $\overline{AB}$ from centre. Prove that $r = \frac{\sqrt{5}}{2} CD$

OR

Q5 E. Answer the following **(4)**

1. In a cyclic quadrilateral ABCD, $\overline{AC} \cap \overline{BD} = \{P\}$. IF m $\angle DBC = 70$ and m $\angle BAC = 30$ then find m $\angle$ BAD
2. Tangents $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ of $\odot$ (P, 5) touch the circle at B and C. If the perimeter of $\square ABCD$ is 34. find PA.
3. O is the circum centre of ABC. If m $\angle$ BOC = 140, and $\overline{AB} \cong \overline{AC}$, then find m $\angle$ ACB
4. Each of $\odot$ (P,a), $\odot$ (Q, b), $\odot$ (R, c) touches the circle externally. If PQ = 4, QR = 6 and PR = 8, then find the value of a + b + c.

Std-10 Subject-Maths Solution

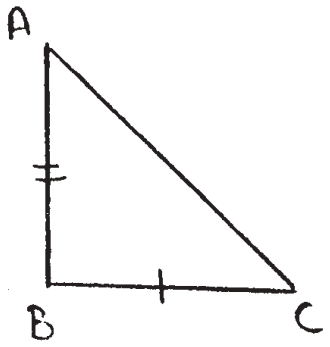
Q 1 A.

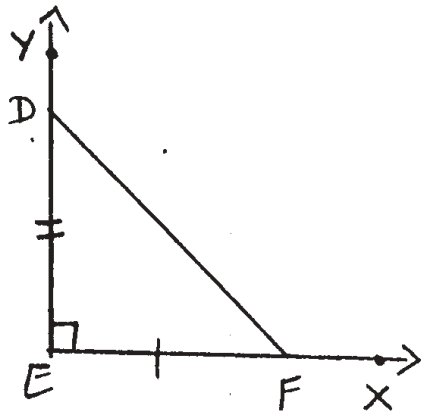
Pythagoras Theorem: In a right angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the remaining sides.

(Converse of the Pythagoras theorem):

Given: In ΔABC , $AC^2 = AB^2 + BC^2$

To prove : $\angle B$ is a right angle.





Proof: Let $\overrightarrow{EX}$ be any ray.

Then we get $\overrightarrow{EY}$ perpendicular to it.

According to the point plotting theorem we get the point F on $\overrightarrow{EX}$ such that EF=BC and the point D on $\overrightarrow{EY}$ such that ED=BA.

In ΔDEF, $\angle E$ being a right angle, according to Pythagoras' theorem,

$DF^2 = DE^2 + EF^2$

$= AB^2 + BC^2$ (since DE=AB, EF=BC)

$= AC^2$ (given)

DF= AC.

Now, in ΔABC and ΔDEF,

$AB \cong DE, BC \cong EF$ and $AC \cong DF$

$ABC \leftrightarrow DEF$ is a congruence . (by SSS theorem)

Therefore $m\angle B = m\angle E$.

But $m\angle E = 90$

Therefore $m\angle B = 90$

Therefore $\angle B$ is a right angle.

Q 1 B)

Q 1 B)

1) $4a^2b+4b^2c+4c^2a+2ab^2+2bc^2+2ca^2+9abc$

$=4a^2b+2a^2c+2ab^2+9abc+4ac^2+4b^2c+2bc^2$

$=2a^2(2b+c)+a(2b^2+9bc+4c^2)+2bc(2b+c)$

$=2a^2(2b+c)+a(2b^2+8bc+bc+4c^2)+2bc(2b+c)$

$=2a^2(2b+c)+a(b+4c)(2b+c)+2bc(2b+c)$

$=(2b+c)(2a^2+ab+4ac+2bc)$

$=(2b+c)[a(2a+b)+2c(2a+b)]$

$=(2a+b)(2b+c)(2c+a)$

2) $\frac{x}{4x^2+6x+2} + \frac{x}{4x^2+10x+6} + \frac{-2x}{4x^2+8x+3}$

$= \frac{x}{4x^2+4x+2x+2} + \frac{x}{4x^2+4x+6x+6} + \frac{-2x}{4x^2+2x+6x+3}$

$= \frac{x}{(4x+2)(x+1)} + \frac{x}{(4x+6)(x+1)} + \frac{-2x}{(2x+1)(2x+3)}$

$= \frac{x}{2(2x+1)(x+1)} + \frac{x}{2(2x+3)(x+1)} + \frac{-2x}{(2x+1)(2x+3)}$

$= \frac{x(2x+3)+x(2x+1)-4x(x+1)}{2(2x+1)(x+1)(2x+3)}$

$= \frac{2x^2+3x+2x^2+x-4x^2-4x}{2(2x+1)(x+1)(2x+3)}$

$=0$

3) $a(b-c)^3+b(c-a)^3+c(a-b)^3$

$=a[b^3-c^3-3bc+3bc]+b[c^3-a^3-3ac+3ac]+c[a^3-b^3-3ab+3ab]$

$=ab^3-ac^3+bc^3-cb^3+ca^3-ba^3$

$=a(b^3-c^3)-bc(b^2-c^2)-a^3(b-c)$

$=(b-c)[a(b^2-bc+c^2)-bc(b+c)-a^3]$

$=(b-c)[ab-a^3+c^2a+bc^2-abc-b^2c]$

$=(b-c)[a(b^2-a^2)+c^2(a+b)-bc(a+b)]$

$=(b-c)[a(b-a)(b+a)+c^2(a+b)-bc(a+b)]$

$=(b-c)(a+b)[c^2-a^2+ab-bc]$

$=(b-c)(a+b)[(c-a)(c+a)+b(a-c)]$

$=(b-c)(a+b)(c-a)(c+a-b)$

Q 1 C)

