

Quadratic EquationsExercise 4.1

$$4.1 (i) x^2 + 6x - 4 = 0$$

$\therefore$ It is a quadratic eq.

$$(ii) \sqrt{3}x^2 - 2x + \frac{1}{2} = 0$$

$\therefore$ It is a quadratic eq.

$$(iii) x^2 + \frac{1}{x^2} = 5$$

$$\Rightarrow x^4 + 1 = 5x^2$$

$$\Rightarrow x^4 - 5x^2 + 1 = 0$$

$\therefore$ It is not a quadratic eq.

$$(iv) x - \frac{3}{x} = x^2$$

$$\Rightarrow x^2 - 3 = x^3$$

$$\Rightarrow x^3 - x^2 + 3 = 0$$

$\therefore$ It is not a quadratic eq.

$$(v) 2x^2 - \sqrt{3}x + 9$$

$\therefore$ It is a quadratic eq.

$$(vi) x^2 - 2x - \sqrt{x} - 5 = 0$$

$\therefore$ It is not a quadratic eq.

$$(vii) 3x^2 - 5x + 9 = x^2 - 7x + 3$$

$$\Rightarrow 3x^2 - x^2 - 5x + 7x + 9 - 3 = 0$$

$$\Rightarrow 2x^2 - 2x + 6 = 0$$

$\therefore$ It is a quadratic eq.

$$(viii) x + \frac{1}{x} = 1$$

$$\Rightarrow x^2 + 1 = x$$

$$\Rightarrow x^2 - x + 1 = 0$$

$\therefore$ It is a quadratic eq.

$$(ix) x^2 - 3x = 0$$

$\therefore$ It is a quadratic eq.

$$(x) \left(x + \frac{1}{x}\right)^2 = 3\left(x + \frac{1}{x}\right)$$

$$\Rightarrow \left(\frac{x^2 + 1}{x}\right)^2 = 3\left(\frac{x^2 + 1}{x}\right)$$

$$\Rightarrow \frac{x^4 + 2x^2 + 1}{x^2} = \frac{3x^2 + 3}{x}$$

$$\Rightarrow x(x^4 + 2x^2 + 1) = x^2(3x^2 + 3)$$

$$\Rightarrow x^5 + 2x^3 + x = 3x^4 + 3x^2$$

$$\Rightarrow x^5 - 3x^4 + 2x^3 + 3x^2 + x = 0$$

$\therefore$ It is not a quadratic eq.

$$(xi) (2x+1)(3x+2) = 6(x-1)(x-2)$$

$$\Rightarrow 6x^2 + 4x + 3x + 2 = 6(x^2 - 2x - x + 2)$$

$$\Rightarrow 6x^2 + 7x + 2 = 6x^2 - 18x + 12$$

$$\Rightarrow 7x + 18x + 2 - 12 = 0$$

$$\Rightarrow 25x - 10 = 0$$

$\therefore$ It is not a quadratic eq.

$$(xii) \quad x + \frac{1}{x} = x^2$$

$$\Rightarrow x^2 + 1 = x^3$$

$$\Rightarrow x^3 - x^2 - 1 = 0$$

$\therefore$ It is not a quadratic eq.

$$(xiii) \quad 16x^2 - 3 = (2x+5)(5x-3)$$

$$\Rightarrow 16x^2 - 3 = 10x^2 - 6x + 25x - 15$$

$$\Rightarrow 16x^2 - 10x^2 - 19x - 3 + 15 = 0$$

$$\Rightarrow 6x^2 - 19x + 12 = 0$$

$\therefore$ It is a quadratic eq.

$$(xiv) \quad (x+2)^3 = x^3 - 4$$

$$\Rightarrow x^3 + 8 + 6x(x+2) = x^3 - 4$$

$$\Rightarrow 8 + 6x^2 + 12x = -4$$

$$\Rightarrow 6x^2 + 12x + 8 + 4 = 0$$

$$\Rightarrow 6x^2 + 12x + 12 = 0$$

$\therefore$ It is a quadratic eq.

$$(xv) \quad x(x+1) + 8 = (x+2)(x-2)$$

$$\Rightarrow x^2 + x + 8 = x^2 - 4$$

$$\Rightarrow x + 8 + 4 = 0$$

$$\Rightarrow x + 12 = 0$$

$\therefore$ It is not a quadratic eq.

$$Q.2(v) \quad x^2 - 3x + 2 = 0$$

$$x = 2, x = -1$$

$$p(2) = (2)^2 - 3(2) + 2$$

$$= 4 - 6 + 2$$

$$= -2 + 0$$

$$= 0$$

$\therefore x = 2$ is a solution

$$p(-1) = (-1)^2 - 3(-1) + 2$$

$$= 1 + 3 + 2$$

$$= 6 \neq 0$$

$\therefore x = -1$ is not a solution.

Quadratic Equations

Exercise 4.1

2(iii) $x^2 - 3\sqrt{3}x + 6 = 0$
 $x = \sqrt{3}, x = -2\sqrt{3}$

$$\begin{aligned} p(\sqrt{3}) &= (\sqrt{3})^2 - 3\sqrt{3}(\sqrt{3}) + 6 \\ &= 3 - 9 + 6 \\ &= -6 + 6 = 0 \end{aligned}$$

$\therefore x = \sqrt{3}$ is a solution

$$\begin{aligned} p(-2\sqrt{3}) &= (-2\sqrt{3})^2 - 3\sqrt{3}(-2\sqrt{3}) + 6 \\ &= 12 + 18 + 6 \\ &= 36 \neq 0 \end{aligned}$$

$\therefore x = -2\sqrt{3}$ is not a solution.

