

12/08/2020

classmate

Date _____
Page _____

Ch-4 (P+X) F- (P+X) F Exercise 4.6 = (F-X) (P+X)

Q.1 (iii) $\frac{3}{5}x^2 - \frac{2}{3}x + 1 = 0$

$$a = \frac{3}{5}, b = -\frac{2}{3}, c = 1$$

$$\begin{aligned} D &= b^2 - 4ac \\ &= \left(-\frac{2}{3}\right)^2 - \left(4 \times \frac{3}{5} \times 1\right) \\ &= \frac{4}{9} - \frac{12}{5} = \frac{20 - 108}{45} \\ &= \frac{-88}{45} \end{aligned}$$

As $D < 0$,

$\therefore$ Roots are not real.

(v) $3x^2 - 2\sqrt{6}x + 2 = 0$

$$a = 3, b = -2\sqrt{6}, c = 2$$

$$\begin{aligned} D &= b^2 - 4ac \\ &= (-2\sqrt{6})^2 - (4 \times 3 \times 2) \\ &= 24 - 24 = 0 \end{aligned}$$

As $D = 0$,

$\therefore$ Roots are real and equal.

(vi) $4x^2 + 4\sqrt{3}x + 3 = 0$

$$a = 4, b = +4\sqrt{3}, c = 3$$

$$D = b^2 - 4ac$$

$$= (4\sqrt{3})^2 - (4 \times 4 \times 3)$$

$$= 48 - 48 = \boxed{0}$$

As $D = 0$,

$\therefore$ Roots are real and equal.

$$(ii) \quad kx^2 - 2\sqrt{5}x + 4 = 0$$

$$a = k, b = -2\sqrt{5}, c = 4$$

$$D = b^2 - 4ac$$

$$= (-2\sqrt{5})^2 - (4 \times k \times 4)$$

$$= 20 - 16k$$

As roots are real & equal, $\therefore D = 0$

ATQ,

$$20 - 16k = 0$$

$$16k = 20$$

$$k = \frac{20}{16}$$

$$k = \frac{5}{4}$$

$$\boxed{k = \frac{5}{4}}$$

$$(iv) \quad 4x^2 + kx + 9 = 0$$

$$a = 4, b = k, c = 9$$

$$D = b^2 - 4ac$$

$$= (k)^2 - (4 \times 4 \times 9)$$

$$= k^2 - 144$$

As roots are real & equal, $\therefore D = 0$

ATQ,

$$k^2 - 144 = 0$$

$$k^2 = 144$$

$$k = \sqrt{144}$$

$$k = \pm 12$$

$$(vi) \quad 9x^2 - 24x + k = 0$$

$$a = 9, b = -24, c = k$$

$$D = b^2 - 4ac$$

$$= (-24)^2 - (4 \times 9 \times k)$$

$$= 576 - 36k$$

As roots are real & equal, $\therefore D = 0$

ATQ,

$$576 - 36k = 0$$

$$36k = 576$$

$$k = \frac{576}{36}$$

$$16$$

$$k = 16$$

x

x

x

Ch-4
Exercise 4.6

(iii) $x^2 - 2(5+2k)x + 3(7+10k) = 0$

$a = 1, b = -2(5+2k), c = 3(7+10k)$

$D = b^2 - 4ac$

$= [-2(5+2k)]^2 - [4 \times 1 \times 3(7+10k)]$

$= 4(25 + 4k^2 + 20k) - 12(7+10k)$

$= 100 + 16k^2 + 80k - 84 - 120k$

$= 16k^2 - 40k + 16$

As roots are real & equal, $\therefore D = 0$

ATQ,

$16k^2 - 40k + 16 = 0$

$2k^2 - 5k + 2 = 0$

$2k^2 - 4k - k + 2 = 0$

$2k(k-2) - 1(k-2) = 0$

$(k-2)(2k-1) = 0$

$k-2 = 0$

$k = 2$

$2k-1 = 0$

$k = \frac{1}{2}$

(iv) $(3k+1)x^2 + 2(k+1)x + k = 0$

$a = 3k+1, b = 2(k+1), c = k$

$D = b^2 - 4ac$

$= [2(k+1)]^2 - [4 \times (3k+1) \times k]$

$= 4(k^2 + 2k + 1) - 4k(3k+1)$

$$= 4k^2 + 8k + 4 - 12k^2 - 4k$$

$$= -8k^2 + 4k + 4$$

As roots are real & equal, $\therefore D = 0$

ATQ,

$$-8k^2 + 4k + 4 = 0$$

$$2k^2 - k - 1 = 0$$

$$2k^2 - 2k + k - 1 = 0$$

$$2k(k-1) + 1(k-1) = 0$$

$$(k-1) \cdot (2k+1) = 0$$

$$k-1 = 0$$

$$\boxed{k=1}$$

$$2k+1 = 0$$

$$\boxed{k = -\frac{1}{2}}$$

x

x

Ch-4

Exercise 4.6

(x) $kx^2 + kx + 1 = -4x^2 - x$
 $kx^2 + 4x^2 + kx + x + 1 = 0$
 $(k+4)x^2 + (k+1)x + 1 = 0$