1) $(x^2-7x+12)^2 + 16x(x-3)^2$

$=(x^2-3x-4x+12)^2 + 16x(x-3)^2$

$=[(x-3)(x-4)]^2 + 16x(x-3)^2$

$=(x-3)^2(x^2-8x+16+16x)$

$=(x-3)^2(x^2+8x+16)$

$=(x-3)^2(x+4)^2$

2) $(9x^2-y^2)-12xy-1$

$=81x^4+18x^2y^2+y^4-36x^2y^2-12xy\ 1$

$=(9x^2+y^2)^2-(6xy+1)^2$

$=(9x^2+y^2-6xy-1)(9x^2+y^2+6xy+1)$

$=(9x^2-6xy+y^2-1)(9x^2+6xy+y^2+1)$

$=[(3x-y)^2-1][(3x+y)^2+1]$

$=(3x-y-1)(3x-y+1)[(3x+y)+1]$

3) $x^3+x^2-3xy+9y^2+27y^3$

$=x^3+27y^3+x^2-3xy+9y^2$

$=(x+3y)(x^2-3xy+9y^2)+1(x^2-3xy+9y^2)$

$=(x^2-3xy+9y^2)(x+3y+1)$

Q 1 D)

1) $X^{-2}+x^{-3}+x^{-4}-x^{-5}+x^{-5}$

$= \frac{1}{x^2} + \frac{1}{x^3} + \frac{1}{x^4} - \frac{x^{-5}}{x^{-5}}$

$= \frac{1}{x^2} + \frac{1}{x^3} + \frac{1}{x^4} - 1$

$= \frac{x^2+x+1-x^4}{x^4}$

$= \frac{(x+1)(-x^3+x^2+1)}{x^4}$

2) zLet $\frac{x}{3} = \frac{y}{4} = \frac{z}{5} = k$

$x=3k, y=4k, z=5k$

$\frac{x+y}{y+z} = \frac{3k+4k}{4k+5k} = \frac{7k}{9k} = \frac{7}{9}$

Q 1 E)

1)f: $N \to R$, $f(x)=\Pi x^2$, then $R_f = \{\Pi n^2 / n \in N\}$

2)f: $R - U \{0\} \to R$, $f(x)= -x$ is a ray.

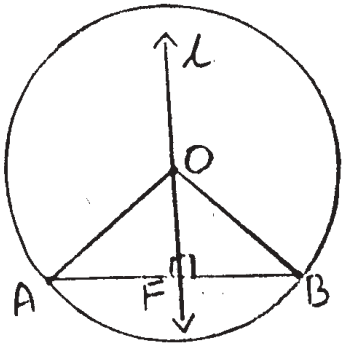
3)Loop is used in a flowchart to indicate repeated steps in short.

4)There are mainly three parts of a computer.

Q2 A)

Given :$\overline{AB}$ is the chord of a circle with center O. l is the perpendicular line from O to $\overleftrightarrow{AB}$.

To prove : l bisects $\overline{AB}$.



Proof: OA=OB=r(radius of the circle)

O is point equidistant from A and B.

Hence, O is a point on the perpendicular bisector of $\overline{AB}$.

Thus, if O$\in \overline{AB}$, then O is the midpoint of $\overline{AB}$.

So the perpendicular from O to $\overline{AB}$ bisects $\overline{AB}$.

If O $\notin \overline{AB}$ and F is the midpoint of AB, then the perpendicular line from F to $\overline{AB}$ is the unique perpendicular bisector of $\overline{AB}$ and O being on the perpendicular bisector of $\overline{AB}$, $\overline{OF}$ is perpendicular to $\overline{AB}$ and bisects $\overline{AB}$. Moreover, the perpendicular line drawn from a point outside a line to the line is unique.

$\overleftrightarrow{OF}$ (=l) bisects $\overline{AB}$.

Q2 B)

1)Each ratio= $\frac{x+y+y+z+z+x}{3a-b+3b-c+3c-a}$

$=\frac{2(x+y+z)}{2(a+b+c)}$

$=\frac{x+y+z}{a+b+c}$ ----------(1)

E.R.= $\frac{x+y+z}{a+b+c}=\frac{y+z}{3b-c}$

E.R.=$\frac{x+y+z-y-z}{a+b+c-3b+c}=\frac{x}{a-2b+2c}$

E.R.=$\frac{x+y+z-x-z}{a+b+c-3c+a}=\frac{y}{b+2a-2c}$

E.R.=$\frac{x+y+z-y-x}{a+b+c-3a+b}=\frac{z}{c+2b-2a}$

Each ratio= $\frac{ax+by+cz}{a(a-2b+2c)+b(b-2c+2a)+c(c-2a+2b)}$

$=\frac{ax+by+cz}{a^2-2ab+2ac+b^2-2bc+2ab+c^2-2ac+2bc}$

$=\frac{ax+by+cz}{a^2+b^2+c^2}$ ----------(2)

From (1) and (2),

$(x+y+z)(a^2+b^2+c^2)=(a+b+c)(ax+by+cz)$

2)$b^2=ac$

LHS= $\frac{a^2+b^2}{b^2+c^2}=\frac{a^2+ac}{ac+c^2}=\frac{a(a+c)}{c(a+c)}=\frac{a}{c}$

RHS = $\frac{a^2+ab}{b^2+bc}=\frac{a^2+ab}{ac+bc}=\frac{a(a+b)}{c(a+b)}=\frac{a}{c}$ =LHS

3) Let height of cylinder be h
The volume be v
The radius be r

$h \propto \frac{v}{r^2}$

$h = \frac{kv}{r^2}$ (constant $k \neq 0$)

Now v=125.6, r=2, h=10

$10 = \frac{k(125.6)}{2^2}$

$k = \frac{40}{125.6}$

hence, $h = \frac{40 * v}{125.6 * r^2}$

→ To find r when v=1413 and h=8

$h = \frac{40 * v}{125.6 * r^2}$

$r^2 = \frac{40*1413}{125.6*8}$

$=56.25$

r=7.5 cm

Thus the radius of cylinder is 7.5 cm.

