Chapter-17

Understanding Shapes-III

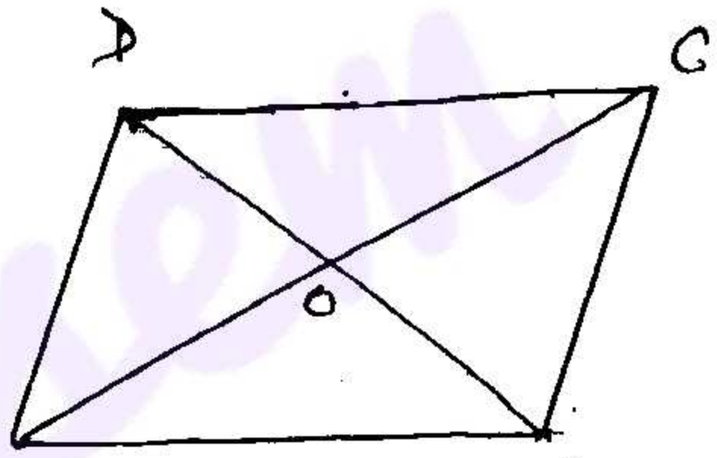
(Special types of quadrilaterals)

Exercise 17-1

Q.1 Given below is a parallelogram ABCD. Complete each statement along with the definition or property used.

(i) AD = (ii) ∠DCB = (iii) OC =

(iv) ∠DAB + ∠CDA =



Sol: (i) AD = BC (opposite sides of ||gm are equal)

(ii) ∠DCB = ∠DAB (opp. angles are equal)

(iii) OC = OA (diagonals bisect each other)

(iv) ∠DAB + ∠CDA = 180° (sum of adjacent angles is 180°)

Q.2 The following figures are parallelograms. Find the degree values of the unknowns x, y and z.

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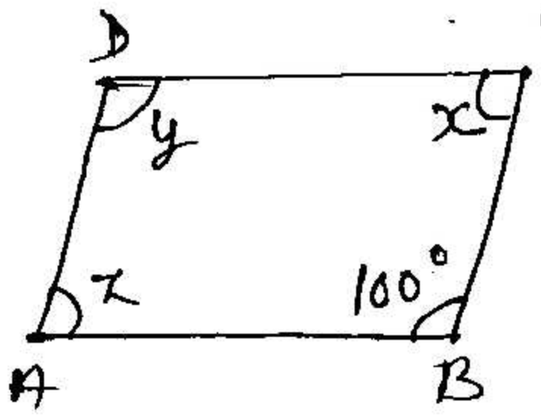
(i)



Sol: $\because x = z$, $y = 100^\circ$ (opp. angles of parallelogram)

also $x + 100^\circ = 180^\circ$ (adjacent $\angle$s)

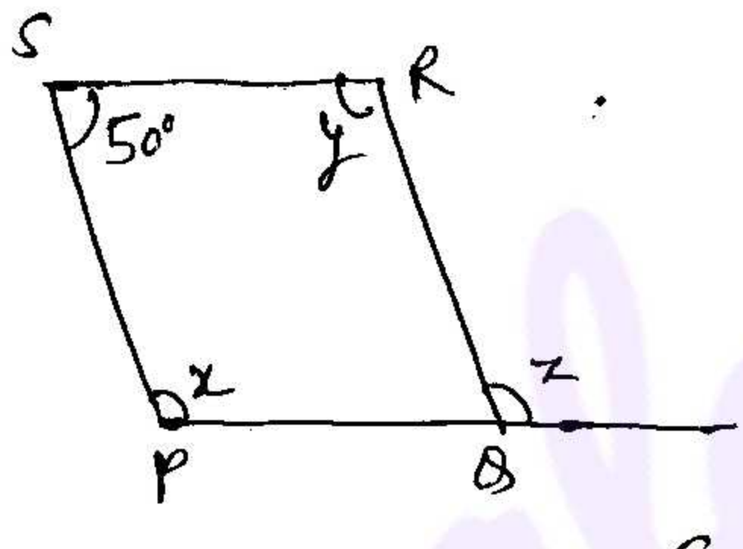
$\Rightarrow x = 180^\circ - 100^\circ$

$= 80^\circ$

$\therefore x = 80^\circ$.

hence $x = 80^\circ$, $y = 100^\circ$, $z = 80^\circ$

(ii)



$\because$ $50^\circ + y = 180^\circ$ (adjacent angles)

$\Rightarrow y = 180 - 50 = 130^\circ$.

also $y = z$, (alternate angles)

$\therefore z = 130^\circ$.

$x = z$ (corresponding angles

$\Rightarrow x = 130^\circ$

$\therefore x = 130^\circ$, $y = 130^\circ$, $z = 130^\circ$.

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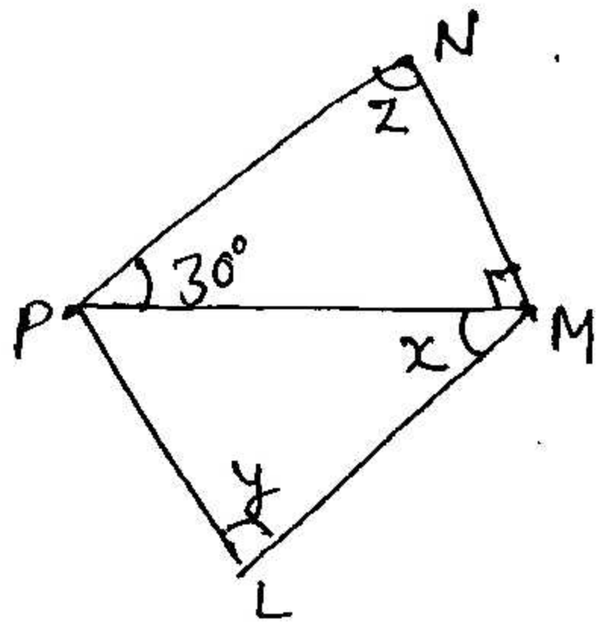
(iii)



$\because \quad 90° + z + 30° = 180°$

$\Rightarrow \quad 120° + z = 180$

$z = 180 - 120° = 60°$

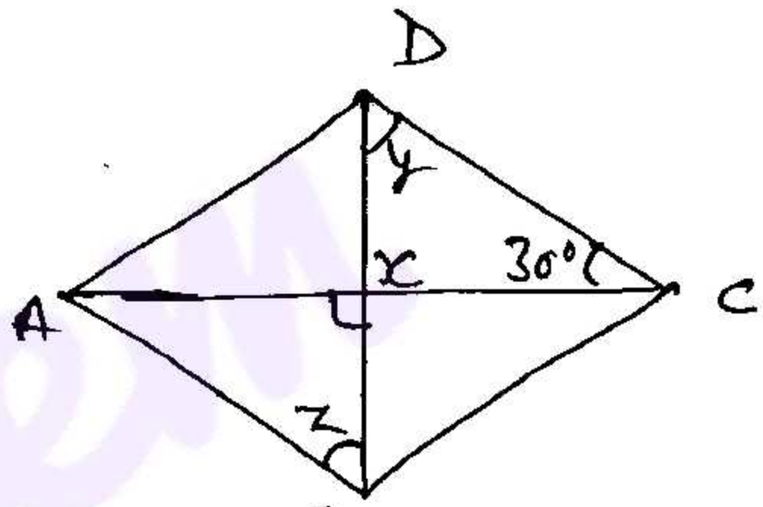
also, $y = z$ (opp. angles)