$$a = k+4, b = k+1, c = 1$$

$$D = b^2 - 4ac$$

$$= (k+1)^2 - [4 \times (k+4) \times 1]$$

$$= k^2 + 2k + 1 - 4k - 16$$

$$= k^2 - 2k - 15$$

As roots are real and equal, $\therefore D = 0$

ATQ,

$$k^2 - 2k - 15 = 0$$

$$k^2 - 5k + 3k - 15 = 0$$

$$k(k-5) + 3(k-5) = 0$$

$$(k-5)(k+3) = 0$$

$$k-5 = 0$$

$$\boxed{k = 5}$$

$$k+3 = 0$$

$$\boxed{k = -3}$$

(xi) $(k+1)x^2 + 2(k+3)x + (k+8) = 0$

$$a = k+1, b = 2(k+3), c = (k+8)$$

$$D = b^2 - 4ac$$

$$= [2(k+3)]^2 - [4 \times (k+1) \times (k+8)]$$

$$= 4(k^2 + 6k + 9) - 4(k^2 + 8k + k + 8)$$

$$= 4(k^2 + 6k + 9) - 4(k^2 + 9k + 8)$$

$$= 4k^2 + 24k + 36 - 4k^2 - 36k - 32$$

$$= -12k + 4$$

As roots are real & equal, $\therefore D = 0$
ATQ,

$$-12k + 4 = 0$$

$$12k = 4$$

$$k = \frac{4}{12}$$

$$\boxed{k = \frac{1}{3}}$$

(xiii) $(k+1)x^2 - 2(3k+1)x + 8k+1 = 0$
 $a = k+1, b = -2(3k+1), c = 8k+1$

$$D = b^2 - 4ac$$

$$= [-2(3k+1)]^2 - [4 \times (k+1)(8k+1)]$$

$$= 4(9k^2 + 6k + 1) - 4(8k^2 + k + 8k + 1)$$

$$= 4(9k^2 + 6k + 1) - 4(8k^2 + 9k + 1)$$

$$= 36k^2 + 24k + 4 - 32k^2 - 36k - 4$$

$$= 4k^2 - 12k$$

As roots are real & equal, $\therefore D = 0$
ATQ,

$$4k^2 - 12k = 0$$

$$k^2 - 3k = 0$$

$$k(k-3) = 0$$

$$\boxed{k = 0}$$

$$k - 3 = 0$$

$$\boxed{k = 3}$$

$$(iv) 5x^2 - 4x + 2 + k(4x^2 - 2x - 1) = 0$$

$$5x^2 - 4x + 2 + 4kx^2 - 2kx - k = 0$$

$$(5+4k)x^2 - (4+2k)x + (2-k) = 0$$

$$a = 5+4k, b = -(4+2k), c = 2-k$$

$$D = b^2 - 4ac$$

$$= [-(4+2k)]^2 - [4 \times (5+4k) \times (2-k)]$$

$$= 16 + 4k^2 + 16k - 4(10 - 5k + 8k - 4k^2)$$

$$= 16 + 4k^2 + 16k - 40 + 20k - 32k + 16k^2$$

$$= 20k^2 + 4k - 24$$

its roots are real & equal, $\therefore D = 0$

ATQ,

$$20k^2 + 4k - 24 = 0$$

$$5k^2 + k - 6 = 0$$

$$5k^2 + 6k - 5k - 6 = 0$$

$$k(5k+6) - 1(5k+6) = 0$$

$$(5k+6)(k-1) = 0$$

$$5k+6 = 0$$

$$\boxed{k = -\frac{6}{5}}$$

$$k-1 = 0$$

$$\boxed{k = 1}$$

$$(v) (4-k)x^2 + (2k+4)x + (8k+1) = 0$$

$$a = 4-k, b = 2k+4, c = 8k+1$$

$$D = b^2 - 4ac$$

$$= (2k+4)^2 - [4 \times (4-k) \times (8k+1)]$$

$$= 4k^2 + 16k + 16 - 4(32k + 4 - 8k^2 - k)$$

$$= 4k^2 + 16k + 16 - 128k - 16 + 32k^2 + 4k$$

$$= 36k^2 - 108k$$

As roots are real & equal, $\therefore D = 0$

ATQ,

$$36k^2 - 108k = 0$$

$$k^2 - 3k = 0$$

$$k(k-3) = 0$$

$$k = 0$$

$$k - 3 = 0$$

$$k = 3$$

(xvi) $(2k+1)x^2 + 2(k+3)x + (k+5) = 0$

$a = 2k+1, b = 2(k+3), c = k+5$

$$D = b^2 - 4ac$$

$$= [2(k+3)]^2 - [4 \times (2k+1) \times (k+5)]$$

$$= 4(k^2 + 6k + 9) - 4(2k^2 + 10k + k + 5)$$

$$= 4k^2 + 24k + 36 - 8k^2 - 40k - 4k - 20$$

$$= -4k^2 - 20k + 16$$

As roots are real & equal, $\therefore D = 0$

ATQ,

$$-4k^2 - 20k + 16 = 0$$

$$k^2 + 5k - 4 = 0$$

$$a = 1, b = 5, c = -4$$

$$D = b^2 - 4ac$$

$$= (5)^2 - [4 \times 1 \times (-4)]$$

$$= 25 + 16 = 41$$

$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$k = \frac{-5 \pm \sqrt{41}}{2}$$