Q2 C)

1) Mean= 19.5

Number of observations =N=25

$\text{Mean} = \frac{\sum x}{N}$

$\sum x = N * \text{Mean} = 25 * 19.5 = 487.5$

Correct $\sum x = \sum x - 40 - 47.5 - 40 + 37.5$

$= 487.5 - 40 - 47.5 - 40 + 37.5$

$= 397.5$

$\text{Correct Mean} = \frac{\text{Correct } \sum x}{N}$

$= \frac{397.5}{25}$

$= 15.9$

2) 3(Mean) = 2(Median) =60
3(Mean) =60
Mean=20
2(Median) =60
Median=30

Mode =3(Median) –2(Mean) =3(30) -2(20) =90-40 =50

3) $\log_{10}10$ =1, $\log_{10}1$=0, $\log_{10}$ 100 =2, $\log_{10}$ 0.1=-1, $\log_{10}$ 0.01=-2
Arranging the observations in ascending order : -2 ,-1, 0 , 1, 2

Median= M = $\frac{(n+1)^{th}}{2}$ observation
= $\frac{(5+1)^{th}}{2}$ observation
=3rd observation
M= $\log_{10}1$

Q2 D)
1) A ={1,3,5,7}
f(x)=$(-1)^x + x$
f(1)=$(-1)^1$ +1 =0
f(3)=$(-1)^1$ +3=2
f(5)=$(-1)^1$ +5=4
f(7)=$(-1)^1$ +7=6

R_f ={0,2,4,6}

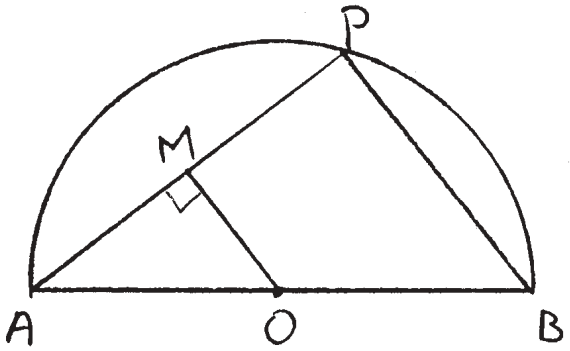
2) (-6, 2] ={x/ -6 < x $\leq$ 2, x $\in$ R}

Q 2 E)
1) If x-y =3 and xy=2. $\therefore$ $x^3 - y^3 = 45$
2) $\frac{x}{2} = \frac{y}{2} = \frac{z}{3} = \frac{3x-5y+2z}{m}$ $\therefore m= 2$
3) If $x \propto \frac{1}{Y^2}$, and $y \propto \frac{1}{\sqrt{z}}$, then $z \propto x$.
4) $\sin^2(2x +45) + \cos^2(3x+20) =1$ then x= 25.

Q 3 A)

Given : $\overset{\frown}{AB}$ is a semicircle with center O. $\angle APB$ is an angle inscribed in the semicircle.

To prove : $m\angle APB = 90$



Proof: $\overset{\frown}{AB}$ is a semicircle. Therefore, its corresponding chord $\overline{AB}$ is a diameter. Thus, O is the midpoint of $\overline{AB}$. If M is the midpoint of $\overline{AP}$, $\overline{OM}$ is the line segment joining the center and the midpoint of the chord $\overline{AP}$.

$\therefore \overline{OM} \perp \overline{AP}$.

Therefore, $m\angle AMO = 90$

M and O are respectively the midpoints of $\overline{AP}$ and $\overline{AB}$ in $\triangle APB$.

$\overline{OM} \parallel \overline{PB}$ and their transversal is $\overleftrightarrow{PA}$.

$\angle AMO \cong \angle APB$ (corresponding angles)

But, $m\angle AMO = 90$

$m\angle APB = 90$

$\angle APB$ is a right angle.

Q 3 B)

1) Radius of hemisphere r=3.5 cm

Radius of cone r=3.5 cm

Height of cone h= 15.5-3.5= 12 cm

Slant height of cone $l = \sqrt{h^2 + r^2} = \sqrt{12^2 + (3.5)^2}$

$= \sqrt{144 + 12.25}$

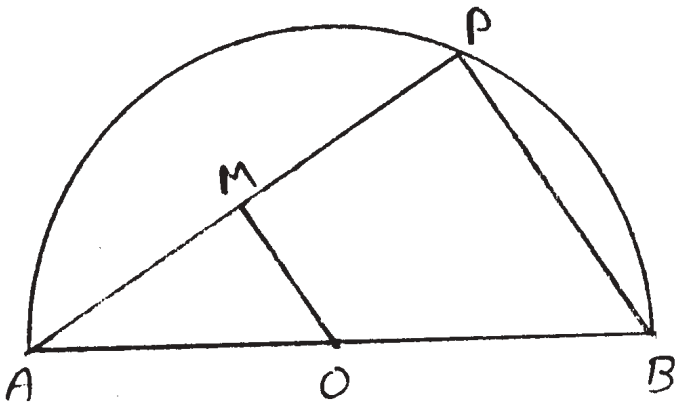
$= \sqrt{156.25}$

$= 12.5$ cm

Q 3 A)

Given : AB is a semicircle with center O. APB is an angle inscribed in the semicircle.