$\Rightarrow \quad y = 60°$

$\because \quad x = 30°$ (alternate Int. $\angle$s)

$\therefore \quad x = 30°, \; y = 60°, \; z = 60°$ Ans

(iv)



$x = 90°$ (vertically opposite angle)

$\because \quad x + y + 30° = 180°$ (Angle sum property of triangle)

$\Rightarrow \quad 90 + 30 + y = 180$

$\Rightarrow \quad 120 + y = 180$

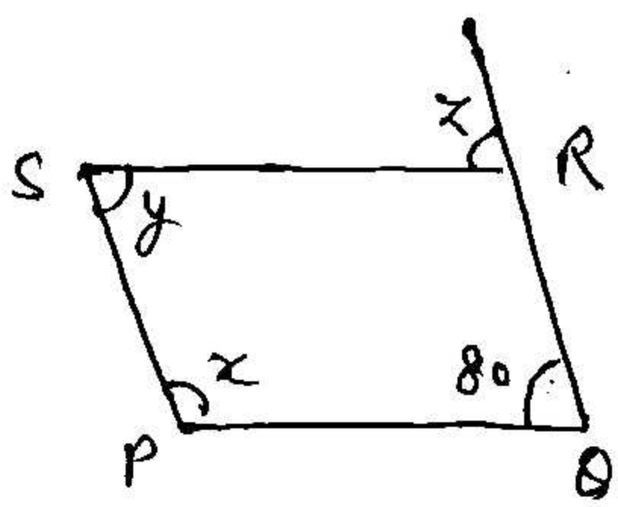
$y = 180 - 120 = 60°$

also $y = z$ (alt. Int. $\angle$s)

$\Rightarrow \quad z = 60°$

$\therefore \quad x = 90°, \; y = 60°, \; z = 60°$ Ans

(v)



$\because \quad y = 80°$ (opp. angles)

also $z = 80°$ (corr. $\angle$s)

$\because \quad x + 80 = 180$

$\Rightarrow x = 180 - 80 = 100°$

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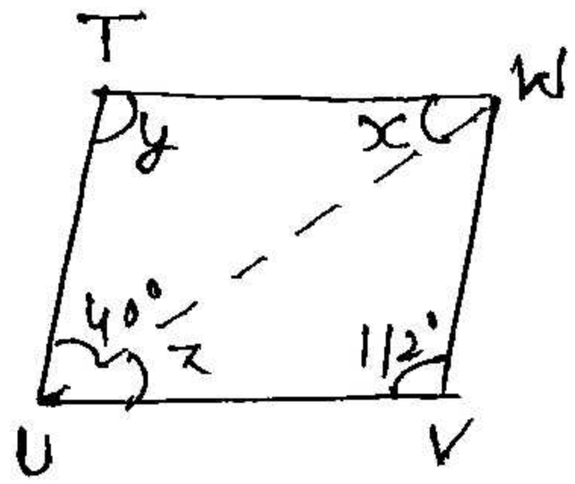
$\therefore$ $x = 100°$, $y = 80°$, $z = 80°$

(vi)



$\therefore$ $y = 112°$ (opp. angles)

Also $40° + z + 112° = 180°$

(Adjacent ∠s)

$\Rightarrow$ $152° + z = 180$

$z = 180 - 152$

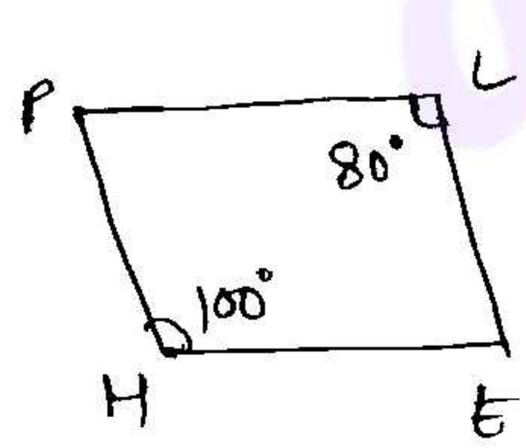
$= 28°$

$\therefore$ $x = z$ (alternate ∠s)

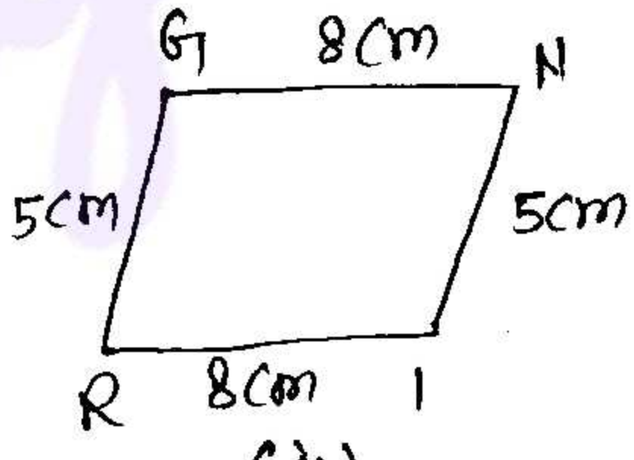
$\therefore$ $x = 28°$

Hence, $x = 28$, $y = 112°$, $z = 28°$

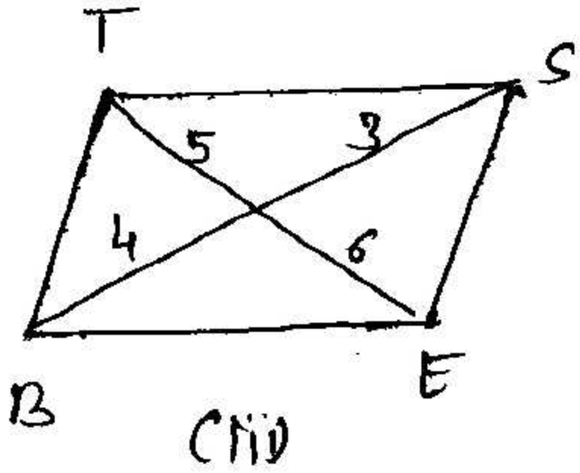
Q.3 Can the following figures be parallelograms. Justify your answer.



