CHAPTER – 39
ALTERNATING CURRENT

1. $f = 50$ Hz

   $I = I_0 \text{ Sin } Wt$

   Peak value $I = \frac{I_0}{\sqrt{2}}$

   $\frac{I_0}{\sqrt{2}} = I_0 \text{ Sin } Wt$

   $\Rightarrow \frac{1}{\sqrt{2}} = \text{Sin } Wt = \text{Sin } \frac{\pi}{4}$

   $\Rightarrow \frac{\pi}{4} = Wt.$ or, $t = \frac{\pi}{400} = \frac{\pi}{4 \times 2\pi f} = \frac{1}{8f} = \frac{1}{8 \times 50} = 0.0025 \text{ s} = 2.5 \text{ ms}$

2. $E_{rms} = 220$ V

   Frequency = 50 Hz

   (a) $E_{rms} = \frac{E_0}{\sqrt{2}}$

   $\Rightarrow E_0 = E_{rms}\sqrt{2} = \sqrt{2} \times 220 = 1.414 \times 220 = 311.08 \text{ V} = 311 \text{ V}$

   (b) Time taken for the current to reach the peak value = Time taken to reach the 0 value from r.m.s

   $I = \frac{I_0}{\sqrt{2}} \Rightarrow \frac{I_0}{\sqrt{2}} = I_0 \text{ Sin } \omega t$

   $\Rightarrow \omega t = \frac{\pi}{4}$

   $\Rightarrow t = \frac{\pi}{4\omega} = \frac{\pi}{4 \times 2\pi f} = \frac{\pi}{8\pi 50} = \frac{1}{400} = 2.5 \text{ ms}$

3. $P = 60$ W $\quad V = 220$ V $= E$

   $R = \frac{v^2}{P} = \frac{220 \times 220}{60} = 806.67$

   $\varepsilon_0 = \sqrt{2}\,E = 1.414 \times 220 = 311.08$

   $I_0 = \frac{\varepsilon_0}{R} = \frac{806.67}{311.08} = 0.385 \approx 0.39 \text{ A}$

4. $E = 12$ volts

   $i^2 Rt = i^2_{rms} RT$

   $\Rightarrow \frac{E^2}{R^2} = \frac{E^2_{rms}}{R^2} \Rightarrow E^2 = \frac{E_0^{\,2}}{2}$

   $\Rightarrow E_0^{\,2} = 2E^2 \Rightarrow E_0^{\,2} = 2 \times 12^2 = 2 \times 144$

   $\Rightarrow E_0 = \sqrt{2 \times 144} = 16.97 \approx 17 \text{ V}$

5. $P_0 = 80$ W (given)

   $P_{rms} = \frac{P_0}{2} = 40 \text{ W}$

   Energy consumed $= P \times t = 40 \times 100 = 4000 \text{ J} = 4.0 \text{ KJ}$

6. $E = 3 \times 10^6$ V/m, $\quad A = 20 \text{ cm}^2$, $\quad d = 0.1$ mm

   Potential diff. across the capacitor $= Ed = 3 \times 10^6 \times 0.1 \times 10^{-3} = 300$ V

   Max. rms Voltage $= \frac{V}{\sqrt{2}} = \frac{300}{\sqrt{2}} = 212 \text{ V}$

7. $i = i_0 e^{-ur}$

   $$\overline{i^2} = \frac{1}{\tau}\int_0^{\tau} i_0^{\,2} e^{-2t/\tau} dt = \frac{i_0^{\,2}}{\tau}\int_0^{\tau} e^{-2t/\tau} dt = \frac{i_0^{\,2}}{\tau} \times \left[\frac{\tau}{2} e^{-2t/\tau}\right]_0^{\tau} = -\frac{i_0^{\,2}}{\tau} \times \frac{\tau}{2} \times \left[e^{-2} - 1\right]$$

$$\sqrt{i^2} = \sqrt{-\frac{i_0^2}{2}\left(\frac{1}{e^2}-1\right)} = \frac{i_0}{e}\sqrt{\left(\frac{e^2-1}{2}\right)}$$