To prove : m∠ APB =90



Proof: AB is a semicircle. Therefore, its corresponding chord AB is a diameter.Thus, O is the midpoint of AB. If M is the midpoint of AP , OM is the line segment joining the center and the midpoint of the chord AP.

OM⊥ AP.

Therefore, m∠ AMO=90

M and O are respectively the midpoints of AP and AB in APB.

OM || PB and their transversal is PA.

∠AMO≅∠ APB (corresponding angles)

But, m ∠ AMO=90

m∠ APB =90

∠ APB is a right angle.

Q 3 B)

1) Radius of hemisphere r=3.5 cm

Radius of cone r=3.5 cm

Height of cone h= 15.5-3.5= 12 cm

Slant height of cone $l = \sqrt{h^2 + r^2} = \sqrt{12^2 + (3.5)^2}$

$= \sqrt{144 + 12.25}$

$= \sqrt{156.25}$

$= 12.5$ cm

Total surface area of the toy $= 2\pi r^2 + \pi rl$

$= 2 \pi (3.5)^2 + \pi (3.5)(12.5)$

$= \pi(3.5)(7 + 12.5)$

$= 22 * 3.5 * 19.5 / 7$

$= 214.5$ sq cm.

2) Radius of metallic sphere $= 10$ cm

Volume of metallic sphere $= 4 \pi r^3 / 3 = 4 \pi (10)^3 / 3$ cu cm

Volume of beads $= 4 \pi r^3 / 3 = 4 \pi (0.25)^3 / 3$ cu cm

No. of beads made $= \frac{\text{Volume of metallic sphere}}{\text{Volume of beads}}$

$= \frac{4 \pi (10)^3 / 3}{4 \pi (0.25)^3 / 3}$

$= 64000$ beads

3) Radius of cone $r = 5$ m

Height of cone $h = 12$ m

Slant height of cone $= l = \sqrt{r^2 + h^2}$

$= \sqrt{5^2 + 12^2}$

$= \sqrt{25 + 144}$

$= \sqrt{169}$

$= 13$ m

Total surface area of cone $= \pi r(l+r)$

$= 22 * 5 * (13+5) / 7$

$= 22 * 5 * 18 / 7$

$= 282.857$ sq m.

Curved surface area $= \pi rl$

$= 22 * 5 * 13 / 7$

$= 204.285$ sq m.

Q3 C)

1) LHS $= \frac{\sec\theta + 2\sin\theta}{\cos\theta + \sin\theta}$

$= \frac{1/\cos\theta + 2\sin\theta}{\cos\theta + \sin\theta}$

$= \frac{1 + 2\sin\theta \cos\theta}{\cos\theta (\cos\theta + \sin\theta)}$

$$= \frac{\sin\theta+\cos\theta+2\sin\theta\cos\theta}{\cos\theta\ (\cos\theta\ +\ \sin\theta\ )}$$

$$= \frac{(\sin\theta\ +\ \cos\theta\ )}{\cos\theta\ (\cos\theta\ +\ \sin\theta\ )}$$

$$= \frac{(\cos\theta\ +\ \sin\theta\ )}{\cos\theta}$$

$= 1 + \tan\theta$

$=$RHS

3) $\cos\theta - \sin\theta = \sqrt{2}\ \sin\theta$

$(\cos\theta - \sin\theta)^2 = 2\sin^2\theta$

$\cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta = 2\sin^2\theta$

$\cos^2\theta - \sin^2\theta = 2\sin\theta\cos\theta$

$(\cos\theta + \sin\theta)(\cos\theta - \sin\theta) = 2\sin\theta\cos\theta$

$$\cos\theta + \sin\theta = \frac{2\sin\theta\cos\theta}{\cos\theta - \sin\theta}$$

$$= \frac{2\sin\theta\cos\theta}{\sqrt{2}\sin\theta}$$

$\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

4) $4\tan^2 30 - 5\ \mathrm{cosec}^2 45 - \frac{1}{3}\sin^4 45$

$= 4(1/\sqrt{3})^2 - 5(\sqrt{2})^2 - 1/3(1/\sqrt{2})^4$

$= 4/3 - 5(2) - 1/12$

$= -105/12$

$= -35/4$

Q 3 D)

1) $a \propto b$, $b \propto c$ $(a \neq -c)$

$a = kb\ (k \neq 0)$, $b = mc\ (m \neq 0)$

$\therefore a = kmc$

So we have to show that $\frac{a^2 + b^2 + c^2}{ab+ac}$ is a non-zero constant.

$$\frac{a^2+b^2+c^2}{ab+ac} = \frac{k^2m^2c^2+m^2c^2+c^2}{km^2c^2+kmc^2}$$

$$= \frac{c^2(k^2m^2+m^2+1)}{c^2(km^2+km)}$$

$$= \frac{k^2m^2+m^2+1)}{(km^2+km)}$$

=non-zero constant

2) LHS= $\frac{\cot 30+1}{\cot 30-1} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$$= \frac{\sqrt{3}+1}{\sqrt{3}-1} * \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$=3+2\sqrt{3}+1 = 4+2\sqrt{3} = 2+\sqrt{3}$$