(i)



(ii)



(iii)

Sol

(i) HELP is not a parallelogram as opposite angles of Parallelogram are not equal.

(ii) RING is a parallelogram as opposite ~~angl~~ sides are equal.

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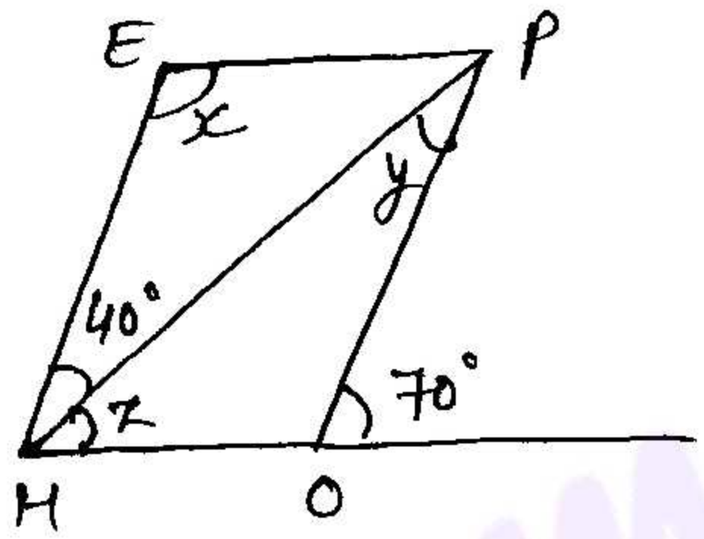
(iii) BEST is not a parallelogram.

∴ The diagonals does not bisect each other.

Q4 In the adjacent figure HOPE is a parallelogram. Find the angle measures x, y and z. State the geometrical truths you use to find them



Sol: ∵ HOPE is a ||gm.

$y = 40^\circ$ (alternate Int. $\angle$s)

∵ $y + z = 70^\circ$ (ext. $\angle$s property of $\triangle$)

$\Rightarrow 40 + z = 70$

$\Rightarrow z = 70 - 40^\circ$

$\Rightarrow z = 30^\circ$

also $x + 40^\circ + z = 180^\circ$ (Adjacent $\angle$s)

$\Rightarrow x + 40 + 30 = 180^\circ$

$\Rightarrow x = 180 - 70 = 110^\circ$

$\Rightarrow x = 110^\circ$

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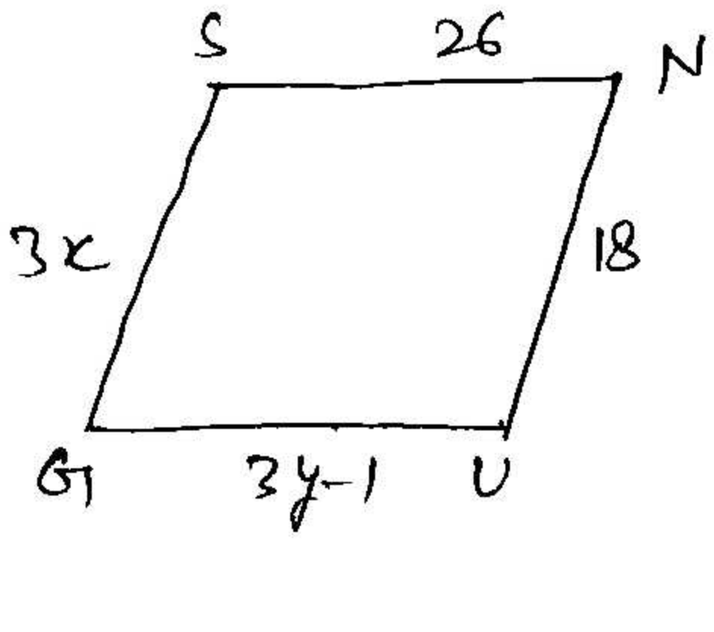
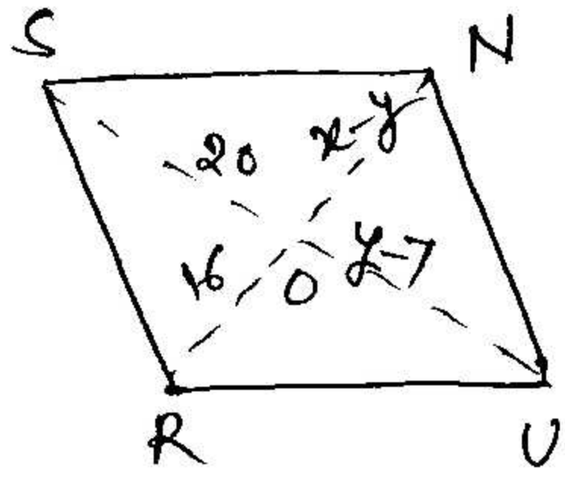
Q.5 In the following figures GUNS and RUNS are parallelograms. Find x and y.

(I)



(II)



Sol: (I) ∵ GUNS is a parallelogram.

$\Rightarrow 3y-1 = 26$ and $3x = 18$

$\Rightarrow 3y = 27$, $x = \frac{18}{3}$

$\Rightarrow y = 9$, $x = 6$

$\therefore x = 6, y = 9$

(II) ∵ RUNS is a parallelogram.

also diagonals of parallelogram bisect each other.

$\Rightarrow 16 = x - y$, $y - 7 = 20$

$\Rightarrow y = 20 + 7$

$= y = 27$

∵ $x - y = 16$

$\Rightarrow x - 27 = 16$

$\Rightarrow x = 16 + 27 = 43$

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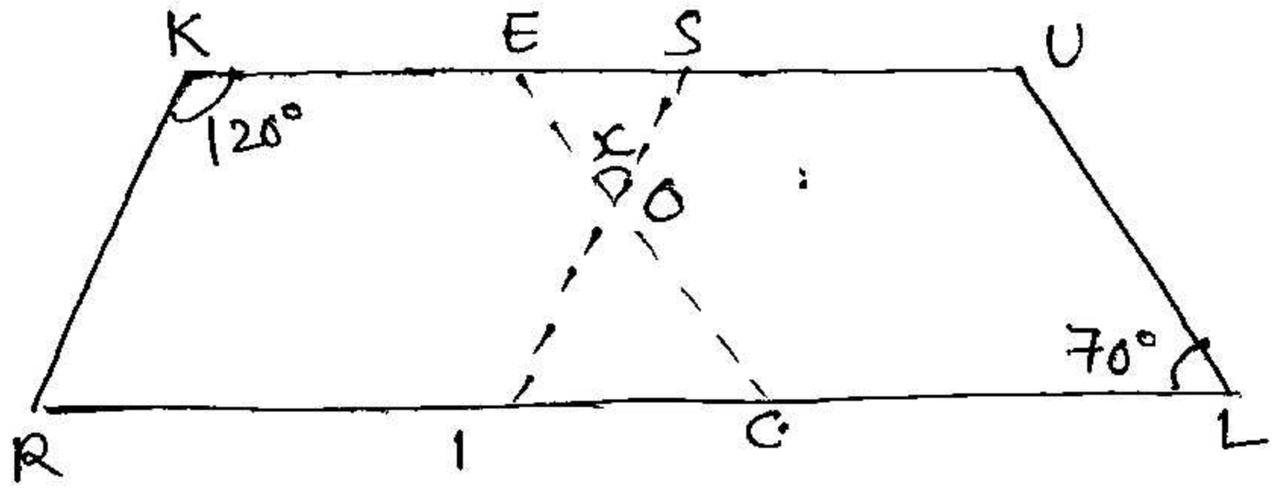
Q6 In the following figure RISK and CLUE are parallelograms. Find the measure of x.