(v) $2x^2 - x + 9 = x^2 + 4x + 3$

$$\Rightarrow 2x^2 - x^2 - x - 4x + 9 - 3 = 0$$

$$\Rightarrow x^2 - 5x + 6 = 0$$

$$x = 2, x = 3$$

$$\begin{aligned} p(2) &= (2)^2 - 5(2) + 6 \\ &= 4 - 10 + 6 \\ &= 10 - 10 = 0 \end{aligned}$$

$\therefore x = 2$ is a solution

$$\begin{aligned} p(3) &= (3)^2 - 5(3) + 6 \\ &= 9 - 15 + 6 \\ &= 15 - 15 = 0 \end{aligned}$$

$\therefore x = 3$ is a solution.

(vii) $a^2x^2 - 3abx + 2b^2 = 0$

$$x = \frac{a}{b}, x = \frac{b}{a}$$

$$\begin{aligned} p\left(\frac{a}{b}\right) &= a^2\left(\frac{a}{b}\right)^2 - 3ab\left(\frac{a}{b}\right) + 2b^2 \\ &= \frac{a^4}{b^2} - 3a^2 + 2b^2 \neq 0 \end{aligned}$$

$\therefore x = \frac{a}{b}$ is not a solution.

$$\begin{aligned}
 f\left(\frac{b}{a}\right) &= a^2 \left(\frac{b^2}{a^2}\right) - 3ab \left(\frac{b}{a}\right) + 2b^2 \\
 &= b^2 - 3b^2 + 2b^2 \\
 &= 3b^2 - 3b^2 = \boxed{0}
 \end{aligned}$$

$\therefore x = \frac{b}{a}$ is a solution.

Q. 3(i) $7x^2 + kx - 3 = 0$, $x = \frac{2}{3}$

$$7\left(\frac{2}{3}\right)^2 + k\left(\frac{2}{3}\right) - 3 = 0$$

$$7\left(\frac{4}{9}\right) + k\left(\frac{2}{3}\right) - 3 = 0$$

$$\frac{28}{9} + k\left(\frac{2}{3}\right) - 3 = 0$$

$$k\left(\frac{2}{3}\right) = 3 - \frac{28}{9}$$

$$k\left(\frac{2}{3}\right) = \frac{27 - 28}{9}$$

$$k = \frac{-1}{9} \times \frac{3}{2} = \frac{-1}{6}$$

$$\boxed{k = -\frac{1}{6}}$$

(ii) $kx^2 + \sqrt{2}x - 4 = 0$, $x = \sqrt{2}$

$$k(\sqrt{2})^2 + \sqrt{2}(\sqrt{2}) - 4 = 0$$

$$2k + 2 - 4 = 0$$

$$2k = 2$$

$$\boxed{k = 1}$$

$$4. \quad \sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 16}$$

$$\Rightarrow \sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} - \sqrt{4x^2 - 14x + 16} = 0$$

$$\text{If } x = 3,$$

$$\begin{aligned} p(3) &= \sqrt{(3)^2 - 4(3) + 3} + \sqrt{(3)^2 - 9} - \sqrt{4(3)^2 - 14(3) + 16} \\ &= \sqrt{9 - 12 + 3} + \sqrt{9 - 9} - \sqrt{36 - 42 + 16} \\ &= \sqrt{0} + \sqrt{0} - \sqrt{16} \\ &= -\sqrt{16} \neq 0 \end{aligned}$$

$\therefore x = 3$ is not its root.

$$5. \quad ax^2 + 7x + b = 0$$

$$x = \frac{2}{3}, \quad x = -3$$

$$p\left(\frac{2}{3}\right) \Rightarrow a\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) + b = 0$$

$$\Rightarrow \frac{4a}{9} + \frac{14}{3} + b = 0 \quad \text{--- (1)}$$

$$p(-3) \Rightarrow a(-3)^2 + 7(-3) + b = 0$$

$$\Rightarrow 9a - 21 + b = 0 \quad \text{--- (2)}$$

Subtract (1) from (2),

$$\left(9a - \frac{4a}{9}\right) - \left(\frac{14}{3} + 21\right) + (\cancel{b} - \cancel{b}) = 0$$

$$\frac{81a - 4a}{9} = \frac{14 + 63}{3}$$

$$\frac{77a}{9} = \frac{77}{3}$$

$$21 + a = 7 \times 8 = 56$$

$$\boxed{a = 35}$$

Putting value of a in ①

$$21 + 35 = 56$$

$$\frac{4a}{9} + \frac{14}{3} + b = 0$$

$$\frac{4 \times 35}{9} + \frac{14}{3} + b = 0$$

$$b = -\frac{4 \times 35}{9} - \frac{14}{3}$$

$$b = -\frac{140}{9} - \frac{42}{9}$$

$$\boxed{b = -\frac{182}{9}}$$

$$\textcircled{1} \quad d = 2 + \frac{1}{2} + \frac{1}{2} = 2$$

$$\textcircled{2} \quad d = 2 + \frac{1}{2} - \frac{1}{2} = 2$$

$$d = (2 - \frac{1}{2}) + (\frac{1}{2} + \frac{1}{2}) = 2$$

$$2 + \frac{1}{2} = 2 + \frac{1}{2}$$

$$2 = 2$$

$$2 = 2$$