RHS =2(1+ cos 30)

$$=2\left(1+\frac{\sqrt{3}}{2}\right) = 2+\sqrt{3} = \text{LHS}$$

Q 3 E)

1) Comparing $9x^2+mx-25=0$ with the standard quadratic equation $ax^2+bx+c=o$ we get, $a=9, b=m, c=-25$

$\therefore \alpha+\beta=0$

$\therefore \frac{-b}{a}=0$

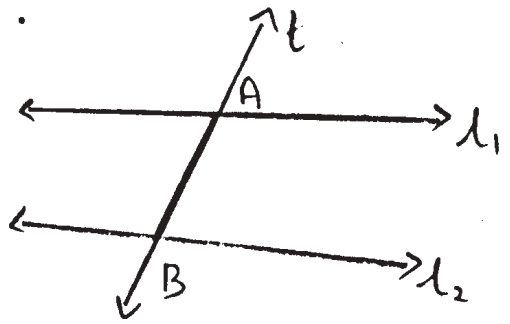
$\therefore -b=0$

$\therefore b=0$

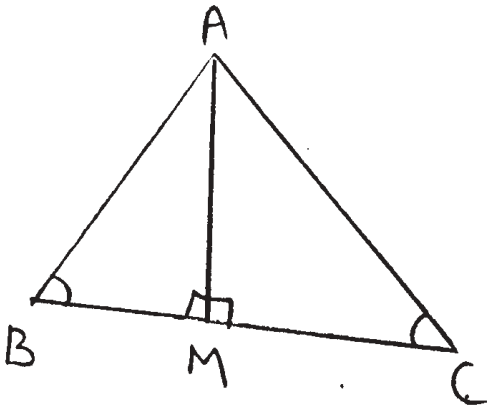
$\therefore m=0$

2) Sum of roots= $\alpha+\beta = -\frac{b}{a} = \frac{-n}{m}$ $(mx^2+nx+1=0)$

3)Intercept made by transversal – If a transversal t of lines l_1 and l_2 intersects them in A and B then AB is called the intercept on t made by lines l_1 and l_2 .



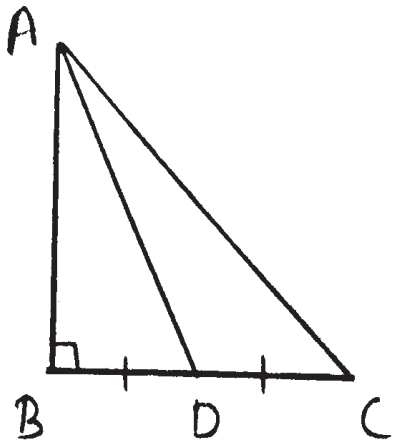
4) Adjacent line segment - In ΔABC, let $\overline{AM}$ be the altitude on $\overline{BC}$. If B and C are acute angles, then B – M – C. Then, $\overline{BM}$ is called the line segment adjacent to $\overline{AB}$ and $\overline{CM}$ is called the line segment adjacent to AC.



Q 4 A)

Given : In ΔABC, $\angle B$ is a right angle and $\overline{AD}$ is its median.

To prove : $AC^2 = AD^2 + 3CD^2$



Proof : In ΔABC, $\angle B$ is a right angle.

$\overline{AD}$ is a median. So, BD=CD.

$\therefore$ BC= 2 CD.

According to Pythagoras theorem,

In ΔABD, $AD^2 = AB^2 + BD^2$ -------(1)

In ΔABC, $AC^2 = AB^2 + BC^2$

$= AB^2 + (2CD)^2$

$= AB^2 + 4CD^2$

$= AB^2 + CD^2 + 3CD^2$

$= AB^2 + BD^2 + 3CD^2$ (as CD = BD)

$AC^2 = AD^2 + 3CD^2$ (from 1)

Q 4 B)

2)

No.of children x_i	No. of families f_i	$f_i x_i$
0	4	0
1	3	3
2	6	12
y	11	11 y
4	1	4
	n = 25	$\Sigma f_i x_i = 19+11y$

$$\text{Mean} = \frac{\Sigma f_i x_I}{n}$$

$$2.08 = \frac{19 + 11y}{25}$$

$2.08 * 25 = 19 + 11y$

$52 - 19 = 11y$

$33 = 11y$

$y = 33/11$

$y = 3$

3)

No.s x_I	Frequency f_i	$f_i x_i$	Cumulative frequency
1	14	14	14
2	18	36	32
3	20	60	52
4	24	96	76
5	20	100	96
6	4	24	100
	n= 100	$\Sigma f_i x_i = 330$	

$$\text{Mean} = \frac{\Sigma f_i x_i}{n}$$

$= \frac{330}{10}$

Mean=3.3

Median= M= $(n/2)^{th}$ observation

= 50 th observation

M = 3

1) $\dfrac{a^2 + b^2 - \frac{1}{a^2} - \frac{1}{b^2}}{ab^2 - \frac{1}{ab^2}} + \dfrac{(a - \frac{1}{a})(b - \frac{1}{b})}{ab + \frac{1}{ab}}$

$= \dfrac{(a^4 b^2 + a^2 b^4 - b^2 - a^2) * a^2 b^2}{a^2 b^2 (a^4 b^4 - 1)} + \dfrac{(a^2 - 1)(b^2 - 1) * ab}{ab (a^2 b^2 + 1)}$

$= \dfrac{a^2 b^2 (a^2 + b^2) - 1(a^2 + b^2)}{(a^4 b^4 - 1)} + \dfrac{(a^2 - 1)(b^2 - 1)}{a^2 b^2 + 1}$