Solution: In parallelogram RISK,

$\angle K + \angle S = 180°$ (adjacent angles)

$\Rightarrow 120° + \angle S = 180$

$\Rightarrow \angle S = 180 - 120° = 60°$

In parallelogram CLUE

$\angle E = \angle L$ (opp. angles)

$\Rightarrow \angle E = 70°$

Now In $\triangle$ EOS

$\angle E + \angle S + \angle x = 180°$

$\Rightarrow 70 + 60 + x = 180$

$\Rightarrow x = 180 - 130°$

$= 50°$

Q.7 Two opposite angles of a parallelogram are $(3x-2)°$ and $(50-x)°$. Find the measure of each angle of parallelogram.

Sol: $\because$ opposite angles of $\parallel^{gm}$ are eqnl.

$\therefore 3x - 2 = 50 - x$

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$\Rightarrow 3x+x = 50+2$

$\Rightarrow 4x = 52$

$x = 13^\circ$

$\therefore$ angles are $3x-2 = 3\times13-2$

$= 39-2 = 37^\circ$

$50-x = 50-13 = 37^\circ$.

$\because$ 37° + adjacent angle $= 180^\circ$

adjacent angle $= 180-37$

$= 143^\circ$.

$\therefore$ Angles are $37^\circ, 143^\circ, 37^\circ, 143^\circ$

Q8 If an angle of a parallelogram is two third of its adjacent angle. Find the angles of the parallelo.

Sol: let one angle $= x$

other angle $= \frac{2}{3}x$

$\because$ $x + \frac{2}{3}x = 180^\circ$ (Adjacent angles)

$\Rightarrow \frac{3x+2x}{3} = 180$

$\Rightarrow 5x = 540^\circ$

$x = 108^\circ$

$\therefore$ angles are $x = 108^\circ$

other angle $= \frac{2}{3}\times 108 = 72^\circ$

Therefore angles are $72^\circ, 108^\circ, 72^\circ, 108^\circ$

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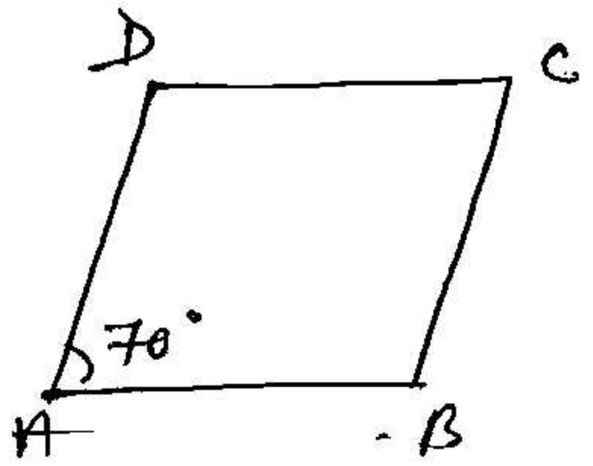
Q.9 The measure of one angle of parallelogram is 70°. What are the measures of remaing angles.

Sol: Let ABCD is a parallelogram



where $\angle A = 70°$

$\because \angle A + \angle B = 180°$ (adjacent ∠s)

$\Rightarrow 70° + \angle B = 180°$

$\Rightarrow \angle B = 180 - 70 = 110°$

Now, $\angle A = \angle C$, $\angle B = \angle D$

$\Rightarrow \angle C = 70°$, $\angle D = 110°$

Q.10 Two adjacent angles of a parallelogram are 1:2. Find the measure of all angles of the parallelogram.

Sol: Let adjacent angles are x, $2x$

$\because x + 2x = 180°$

$\Rightarrow 3x = 180°$

$\Rightarrow x = \frac{180}{3} = 60°$

Then $2x = 2 \times 60° = 120°$

$\therefore$ angles are $60°, 120°, 60°, 120°$

Q.11 In a parallelogram ABCD, $\angle D = 135°$. determine the measure of $\angle A$ and $\angle B$.

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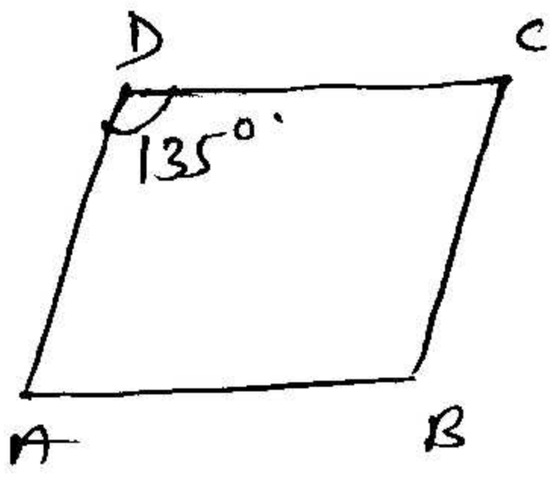


Sol:

$\because \angle A + \angle D = 180^\circ$ (Adjacent $\angle$s)

$\Rightarrow \angle A + 135 = 180^\circ$

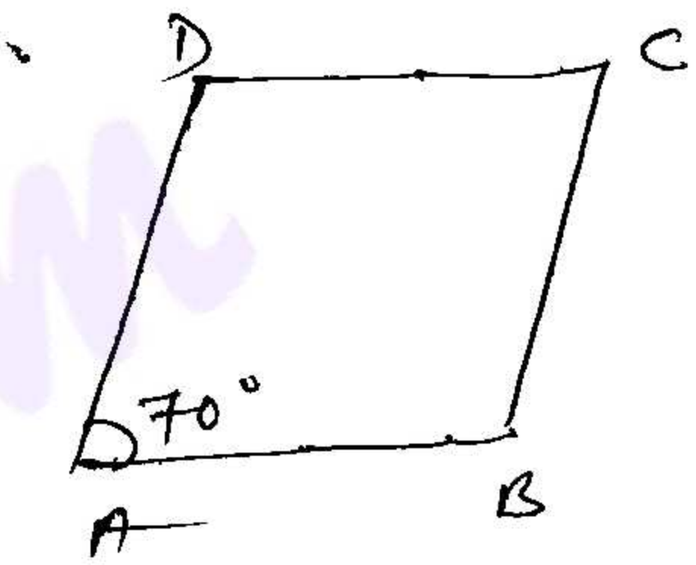
$\Rightarrow \angle A = 180 - 135$

$= 45^\circ$

also $\angle B = \angle D$ (opposite angles)

$\Rightarrow \angle B = 135^\circ$

Q.12 ABCD is a parallelogram in which $\angle A = 70^\circ$. Compute $\angle B$, $\angle C$ and $\angle D$.



