Total No. of Printed Pages—12

HS/XII/A. Sc. Com/M/NC/21

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MATHEMATICS

(New Course)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

General Instructions :

(i) All questions are compulsory.

(ii) This question paper contains 36 questions divided into four Sections A, B, C and D. Section—A comprises of 20 questions of *1* mark each, Section—B comprises of 6 questions of *2* marks each, Section—C comprises of 6 questions of *4* marks each and Section—D comprises of 4 questions of *6* marks each.

(iii) There is no overall choice. However, internal choice has been provided in 9 questions of Section—A, 5 questions of Section—B, 5 questions of Section—C and 2 questions of Section—D. You have to attempt only one of the alternatives in all such questions.

(iv) Use of calculator is not permitted.

SECTION—A

1. If $R = \{(1, -1), (2, -2), (3, -1)\}$ is a relation, then find the domain and range of R. 1

Or

Find the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$. 1

2. If $f : \mathbb{R} \to \mathbb{R}$ is a function defined by $f(x) = x^2, \ \forall x \in \mathbb{R}$, then show that f is not one-one. 1

3. Construct a 2×2 matrix $A = [a_{ij}]$, whose elements are given by

$$a_{ij} = \frac{(i + j)^2}{2}$$ 1

Or

Find the value of AB when $A = [1\ 2\ 3\ 4]$ and

$$B = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$ 1

4. Use determinant to find the value of K for which the points $A(3, -2)$, $B(K, 2)$ and $C(8, 8)$ are collinear. 1

Or

Find the value of λ so that the matrix

$$\begin{bmatrix} 5-\lambda & \lambda+1 \\ 2 & 4 \end{bmatrix}$$

is singular. 1

5. If

$$\begin{vmatrix} 4 & m \\ -3 & 5 \end{vmatrix} = 8$$

find the value of m. 1

Or

If

$$\begin{vmatrix} x-2 & -3 \\ 3x & 2x \end{vmatrix} = 3$$

find the value of x. 1

6. Show that the matrix

$$A = \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}$$

is skew-symmetric. 1

7. If $y = e^{3\log x}$, then find $\frac{dy}{dx}$. 1

Or

Find the value of $\frac{d}{dx}(\sin^2 x^4 + \cos^2 x^4)^4$. 1

8. Find the value of $\int_2^3 |x|\,dx$. 1

Or

Find the value of $\int_0^{\pi/2} \cos 2x\,dx$. 1

9. Find the slope of the tangent to the curve

$$y = 2x^2 + 3\sin x$$

at $x = 0$. 1

10. What is the order and the degree of the following differential equation? 1

$$\left(\frac{d^2y}{dx^2}\right)^3 + 2\left(\frac{dy}{dx}\right)^5 + 9y = \sin x$$

11. If $\bar{a}$ and $\bar{b}$ are two vectors such that $|\bar{a}| = \sqrt{2}$, $|\bar{b}| = 2$ and $\bar{a} \cdot \bar{b} = \sqrt{6}$, find the angle between $\bar{a}$ and $\bar{b}$. 1

Or

Find the dot product of the vectors $\bar{a} = \hat{i} - \hat{j} + \hat{k}$ and $\bar{b} = \hat{i} - \hat{k}$. 1

12. If $P(1, 3, 4)$ and $Q(2, 5, 3)$ be two points in space, find the unit vector along $\overrightarrow{PQ}$. 1

13. A and B appear for an interview for two vacancies in a company. The probability of A's selection is $\frac{1}{5}$ and that of B's selection is $\frac{1}{6}$. What is the probability that both of them got selected? 1

14. If A and B are events such that

$$P(A) = \frac{5}{11},\ P(B) = \frac{6}{11} \text{ and } P(A \cup B) = \frac{7}{11}$$

find $P\left(\frac{B}{A}\right)$. 1

15. Find $\bar{a} \times \bar{b}$ where $\bar{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\bar{b} = \hat{i} + 2\hat{j} - \hat{k}$. 1

Or

Find the vector equation of the straight line joining the points (1, 2, 3) and (2, 1, 4). 1