$= \dfrac{(a^2 + b^2)}{a^2 b^2 + 1} + \dfrac{(a^2 - 1)(b^2 - 1)}{a^2 b^2 + 1}$

$= \dfrac{a^2 + b^2 + a^2 b^2 - a^2 - b^2 + 1}{a^2 b^2 + 1}$

$= \dfrac{a^2 b^2 + 1}{a^2 b^2 + 1}$

=1

Q4 C)

1) AB= 8 , DM =2

Radius = r =PD

PM= PD – DM = r – 2

MB = (1/2) AB =4

In Δ PMB, m ∠ PMB=90

According to Pythagoras theorem, $PB^2 = PM^2 + MB^2$

$r^2 = (r-2)^2 + 4^2$

$r^2 = r^2 - 4r + 4 + 16$

$r^2 - r^2 = -4r + 20$

$4r = 20$

$r = 5$

2) Radius r= 7.5

PR = 2r = 15

$\angle PQR$ is angle inscribed in semicircle.

$\angle PQR = 90$.

According to Pythagoras theorem,

$PR^2 = PQ^2 + QR^2$

$QR^2 = PR^2 - PQ^2$

$=15^2 - 9^2$

$=225 - 81 = 144$

$QR = 12$

3) CD = 1.6 (Radius of inner circle)

$\overline{BC}$ =3.4 (Radius of outer circle)

$CD \perp BA$

In ΔCDB, $CB^2 = CD^2 + DB^2$

$(3.4)^2 = (1.6)^2 + DB^2$

$(3.4)^2 - (1.6)^2 = BD^2$

$(3.4 - 1.6)(3.4 + 1.6) = BD^2$

$(1.8)(5) = BD^2$

$BD^2 = 9$

$BD = 3$

Length of chord AB= 2 BD = 2*3

= 6

Q 4 D)

1) $x^2 - 5x + 6 = 0$

$x^2 - 2x - 3x + 6 = 0$

$(x-2)(x-3) = 0$

$x - 2 = 0$ or $x - 3 = 0$

$x = 2$ or $x = 3$

If x=2, $2^2 + 3(2) + k = 0$

$4 + 6 + k = 0$

$k = -10$

If x=3, $3^2 + 3(3) + k = 0$

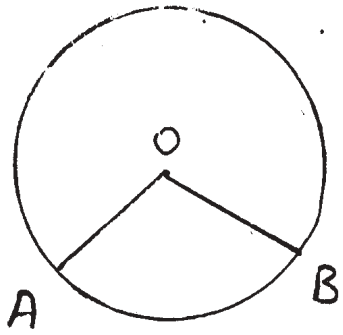
$9 + 9 + k = 0$

$k = -18$

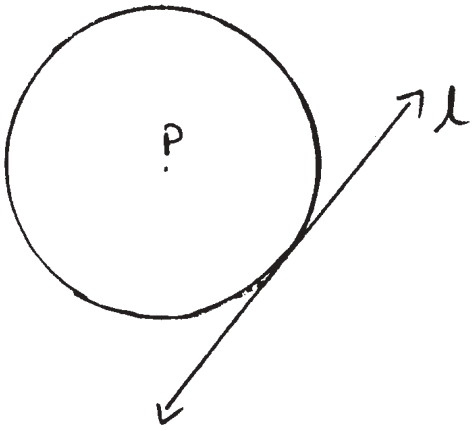
Thus ,k= -10 or k=-18

2)Comparing $2x^2 - 11x - 16 = 0$ with standard equation
$ax^2 + bx + c = 0$ we get, a=2, b= -11, c= -16
Product of roots $= \alpha\beta = c/a$
$= -16/2$
$= -8$

Q 4 E)
1) Major arc of the circle : If A and B are the points of a circle with center O, then the set of the points on the circle lying in the exterior of∠AOB ,together with the points A and B is called the major arc AB.