Sol: $\because \angle A + \angle D = 180^\circ$

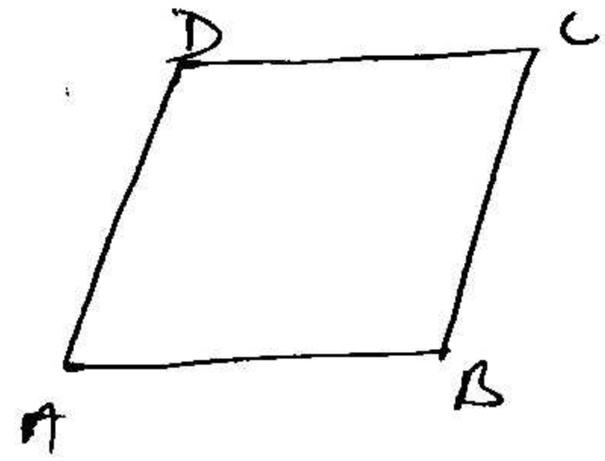
$\Rightarrow 70 + \angle D = 180^\circ$

$\Rightarrow \angle D = 180 - 70 = 110^\circ$

also $\angle A = \angle C = 70^\circ$

$\angle B = \angle D = 110^\circ$ {opp. angles}.

Q.13 The sum of two opposite angles of parallelogram is 130°. Find all the angles of the parallelogram.



Sol: We are given

$\angle A + \angle C = 130^\circ$ {$\angle A = \angle C$}

$\Rightarrow 2\angle A = 130^\circ$

$\Rightarrow \angle A = \frac{130}{2} = 65^\circ$

$\therefore \angle A = \angle C = 65^\circ$

Now, $\angle A + \angle B = 180^\circ$

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$\Rightarrow \quad 65^\circ + \angle B = 180^\circ$

$\Rightarrow \angle B = 180 - 65 = 115^\circ$

$\therefore \angle B = \angle D = 115^\circ$

Q.14 All the angles of a quadrilateral are equal to each other. Find the measure of each. Is the quadrilateral a parallelogram? What special type of parallelogram is it.

Sol: Let all angles measures x°.

Then $x + x + x + x = 360^\circ$

$\Rightarrow 4x = 360$

$\Rightarrow x = \frac{360}{4} = 90^\circ$

$\therefore$ It is a rectangle. Yes it is a parallelogram

Q.15 Two adjacent sides of a parallelogram are 4cm and 3 cm respectively. Find its perimeter.?

Sol: $P = 2(l+b)$

or

2 (Sum of adjacent sides)

$\Rightarrow P = 2(4+3) = 2 \times 7$

$= 14 cm.$

Q.16 The perimeter of parallelogram is 150 cm. One of its sides is greater than the other by 25 cm. Find the length of the sides of the parallelogram?

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Sol Let sides are x cm and $(x+25)$ cm.

$\because P = 2$ (sum of adjacent sides)

$\Rightarrow P = 2(x+x+25)$

$\Rightarrow 150 = 2(2x+25)$

$\Rightarrow 150 = 4x+50$

$\Rightarrow 150-50 = 4x$

$\Rightarrow 4x = 100$

$\Rightarrow x = \frac{100}{4} = 25$

$\therefore$ Sides are 25 cm and 50 cm.

Q17 The shorter side of parallelogram is 4.8 cm and the longer side is half as much again as the shorter side. And the perimeter of parallelogram.

Sol: Shorter side $= 4.8$ cm

longer side $= 4.8 + \frac{1}{2} \times 4.8$

$= 4.8 + 2.4 = 7.2$ cm

$\therefore P = 2$ (sum of adjacent sides)

$= 2(4.8+7.2) = 2 \times 12$

$= 24$ cm.

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angles.

Q.18 Two adjacent ~~sides~~ of parallelogram are $(3x-4)^\circ$ and $(3x+10)^\circ$. Find the angles of the parallelogram

Sol: $\because$ Sum of adjacent angles is 180°.

$$3x-4+3x+10=180$$

$$\Rightarrow 6x+6=180$$

$$\Rightarrow 6x=180-6=174$$

$$\Rightarrow x=\frac{174}{6}=29$$

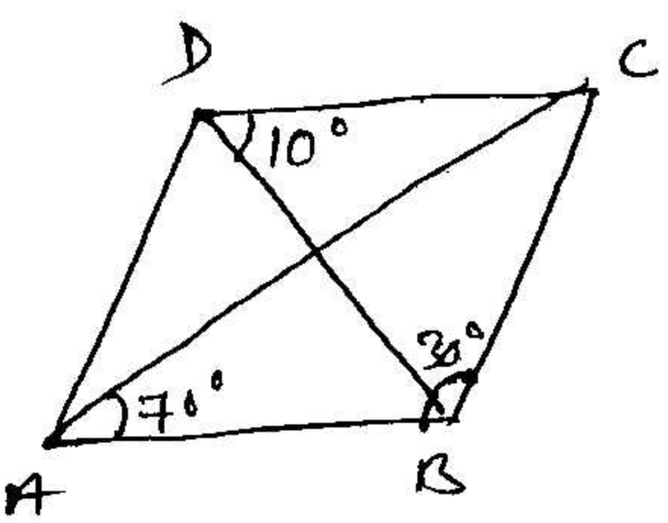
$\therefore$ angles are $3x-4 = 3\times29-4 = 87-4 = 83^\circ$

$3x+10 = 3\times29+10 = 87+10 = 97^\circ$.

Q.19 In a parallelogram ABCD, the diagonals bisect each other at O. If $\angle ABC=30^\circ$, $\angle BDC=10^\circ$, $\angle CAB=70^\circ$, find

$\angle DAB$, $\angle ADC$, $\angle BCD$, $\angle AOD$, $\angle DOC$, $\angle BOC$, $\angle AOB$, $\angle ACD$

$\angle CAB$, $\angle ADB$, $\angle ACB$, $\angle DBC$, $\angle DBA$



Sol: Given $\angle ABC=30^\circ$.