Choose the correct answer :

16. The value of $\int 2^x dx$ is

(a) $\frac{2^{x+1}}{x+1} + C$

(b) $2^x \log 2 + C$

(c) $\frac{2^x}{\log 2} + C$

(d) None of the above 1

17. The derivative of a constant function is

(a) a non-zero constant

(b) zero

(c) the function itself

(d) None of the above 1

Or

The second-order derivative of $\log x$ with respect to x is

(a) $\frac{1}{x}$

(b) $\frac{1}{x^2}$

(c) $-\frac{1}{x^2}$

(d) 1 1

18. If $f(x) = 2$, then the value of $f(2)$ is

(a) 2

(b) x

(c) x^2

(d) 2^x 1

19. If $\bar{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\bar{b} = \hat{i} - 9\hat{j} - 3\hat{k}$, then $\bar{a}$ and $\bar{b}$ are

(a) perpendicular vectors

(b) parallel vectors

(c) equal vectors

(d) None of the above 1

20. The maximum value of $Z = 4x + 3y$, subject to the constraints $x + y \leq 4$, $x \geq 0$, $y \geq 0$ is

(a) 8

(b) 10

(c) 12

(d) 16 1

SECTION—B

21. Show that the relation R in the set of real numbers $\mathbb{R}$ defined by $R = \{(a, b) : a \leq b\}$ is transitive but not symmetric. 2

22. Show that

$$\tan^{-1}\left(\frac{x}{\sqrt{a^2 - x^2}}\right) = \sin^{-1}\left(\frac{x}{a}\right)$$ 2

Or

Evaluate $\sin\left\{\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right\}$. 2

23. If

$$A = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix}$$

find the matrix X such that $2A + X = B$. 2

Or

Find the cofactors of the elements of the second column of the determinant

$$\begin{vmatrix} 8 & 4 & 2 \\ 2 & 9 & 4 \\ 1 & 2 & 8 \end{vmatrix}$$ 2

24. Is the function defined by

$$f(x) = \begin{cases} 2x + 3 & , \text{ if } x \leq 2 \\ 2x - 3 & , \text{ if } x > 2 \end{cases}$$

continuous at $x = 2$? Justify. 2

Or

Find $\frac{dy}{dx}$, when $x = \sin t$ and $y = \cos 2t$. 2

25. By using the properties of definite integral, show that

$$\int_0^1 x(1-x)^5\,dx = \frac{1}{42}$$ 2

Or

Evaluate $\int \frac{(\tan^{-1} x)^2}{4+4x^2}\,dx.$ 2

26. If $y=\sqrt{x+\sqrt{x+\sqrt{x+\sqrt{x+\cdots}}}}$ to ∞, then prove that

$$\frac{dy}{dx} = \frac{1}{2y-1}$$ 2

Or

Solve the following equation : 2

$$(x^2+1)\frac{dy}{dx} = xy$$

SECTION—C

27. *(a)* Find the value of k, if the function defined by

$$f(x) = \begin{cases} kx+1 & , \text{ if } x \le 5 \\ 3x-5 & , \text{ if } x > 5 \end{cases}$$

is continuous at $x=5$. 2

(b) Use definition to find the derivative of x^2. 2

28. If $y = \sin^{-1} x$, then prove that

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$$ 4

Or

Find the interval in which the function

$$f(x) = 2x^3 - 3x^2 - 36x + 7$$

is

(a) strictly increasing;

(b) strictly decreasing. 4

29. Prove that

$$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx = \frac{\pi}{4}$$ 4

Or

Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is parallel to the line $2x - y + 9 = 0$. 4

30. Find the Cartesian and vector equation of the line which passes through the point $(-2, 4, -5)$ and parallel to the line given by

$$\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$$ 4

Or

Find the vector equation of the plane passing through the intersection of the planes $\bar{r} \cdot (\hat{i} + \hat{j} + \hat{k}) - 6 = 0$ and $\bar{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) + 5 = 0$ and the point (1, 1, 1). 4