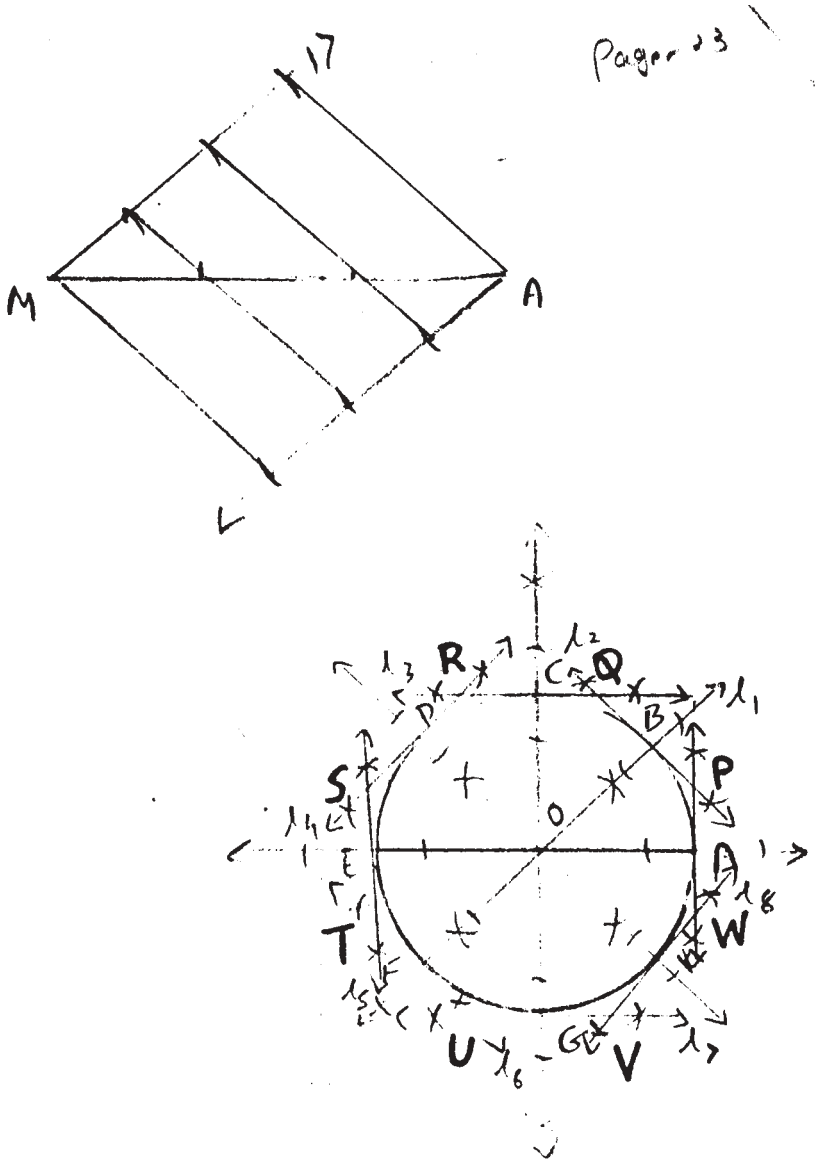


2) Tangent of a circle : A line in the plane of circle intersecting the circle at one and only one point is called a tangent of the circle at that point.



3)If the surface area of the sphere is 2464 sq m. the radius of it is 14 m.
4)The volume of a cone and the sphere of the same radius x are equal. Then the height of the cone is 4x.

Q5 A)



Steps of Construction :

1. **Construct O(O, 1/3 MA) and a diameter EA of O(O,1/3 MA).**
2. **Through O, construct a line perpendicular to EA which intersects the circle in two distinct points C and G.**
3. **Construct the bisectors of $\angle AOC$ and $\angle EOC$ which intersect the circle respectively at B and D.**
4. **Construct the rays opposite to OB and OD which intersect the circle at F and H respectively.**
5. **Through points A,B,C,D,E,F,G and H respectively construct lines l_1 ,l_2 ,l_3 ,l_4 ,l_5 ,l_6 ,l_7 and l_8 which are perpendicular to OA, OB ,OC, OD, OE, OF, OG and OH respectively so that l_1 and l_2 ,l_2 and l_3 , l_3 and l_4 ,l_4 and l_5 , l_5 and l_6 ,l_6 and l_7 , l_7 and l_8 ,l_8 and l_1 intersect repectively at P,Q,R,S,T,U,V,W.**
6. **Thus, octagon PQRSTUVW is the required octagon circumscribed about O (O,1/3MA).**

Q 5 B)

1) $8\left(x^2 + \frac{1}{x^2}\right) - 14\left(x + \frac{1}{x}\right) - 69 = 0$

Let $\left(x + \frac{1}{x}\right) = m$

$\therefore x^2 + \frac{1}{x^2} + 2 = m^2$

$\therefore x^2 + \frac{1}{x^2} = m^2 - 2$

$\therefore 8(m^2 - 2) - 14m - 69 = 0$

$\therefore 8m^2 - 16 - 14m - 69 = 0$

$\therefore 8m^2 - 14m - 85 = 0$

$\therefore 8m^2 + 20m - 34m - 85 = 0$

$\therefore (2m+5)(4m-17) = 0$

$\therefore 2m + 5 = 0$ or $4m - 17 = 0$

$\therefore m = -5/2$ or $m = 17/4$

$\therefore x + \frac{1}{x} = -5/2$ $\quad x + \frac{1}{x} = 17/4$

$\frac{x^2+1}{x} = -5$ $\quad \frac{x^2+1}{x} = -5$

$2x^2 + 2 = -5x$ $\quad 4x^2 + 4 = 17x$

$2x^2 + 5x + 2 = 0$ $\quad 4x^2 - 17x + 4 = 0$

$2x^2 + 4x + x + 2 = 0$ $\quad 4x^2 - 16x - x + 4 = 0$

$(2x+1)(x+2) = 0$ $\quad (4x-1)(x-4) = 0$

$2x + 1 = 0$ or $x + 2 = 0$ $\quad 4x - 1 = 0$ or $x - 4 = 0$

$x = \frac{-1}{2}$ or $x = -2$ $\quad x = \frac{1}{4}$ or $x = 4$

Roots are: $\frac{-1}{2}, -2, \frac{1}{4}, 4$

2) Let the speed of the current of river be x km/hr.

Speed of boat during down-stream $= 15 + x$

Time taken for down stream $= \frac{60}{15+x}$ hrs.

Speed of boat during up-stream $= 15 - x$

Time taken for up stream $= \frac{60}{15-x}$ hrs.

Total time for round journey is 9 hrs.

$\frac{60}{15+x} + \frac{60}{15-x} = 9$

$60(15-x) + 60(15+x) = (15+x)(15-x)$
$900-60x+900+60x = 9(225-x^2)$
$1800 = 9(225-x^2)$
$x^2 - 225 + 200 = 0$
$x^2 - 25 = 0$
$(x-5)(x+5) = 0$
$x = 5$ or $x = -5$
$x = -5$ is impossible.
The speed of current of river is 5 km/hr.