$\angle ABC=\angle ADC=30^\circ$ (opp. angles)

$\angle BDC=10$, $\angle CAB=70^\circ$

$\therefore \angle BDA=\angle ADB=\angle ADC-\angle BDC=30^\circ-10^\circ=20^\circ$

$\angle DAB=180-30=150^\circ$

$\angle ADB=\angle DBC=20^\circ$ (alternate $\angle$s)

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$\angle BCD = \angle DAB = 150^\circ$ (opp. angles are equal)

$\angle DBA = \angle BDC = 10^\circ$ (alternate angles)

In $\triangle ABC$

$\angle CAB + \angle ABC + \angle BCA = 180^\circ$

$\Rightarrow 70^\circ + 30^\circ + \angle BCA = 180^\circ$

$\Rightarrow \angle BCA = 180 - 100^\circ$

$= 80^\circ$

$\angle DAB = \angle DAC + \angle CAB = 70^\circ + 80^\circ = 150^\circ$

$\angle BCD = 150^\circ$ (opp. angles)

$\angle DCA = \angle CAB = 70^\circ$

In $\triangle DOC$

$\angle BDC + \angle ACD + \angle DOC = 180^\circ$

$10^\circ + 70^\circ + \angle DOC = 180^\circ$

$\angle DOC = 180 - 80 = 100^\circ$

$\angle DOC = \angle AOB = 100^\circ$ (Vertically opp. angles)

So, $\angle DOC + \angle AOD = 180^\circ$ (L.P)

$\Rightarrow 100 + \angle AOD = 180$

$\Rightarrow \angle AOD = 80^\circ$.

So, $\angle AOD = \angle BOC = 80^\circ$ (vertically opp. angles)

$\angle CAB = 70^\circ$

$\angle ABC + \angle BCD = 180^\circ$ (adjacent angles)

$\Rightarrow 30^\circ + \angle ACB + \angle ACD = 180^\circ$

$\Rightarrow 30^\circ + \angle ACB + 70^\circ = 180$

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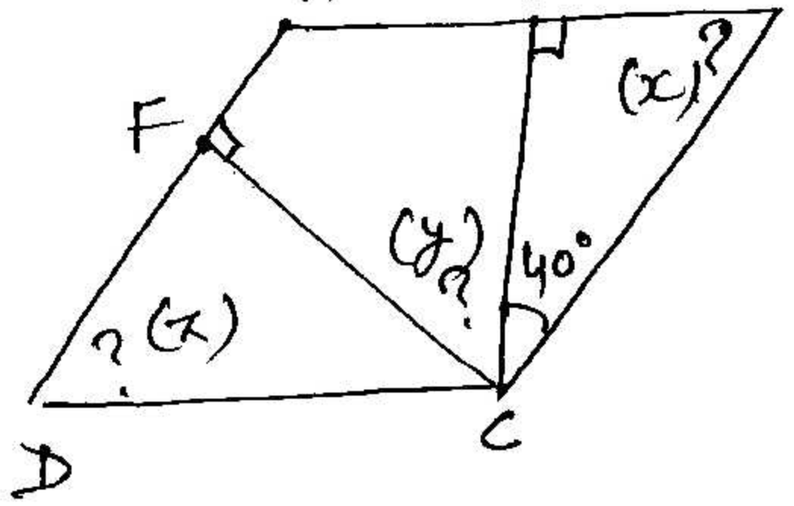
$\therefore \angle ACB = 180 - 100$

$= 80°$

Q.20 Find the angles marked with a question mark shown in fig 17.27.



(i)

In $\triangle BEC$

$40° + 90° + x = 180°$

$\Rightarrow 130° + x = 180°$

$\Rightarrow x = 180 - 130° = 50°$

$\therefore x = z = 50°$ (opp. angles)

$\therefore \angle D + \angle C = 180°$ (adjacent angles)

$\Rightarrow z + y + 40° = 180$

$\Rightarrow 50 + y + 40° = 180$

$\Rightarrow 90 + y = 180 \Rightarrow y = 180 - 90$

$= \underline{90°}$

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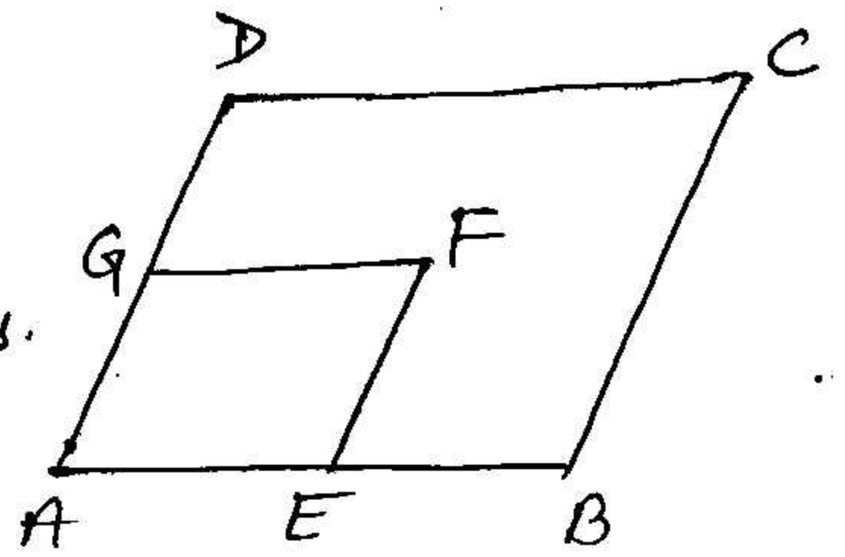
Q29. In fig. ABCD and AEFG are parallelogram. If $\angle C = 55^\circ$. what is the measure of $\angle F$?



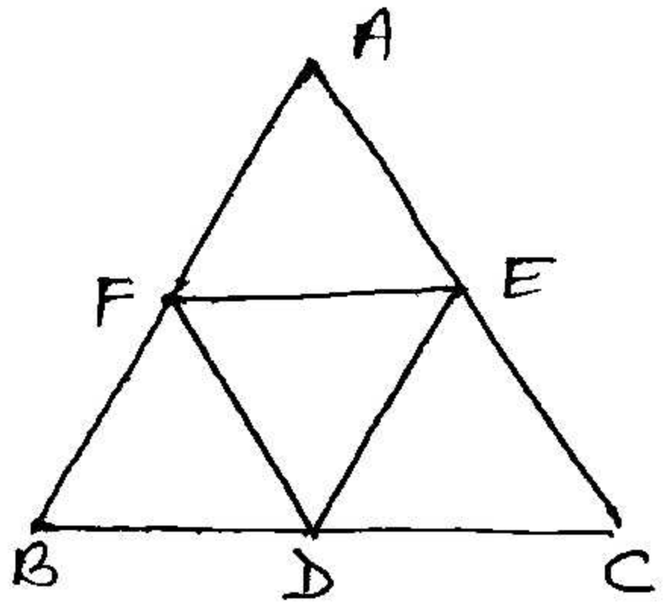
Sol: $\because$ ABCD is a 11^{gm}.