31. Find two positive numbers whose product is 49 and the sum is minimum. 4

Or

If $\bar{a} = 3\hat{i} - \hat{j}$ and $\bar{b} = 2\hat{i} + \hat{j} - 3\hat{k}$, then express $\bar{b}$ in the form $\bar{b} = \bar{c} + \bar{d}$ where $\bar{c}$ is parallel to $\bar{a}$ and $\bar{d}$ is perpendicular to $\bar{a}$. 4

32. If A and B are two events such that

$$2P(A) = P(B) = \frac{5}{13} \text{ and } P\left(\frac{A}{B}\right) = \frac{2}{5}$$

find P(not A and not B). 4

Or

Solve the following LPP graphically : 4

Maximize $Z = 4x + y$

subject to the constraints

$$x + y \leq 50$$
$$3x + y \leq 90$$
$$x \geq 0,\ y \geq 0$$

SECTION—D

33. If

$$A = \begin{bmatrix} 4 & 5 & 3 \\ 1 & 0 & 6 \\ 2 & 7 & 9 \end{bmatrix}$$

verify that $A \cdot (\text{adj}\, A) = (\text{adj}\, A) \cdot A = |A| I_3$. 6

Or

Solve the system equations by matrix method : 6

$$5x - y + z = 4$$
$$3x + 2y - 5z = 2$$
$$x + 3y - 2z = 5$$

34. Integrate the following : 3×2=6

(i) $\int \frac{(x+1)e^x}{\cos^2(xe^x)}dx$

(ii) $\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx$

35. Find the shortest distance between the lines

$$\bar{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$$
$$\bar{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$$ 6

36. State Bayes' theorem on probability. Use this theorem to solve the following [*(a)* or *(b)*] : 2+4=6

(a) An insurance company insured 2000 scooty drivers, 4000 taxi drivers and 6000 bus drivers in a particular year. The probability of their accidents are 0·01, 0·03 and 0·15 respectively. One of the insured drivers meets with an accident. What is the probability that the person drives a scooty?

Or

(b) First bag contains 3 red and 4 black balls, and second bag contains 5 red and 6 black balls. A ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from the second bag.

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2021

MATHEMATICS

(Old Course)

Full Marks : 100

Time : 3 hours

The figures in the margin indicate full marks for the questions

General Instructions :

(i) Write all the answers in the Answer Script.

(ii) The question paper consists of three Sections—A, B and C.

(iii) Section—A consists of 15 questions, carrying *2* marks each.

(iv) Section—B consists of 10 questions, carrying *4* marks each, out of which 2 questions have internal choices.

(v) Section—C has 5 questions, carrying *6* marks each, out of which 2 questions have internal choices.

SECTION—A

1. Let $R = \{(a, b) : a, b \in N \text{ and } a + 3b = 12\}$. Find *(i)* dom($R$) and *(ii)* range($R$). 2

2. If $f(x) = \frac{4x+3}{6x-4}$, $x \neq \frac{2}{3}$, show that $(f \circ f)(x) = x$. 2

3. Find the matrix X such that $2A - B + X = 0$, where

$$A = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix}$$ 2

4. Find adj A, if $A = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$. 2

5. If $y = \sin^{-1}(\cos x) + \cos^{-1}(\sin x)$, then prove that

$$\frac{dy}{dx} + 2 = 0$$ 2

6. Evaluate : 2

$$\int x \log x dx$$

7. Form the differential equation of the family of curves given by $y = A\cos 2x + B\sin 2x$, where A and B are arbitrary constants. 2

8. If $y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \cdots}}}\ \infty$, then prove that

$$\frac{dy}{dx} = \frac{\sin x}{(1 - 2y)}$$ 2

9. Find the value of K if the function

$$f(x) = \begin{cases} \dfrac{\sin 2x}{5x}, & \text{when } x \neq 0 \\ K \quad , & \text{when } x = 0 \end{cases}$$

is continuous at $x = 0$. 2

10. Prove that

$$\int_0^{\pi/2} \frac{\cos x}{\sin x + \cos x} dx = \frac{\pi}{4}$$ 2

11. Find the unit vector perpendicular to both $\vec{a}$ and $\vec{b}$, where $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$. 2