8. $C = 10\ \mu F = 10 \times 10^{-6}\ F = 10^{-5}\ F$

$E = (10\ V)\ \text{Sin}\ \omega t$

a) $I = \frac{E_0}{Xc} = \frac{E_0}{\left(\frac{1}{\omega C}\right)} = \frac{10}{\left(\frac{1}{10\times10^{-5}}\right)} = 1 \times 10^{-3}\ A$

b) $\omega = 100\ s^{-1}$

$$I = \frac{E_0}{\left(\frac{1}{\omega C}\right)} = \frac{10}{\left(\frac{1}{100\times10^{-5}}\right)} = 1 \times 10^{-2}\ A = 0.01\ A$$

c) $\omega = 500\ s^{-1}$

$$I = \frac{E_0}{\left(\frac{1}{\omega C}\right)} = \frac{10}{\left(\frac{1}{500\times10^{-5}}\right)} = 5 \times 10^{-2}\ A = 0.05\ A$$

d) $\omega = 1000\ s^{-1}$

$$I = \frac{E_0}{\left(\frac{1}{\omega C}\right)} = \frac{10}{\left(\frac{1}{1000\times10^{-5}}\right)} = 1 \times 10^{-1}\ A = 0.1\ A$$

9. Inductance = 5.0 mH = 0.005 H

a) $\omega = 100\ s^{-1}$

$$X_L = \omega L = 100 \times \frac{5}{1000} = 0.5\ \Omega$$

$$i = \frac{\varepsilon_0}{X_L} = \frac{10}{0.5} = 20\ A$$

b) $\omega = 500\ s^{-1}$

$$X_L = \omega L = 500 \times \frac{5}{1000} = 2.5\ \Omega$$

$$i = \frac{\varepsilon_0}{X_L} = \frac{10}{2.5} = 4\ A$$

c) $\omega = 1000\ s^{-1}$

$$X_L = \omega L = 1000 \times \frac{5}{1000} = 5\ \Omega$$

$$i = \frac{\varepsilon_0}{X_L} = \frac{10}{5} = 2\ A$$

10. $R = 10\ \Omega$, $L = 0.4$ Henry

$E = 6.5\ V$, $f = \frac{30}{\pi}\ Hz$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (2\pi fL)^2}$$

Power $= V_{rms}\ I_{rms} \cos\phi$

$$= 6.5 \times \frac{6.5}{Z} \times \frac{R}{Z} = \frac{6.5\times6.5\times10}{\left[\sqrt{R^2+(2\pi fL)^2}\right]^2} = \frac{6.5\times6.5\times10}{10^2+\left(2\pi\times\frac{30}{\pi}\times0.4\right)^2} = \frac{6.5\times6.5\times10}{100+576} = 0.625 = \frac{5}{8}\omega$$

11. $H = \frac{V^2}{R}T$, $E_0 = 12\ V$, $\omega = 250\ \pi$, $R = 100\ \Omega$

$$H = \int_0^H dH = \int \frac{E_0^2 \text{Sin}^2\omega t}{R}\,dt = \frac{144}{100}\int \sin^2\omega t\ dt = 1.44\int\left(\frac{1-\cos 2\omega t}{2}\right)dt$$

$$= \frac{1.44}{2}\left[\int_0^{10^{-3}} dt - \int_0^{10^{-3}} \text{Cos}2\omega t\ dt\right] = 0.72\left[10^{-3} - \left(\frac{\text{Sin}2\omega t}{2\omega}\right)_0^{10^{-3}}\right]$$

$= 0.72\left[\frac{1}{1000} - \frac{1}{500\pi}\right] = \frac{(\pi - 2)}{1000\pi} \times 0.72 = 0.0002614 = 2.61 \times 10^{-4}$ J

12. $R = 300\Omega$, $C = 25\ \mu F = 25 \times 10^{-6}$ F, $\varepsilon_0 = 50$ V, $f = 50$ Hz

$$X_c = \frac{1}{\omega c} = \frac{1}{\frac{50}{\pi} \times 2\pi \times 25 \times 10^{-6}} = \frac{10^4}{25}$$

$$Z = \sqrt{R^2 + X_c^{\ 2}} = \sqrt{(300)^2 + \left(\frac{10^4}{25}\right)^2} = \sqrt{(300)^2 + (400)^2} = 500$$

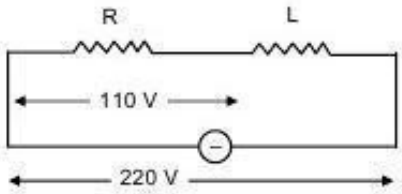
(a) Peak current $= \frac{E_0}{Z} = \frac{50}{500} = 0.1$ A