$\angle C = \angle A = 55^\circ$ (opposite angles of 11^{gm})

also AEFG is a 11^{gm}

$\angle F = \angle A = 55^\circ$ (opposite angles of parallelogram)

Q.23 In fig 17.29, BDEF and DCEF are each a parallelogram. Is it true that BD=DC. Why or why not?



Sol: $\because$ BDEF is a 11^{gm}.

$\therefore$ BD = EF (opposite sides of parallelogram are equal) — (1)

also DCEF is a 11^{gm} — (2)

$\therefore$ DC = EF (opposite sides are equal)

By eqn (1) and (2)

BD = DC

H.P

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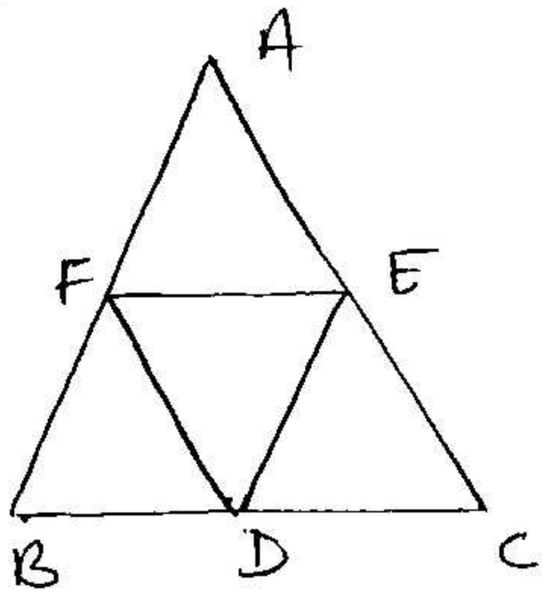
Q 24 In fig. 17.29, suppose it is known that DE = DF. Then, is △ ABC isosceles. why or why not?



Proof:

As we know by Mid point theorem, a line joining the mid points of sides of triangle is parallel to the third side and half of third side.

$\therefore \quad DE = \frac{1}{2} AB$

$DF = \frac{1}{2} AC$

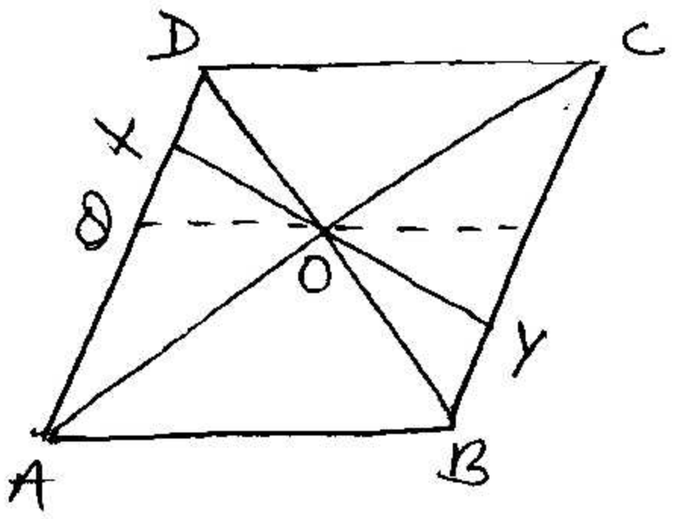
$\because \quad DE = DF$

$\Rightarrow \frac{1}{2} AB = \frac{1}{2} AC$

$\Rightarrow AB = AC$

$\therefore$ $\triangle ABC$ is an isosceles triangle.

Q.25 Diagonals of parallelogram ABCD intersect at O as shown in fig. 17.30. XY contains O, and X, Y are points on opposite sides of the parallelogram. Give reasons for each of the following.



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(i) $OB = OD$ (ii) $\angle OBY = \angle ODX$

(iii) $\angle BOY = \angle DOX$ (iv) $\triangle BOY \cong \triangle DOX$

Now, State If XY is bisected at O.

Solution : (i) $OB = OD$

$\because$ diagonals of parallelogram bisect each other.

(ii) $\angle OBY = \angle ODX$ { $\because$ $AD \parallel BC$ alternate interior angles}

(iii) $\angle BOY = \angle DOX$ (vertically opposite angles)

(iv) $\triangle BOY \cong \triangle DOX$

In $\triangle BOY$ and $\triangle DOX$

$OB = OD$

$\angle BOY = \angle DOX$

$\angle OBY = \angle ODX$ (alternate int. angles)

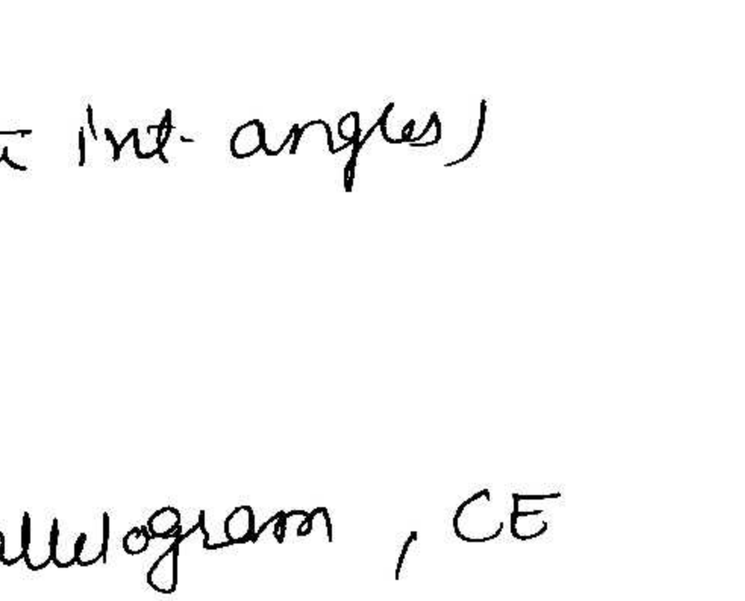
By AAS $\triangle BOY \cong \triangle DOX$

Q.26 In fig 17.31 ABCD is a parallelogram, CE bisects $\angle C$ and AF bisects $\angle A$. In each of the following statement is true, give a reason for the same.

(i) $\angle A = \angle C$ (ii) $\angle FAB = \frac{1}{2}\angle A$

(iii) $\angle DCE = \frac{1}{2}\angle C$ (iv) $\angle CEB = \angle FAB$

(v) $CE \parallel AF$.