12. Find the angle between the vectors $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$. 2

13. Prove that

$$\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right) = \frac{x}{2}$$ 2

14. Find the angle between the lines

$$\frac{x+1}{1} = \frac{4-y}{-1} = \frac{z-5}{2} \text{ and } \frac{x+3}{3} = \frac{y-2}{5} = \frac{z+5}{4}$$ 2

15. If A and B are independent events such that $P(A) = 0 \cdot 3$ and $P(B) = 0 \cdot 4$, then find—

(i) $P(A \text{ and } B)$

(ii) $P(A \text{ or } B)$ 2

SECTION—B

16. Prove that

$$\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$$ 4

17. Using properties of determinant, prove that

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$ 4

18. Verify Rolle's theorem for the function $f(x) = x^2 - 5x + 6$ in [2, 3]. 4

19. Evaluate : 4

$$\int \frac{xe^x}{(1+x)^2}\,dx$$

20. Evaluate : 4

$$\int \frac{(2x-1)}{(x-1)(x+2)(x-3)}\,dx$$

21. Solve the differential equation : 4

$$(1+x^2)\frac{dy}{dx} + 2xy = \cos x$$

22. Evaluate $\int_0^2 (x^2 + x)dx$ as the limit of a sum. 4

23. Prove that

$$\int_0^{\pi/4} \log(1+\tan x)\,dx = \frac{\pi}{8}\log 2$$ 4

Or

Evaluate : 4

$$\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}\, dx$$

24. Find the equation of the plane through the line of intersection of the planes $x + y + z = 6$ and $2x + 3y + 4z + 5 = 0$ and passing through the point $(1, 1, 1)$. 4

Or

Find the image of the point $(1, 6, 3)$ in the line

$$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$$ 4

25. Find the shortest distance between the lines

$$\vec{r} = (6\hat{i} + 3\hat{k}) + \lambda(2\hat{i} - \hat{j} + 4\hat{k})$$

and $\vec{r} = (-9\hat{i} + \hat{j} - 10\hat{k}) + \mu(4\hat{i} + \hat{j} + 6\hat{k})$ 4

SECTION—C

26. Solve the following system of equations using matrix method : 6

$$2x - 3y + 5z = 16$$
$$3x + 2y - 4z = -4$$
$$x + y - 2z = -3$$

27. Using integration, find the area of ΔABC, whose vertices are $A(2, 0)$, $B(4, 5)$ and $C(6, 3)$. 6

28. In a bolt factory, three machines *A*, *B* and *C* manufacture 25%, 35% and 40% of the total production respectively. Of their respective outputs, 5%, 4% and 2% are defective. A bolt is drawn at random from the total product and it is found to be defective. Find the probability that it was manufactured by the machine *C*. 6

29. A square piece of tin of side 18 cm is to be made into a box without the top, but cutting a square piece from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum? Also, find the maximum volume of the box. 6

Or

Show that the maximum volume of the cylinder which can be inscribed in a sphere of radius $5\sqrt{3}$ cm is (500π) cm^3. 6

30. A manufacturer produces two types of steel trunk. He has two machines, *A* and *B*. The first type of trunk requires 3 hours on machine *A* and 3 hours on machine *B*. The second type of trunk requires 3 hours on machine *A* and 2 hours on machine *B*. Machines *A* and *B* can work at most for 18 hours and 15 hours per day respectively. He earns a profit of ₹ 30 and ₹ 25 per trunk of the first type and second type resepctively. How many trunks of each type must he make each day to make the maximum profit? 6

Or

If a young man rides his motorcycle at 25 km per hour, he has to spend ₹ 2 per km on petrol. If he rides it at a faster speed of 40 km per hour, the petrol cost increases to ₹ 5 per km. He has ₹ 100 to spend on petrol and wishes to find the maximum distance he can travel within one hour. Express this as a linear programming problem and then solve it. 6

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