(b) Average Power dissipitated, $= E_{rms}\, I_{rms} \cos\phi$

$$= \frac{E_0}{\sqrt{2}} \times \frac{E_0}{\sqrt{2}Z} \times \frac{R}{Z} = \frac{E_0^{\ 2}}{2Z^2} = \frac{50 \times 50 \times 300}{2 \times 500 \times 500} = \frac{3}{2} = 1.5\ \omega.$$

13. Power = 55 W, Voltage = 110 V, Resistance $= \frac{V^2}{P} = \frac{110 \times 110}{55} = 220\ \Omega$

frequency (f) = 50 Hz, $\omega = 2\pi f = 2\pi \times 50 = 100\ \pi$



Current in the circuit $= \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (\omega L)^2}}$

Voltage drop across the resistor = ir $= \frac{VR}{\sqrt{R^2 + (\omega L)^2}}$

$$= \frac{220 \times 220}{\sqrt{(220)^2 + (100\pi L)^2}} = 110$$

$\Rightarrow 220 \times 2 = \sqrt{(220)^2 + (100\pi L)^2} \quad \Rightarrow (220)^2 + (100\pi L)^2 = (440)^2$

$\Rightarrow 48400 + 10^4\pi^2 L^2 = 193600 \quad \Rightarrow 10^4\pi^2 L^2 = 193600 - 48400$

$\Rightarrow L^2 = \frac{142500}{\pi^2 \times 10^4} = 1.4726 \quad \Rightarrow L = 1.2135 \approx 1.2$ Hz

14. $R = 300\ \Omega$, $C = 20\ \mu F = 20 \times 10^{-6}$ F

L = 1 Henry, E = 50 V, $V = \frac{50}{\pi}$ Hz

(a) $I_0 = \frac{E_0}{Z}$,

$$Z = \sqrt{R^2 + (X_c - X_L)^2} = \sqrt{(300)^2 + \left(\frac{1}{2\pi f C} - 2\pi f L\right)^2}$$

$$= \sqrt{(300)^2 + \left(\frac{1}{2\pi \times \frac{50}{\pi} \times 20 \times 10^{-6}} - 2\pi \times \frac{50}{\pi} \times 1\right)^2} = \sqrt{(300)^2 + \left(\frac{10^4}{20} - 100\right)^2} = 500$$

$I_0 = \frac{E_0}{Z} = \frac{50}{500} = 0.1$ A

(b) Potential across the capacitor = $i_0 \times X_c = 0.1 \times 500 = 50$ V

Potential difference across the resistor = $i_0 \times R = 0.1 \times 300 = 30$ V

Potential difference across the inductor = $i_0 \times X_L = 0.1 \times 100 = 10$ V

Rms. potential = 50 V

Net sum of all potential drops = 50 V + 30 V + 10 V = 90 V

Sum or potential drops > R.M.S potential applied.

15. $R = 300\ \Omega$

$C = 20\ \mu F = 20 \times 10^{-6}$ F

L = 1H, Z = 500 (from 14)

$\varepsilon_0 = 50$ V, $I_0 = \frac{E_0}{Z} = \frac{50}{500} = 0.1$ A

Electric Energy stored in Capacitor = (1/2) CV^2 = (1/2) × 20 × 10^{-6} × 50 × 50 = 25 × 10^{-3} J = 25 mJ

Magnetic field energy stored in the coil = (1/2) L I_0^2 = (1/2) × 1 × $(0.1)^2$ = 5 × 10^{-3} J = 5 mJ

16. (a)For current to be maximum in a circuit

$X_l = X_c$ (Resonant Condition)

$$\Rightarrow WL = \frac{1}{WC}$$

$$\Rightarrow W^2 = \frac{1}{LC} = \frac{1}{2\times18\times10^{-6}} = \frac{10^6}{36}$$

$$\Rightarrow W = \frac{10^3}{6} \Rightarrow 2\pi f = \frac{10^3}{6}$$

$$\Rightarrow f = \frac{1000}{6\times2\pi} = 26.537 \text{ Hz} \approx 27 \text{ Hz}$$