3)Let the cost of printing be y.
No. of copies be b.
$Y = a + kb$ $(k \neq 0)$
Now, for $b=120, y=2000$
$2000 = a+120k$ ---------(1)

For $b=100, y=1700$
$1700 = a+100k$ -------(2)

From 1 and 2 we get, $2000 = a+120k$
$1700 = a+100k$
$- \quad - \quad -$

$300 = 20k$
$k = 15$

Now, $2000 = a + 120(15)$
$a = 2000-1800$
$a = 200$

Now, $y=1880$, $a=200$, $k=15$ find x.
$y = a+kb$
$1880 = 200+15b$
$1880-200 = 15b$
$\frac{1680}{15} = b$
$b = 112$
No. of copies are 112.

Q 5 C)

1) $AB^2 + AD^2 = 260$

BD=18

DM=1/2 BD

According to Appollonius theorem,

$AD^2 + AB^2 = 2(DM^2 + AM^2)$

$260 = 2(9^2 + AM^2)$

$130 = 2(81 + AM^2)$

$AM^2 = 49$

AM=7

AC=2 AM= 2*7 =14

2) In ΔABC ,BC : BA = 4: 5, CD =12

According to corollary, BC:BA = CD:AD

4:5 = 12: AD

AD= 12 * 5/4

AD= 15

CA=AD+CD

=15+12

=27

3) In $\Delta ABC, m\angle B = 90$,$BM \perp AC$

AM=10,AC=18, CM=AC-AM =18-10 =8

$BC^2 = CM * AC$

=8*18

=144

BC=12

Q 5 D)

1) In ABC, AB=15, AC= 15, BC =24

BD= ½ BC = 12

According to Apollonius theorem, $AB^2 + AC^2 = 2(AD^2 + BD^2)$

$2(15)^2 = 2(AD^2 + 12^2)$

$225 = AD^2 + 144$

$AD^2 = 81$

AD= 9

AG=2/3 (AD)

=2/3 (9) = 6

2)Curved surface area of planet = $4\pi r^2$

$4\pi r^2$ =498960000 km. Sq.

$r^2 = (498960000 * 7) / (22 * 4)$

$= 39690000$

$r = 6300$

Diameter of the planet = $2r = 2(6300)$

=12600 km.

Q 5 E)

1) In cyclic quadrilateral ABCD, $m\angle DBC = 70$, $m\angle BAC = 30$.

$\therefore m\angle DAC = 70$ (angles of the same segment)

$m\angle BAD = m\angle DAC + m\angle BAC$

$= 70 + 30$

$= 100$

2)Lines AB and AC are tangents of O(P , 5)

Perimeter of $\square ABCD = AB+BC+CD+DA$

$\therefore$ 34 =AB+AC+5+5

$\therefore$ AB+AC=34-10

$\therefore$ 2AC =24 (as lines AB and AC are tangents)

$\therefore$ AC=12

In ΔAPC, $m\angle ACP = 90$

$AP^2 = PC^2 + AC^2 = 5 + 12 = 25 + 144$

$=169$

$\therefore AP = 13$

3) In ΔABC, $m\angle BOC = 140$

$\therefore m\angle CAB = 1/2\ (m\angle BOC)$

In ABC, $AB \cong AC$.

$\therefore m\angle ACB = m\angle ABC$

$m\angle ABC + m\angle ACB + m\angle BAC = 180$

$2\ m\angle ACB = 180-70 = 110$

$\therefore m\angle ACB = 55$.

4)PQ=4, QR= 6, PR= 8

PQ+QR+PR= 2(a+b+c)

4+6+8 =2(a+b+c)

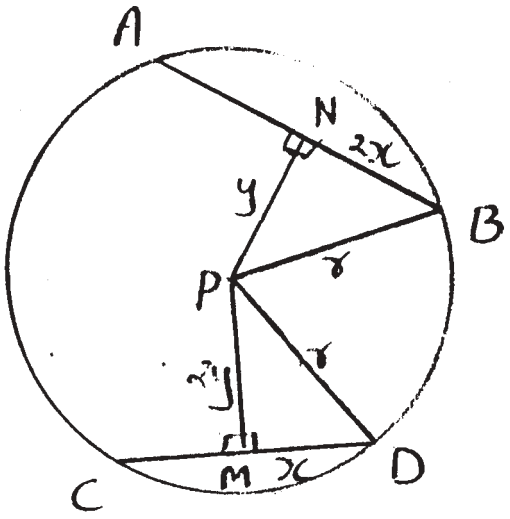
a+b+c = 18/2 = 9

OR

) 5 E)

iven : AB and CD are two chords of O(P, r) and AB = 2CD.
Also, PM CD , M CD and PN AB, N AB such that PM = 2 PN

'o prove : r = 5 /2 (CD)



Proof : Let CD =2x
AB =2CD =4x
MD = ½(CD) =x
NB= ½(AB) = ½(4x) =2x

Let PN =y
PM = 2PN =2y

In PNB, m N is a right angle.
$PB^2 = PN^2 + NB^2$
$r^2 = y^2 + (2x)^2$
$r^2 = y^2 + 4x^2$ -------(1)

I n PMD, m M is a right angle.
$PD^2 = PM^2 + MD^2$
$r^2 = (2y)^2 + x^2$
$r^2 = 4y^2 + x^2$ ---------(2)
From (1) and (2) we get,
$y^2 + 4x^2 = 4y^2 + x^2$
$3x^2 = 3y^2$
$x^2 = y^2$
Substituting $x^2 = y^2$ in (1) we get,
$r^2 = x^2 + 4x^2$
$r^2 = 5x^2$
$r = \sqrt{5}x$
$r = (\sqrt{5}\ CD)/2$ (CD =2x)
$r = \frac{\sqrt{5}\ CD}{2}$