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Sol: (i) $\angle A = \angle C$ (opposite angles of parallelogram)

(ii) $\angle FAB = \frac{1}{2}\angle A$ ($\because$ AF is bisector of $\angle A$)

(iii) $\angle DCE = \frac{1}{2}\angle C$ ($\because$ EC is bisector of $\angle C$)

(iv) $\because \angle A = \angle C$ (opposite angles of $\parallel^{gm}$)

$\Rightarrow \frac{1}{2}\angle A = \frac{1}{2}\angle C$

$\Rightarrow \angle FAB = \angle ECF$ — (1)

also $\angle ECF = \angle CEB$ — (2) (alternate angle)

$\therefore \angle FAB = \angle CEB$

(v) $\because \angle FAB = \angle CEB$

These are corresponding angles, as we know if corresponding angles are equal, lines are parallel

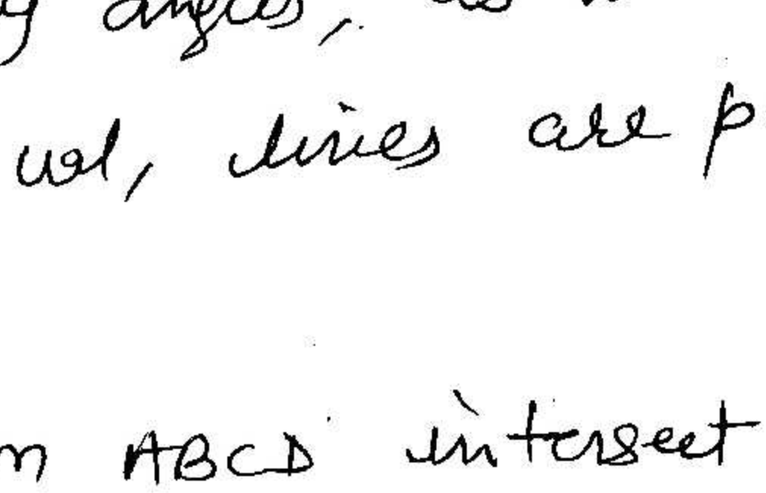
Q.27 Diagonals of a parallelogram ABCD intersect at O. AL and CM are drawn perpendiculars to BD such that L and M lee on BD. Is AL=CM? Why or why not?

Sol: In $\triangle AOL$ and $\triangle COM$

$AO = OC$ (diagonals bisect each other)

$\angle AOL = \angle COM$ (vertically opp)

$\angle ALO = \angle CMO$ (90°)



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By AAS

$\Delta AOL \cong \Delta COM$

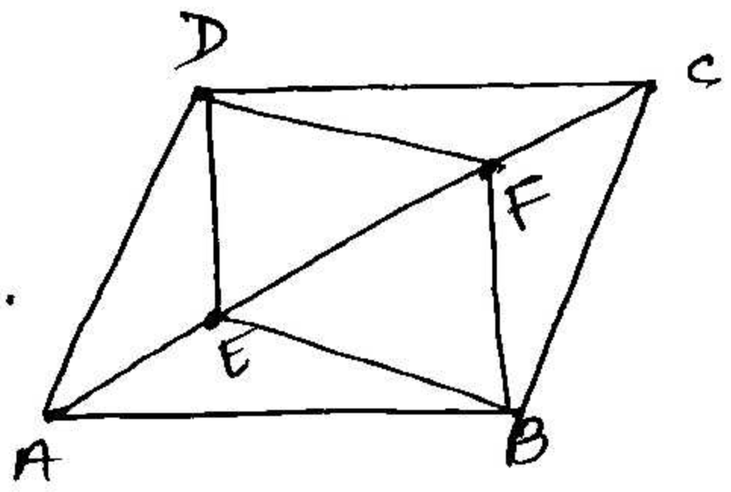
$\Rightarrow AL = CM$ (By CPCT)

Q.28 Points E and F lie on diagonal AC of parallelogram ABCD such that $AE = CF$. What type of quadrilateral is BFDE?

Sol:

Given: ABCD is a parallelogram.

$AE = CF$



Sol: In ΔADE and ΔCBF

$AD = BC$

$AE = CF$

$\angle DAE = \angle BCF$ (alternate angles)

$\therefore$ By SAS $\Delta ADE \cong \Delta CBF$

$\therefore DE = BF$ (By CPCT) — (1)

Similarly $\Delta CDF \cong \Delta ABE$

$\Rightarrow DF = BE$ (By CPCT) — (2)

Now By (1) and (2), opposite sides of quadrilateral BEFD are equal.

$\therefore$ BFDE is a parallelogram.

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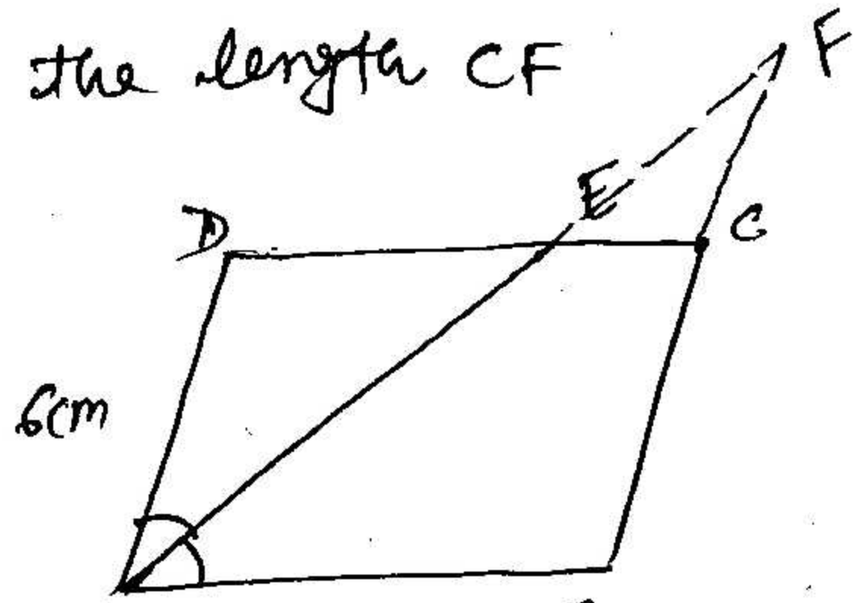
Q.29 In a parallelogram ABCD, AB = 10 cm, AD = 6 cm, The bisectors of ∠A meets DC in E, AE and BC Produced meet at F. Find the length CF



Sol:

here,

$\angle DAE = \angle FAB$ — (1)

(∵ AE is bisector of ∠A)

also $\angle DAE = \angle AFB$ (alternate int. ∠s) — (2)

By (1) and (2)

$\angle FAB = \angle AFB$

∴ $AB = BF$ (opp. sides of equal angles are equal)

⇒ $BF = 10$ cm

Now $CF = BF - BC$

$= 10 - 6$ {∵ $AD = BC = 6$cm}

$= 4$ cm.

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