(b) Maximum Current = $\frac{E}{R}$ (in resonance and)

$$= \frac{20}{10\times10^3} = \frac{2}{10^3} \text{ A} = 2 \text{ mA}$$

17. E_{rms} = 24 V

r = 4 Ω, I_{rms} = 6 A

$$R = \frac{E}{I} = \frac{24}{6} = 4\ \Omega$$

Internal Resistance = 4 Ω

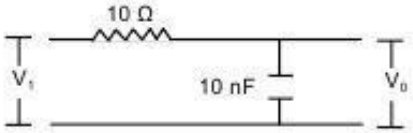
Hence net resistance = 4 + 4 = 8 Ω

$$\therefore \text{ Current} = \frac{12}{8} = 1.5 \text{ A}$$

18. $V_1 = 10 \times 10^{-3}$ V

$R = 1 \times 10^3\ \Omega$

$C = 10 \times 10^{-9}$ F



$$\text{(a) } X_c = \frac{1}{WC} = \frac{1}{2\pi fC} = \frac{1}{2\pi\times10\times10^3\times10\times10^{-9}} = \frac{1}{2\pi\times10^{-4}} = \frac{10^4}{2\pi} = \frac{5000}{\pi}$$

$$Z = \sqrt{R^2 + X_c^2} = \sqrt{\left(1\times10^3\right)^2 + \left(\frac{5000}{\pi}\right)^2} = \sqrt{10^6 + \left(\frac{5000}{\pi}\right)^2}$$

$$I_0 = \frac{E_0}{Z} = \frac{V_1}{Z} = \frac{10\times10^{-3}}{\sqrt{10^6 + \left(\frac{5000}{\pi}\right)^2}}$$

$$\text{(b) } X_c = \frac{1}{WC} = \frac{1}{2\pi fC} = \frac{1}{2\pi\times10^5\times10\times10^{-9}} = \frac{1}{2\pi\times10^{-3}} = \frac{10^3}{2\pi} = \frac{500}{\pi}$$

$$Z = \sqrt{R^2 + X_c^2} = \sqrt{\left(10^3\right)^2 + \left(\frac{500}{\pi}\right)^2} = \sqrt{10^6 + \left(\frac{500}{\pi}\right)^2}$$

$$I_0 = \frac{E_0}{Z} = \frac{V_1}{Z} = \frac{10\times10^{-3}}{\sqrt{10^6 + \left(\frac{500}{\pi}\right)^2}}$$

$$V_0 = I_0 X_c = \frac{10\times10^{-3}}{\sqrt{10^6 + \left(\frac{500}{\pi}\right)^2}} \times \frac{500}{\pi} = 1.6124 \text{ V} \approx 1.6 \text{ mV}$$

(c) f = 1 MHz = 10^6 Hz

$$X_c = \frac{1}{WC} = \frac{1}{2\pi fC} = \frac{1}{2\pi\times10^6\times10\times10^{-9}} = \frac{1}{2\pi\times10^{-2}} = \frac{10^2}{2\pi} = \frac{50}{\pi}$$

$$Z = \sqrt{R^2 + X_c^2} = \sqrt{(10^3)^2 + \left(\frac{50}{\pi}\right)^2} = \sqrt{10^6 + \left(\frac{50}{\pi}\right)^2}$$

$$I_0 = \frac{E_0}{Z} = \frac{V_1}{Z} = \frac{10 \times 10^{-3}}{\sqrt{10^6 + \left(\frac{50}{\pi}\right)^2}}$$

$$V_0 = I_0 X_c = \frac{10 \times 10^{-3}}{\sqrt{10^6 + \left(\frac{50}{\pi}\right)^2}} \times \frac{50}{\pi} \approx 0.16 \text{ mV}$$

(d) $f = 10 \text{ MHz} = 10^7 \text{ Hz}$

$$X_c = \frac{1}{WC} = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 10^7 \times 10 \times 10^{-9}} = \frac{1}{2\pi \times 10^{-1}} = \frac{10}{2\pi} = \frac{5}{\pi}$$

$$Z = \sqrt{R^2 + X_c^2} = \sqrt{(10^3)^2 + \left(\frac{5}{\pi}\right)^2} = \sqrt{10^6 + \left(\frac{5}{\pi}\right)^2}$$

$$I_0 = \frac{E_0}{Z} = \frac{V_1}{Z} = \frac{10 \times 10^{-3}}{\sqrt{10^6 + \left(\frac{5}{\pi}\right)^2}}$$

$$V_0 = I_0 X_c = \frac{10 \times 10^{-3}}{\sqrt{10^6 + \left(\frac{5}{\pi}\right)^2}} \times \frac{5}{\pi} \approx 16 \ \mu\text{V}$$

19. Transformer works upon the principle of induction which is only possible in case of AC.

Hence when DC is supplied to it, the primary coil blocks the Current supplied to it and hence induced current supplied to it and hence induced Current in the secondary coil is zero.