$$k = \frac{-5 \pm \sqrt{41}}{2}$$

$$(vi) 4x^2 - 2(k+1)x + (k+4) = 0$$

$$a = 4, b = -2(k+1), c = k+4$$

$$D = b^2 - 4ac$$

$$= [-2(k+1)]^2 - [4 \times 4 \times (k+4)]$$

$$= 4(k^2 + 2k + 1) - 16(k+4)$$

$$= 4k^2 + 8k + 4 - 16k - 64$$

$$= 4k^2 - 8k - 60$$

As roots are real & equal, $\therefore D = 0$

ATQ,

$$4k^2 - 8k - 60 = 0$$

$$k^2 - 2k - 15 = 0$$

$$k^2 - 5k + 3k - 15 = 0$$

$$k(k-5) + 3(k-5) = 0$$

$$(k-5)(k+3) = 0$$

$$k - 5 = 0$$

$$\boxed{k = 5}$$

$$k + 3 = 0$$

$$\boxed{k = -3}$$

$$(vii) 4x^2 - 2(k+1)x + (k+1) = 0$$

$$a = 4, b = -2(k+1), c = k+1$$

$$D = b^2 - 4ac$$

$$= [-2(k+1)]^2 - [4 \times 4 \times (k+1)]$$

$$= 4(k^2 + 2k + 1) - 16(k+1)$$

$$= 4k^2 + 8k + 4 - 16k - 16$$

$$= 4k^2 - 8k - 12$$

As roots are real & equal, $\therefore D = 0$

ATQ,

$$4k^2 - 8k - 12 = 0$$

$$k^2 - 2k - 3 = 0$$

$$k^2 - 3k + k - 3 = 0$$

$$k(k-3) + 1(k-3) = 0$$

$$(k+1)(k-3) = 0$$

$$(k+1) = 0$$

$$k = -1$$

$$k-3 = 0$$

$$k = 3$$

Ch-4
Exercise 4.6

3(i) $2x^2 + 3x + k = 0$

$a = 2, b = 3, c = k$

$D = b^2 - 4ac$

$= (3)^2 - (4 \times 2 \times k)$

$= 9 - 8k$

As the roots are real, $\therefore D \geq 0$

ATQ,

$9 - 8k \geq 0$

$9 \geq 8k$

$k \leq \frac{9}{8}$

4(ii) $2x^2 - 5x - k = 0$

$a = 2, b = -5, c = -k$

$D = b^2 - 4ac$

$= (-5)^2 - [4 \times 2 \times (-k)]$

$= 25 + 8k$

As the roots are real, $\therefore D \geq 0$

ATQ,

$25 + 8k \geq 0$

$8k \geq -25$

$k \geq \frac{-25}{8}$

$$(N) \quad 3x^2 + 2x + k = 0$$

$$a = 3, b = 2, c = k$$

$$\begin{aligned} D &= b^2 - 4ac \\ &= (2)^2 - (4 \times 3 \times k) \\ &= 4 - 12k \end{aligned}$$

As the roots are real, $\therefore D \geq 0$

ATQ,

$$4 - 12k \geq 0$$

$$12k \geq 4$$

$$k \geq \frac{4}{12}$$

$$k \geq \frac{1}{3}$$

$$Q.4 (i) \quad x^2 - 2(k+1)x + k^2 = 0$$

$$a = 1, b = -2(k+1), c = k^2$$

$$D = b^2 - 4ac$$

$$= [-2(k+1)]^2 - (4 \times 1 \times k^2)$$

$$= 4(k+2k+1) - 4k^2$$

$$= 4k^2 + 8k + 4 - 4k^2$$

$$= 8k + 4$$

As the roots are real and equal, $\therefore D = 0$

ATQ,

$$8k + 4 = 0$$

$$8k = -4$$

$$k = \frac{-4}{8}$$

$$k = \frac{-1}{2}$$

$$(i) k^2 x^2 - 2(2k-1)x + 4 = 0$$

$$a = k^2, b = -2(2k-1), c = 4$$

$$D = b^2 - 4ac$$

$$= [-2(2k-1)]^2 - [4 \times k^2 \times 4]$$

$$= 4(4k^2 - 4k + 1) - 16k^2$$

$$= \cancel{16k^2} - 16k + 4 - \cancel{16k^2}$$

$$= -16k + 4$$

As the roots are real & equal, $\therefore D = 0$

ATQ,

$$-16k + 4 = 0$$

$$4 = 16k$$

$$k = \frac{4}{16}$$

$$\boxed{k = \frac{1}{4}}$$

$$(ii) (k+1)x^2 - 2(k-1)x + 1 = 0$$

$$a = k+1, b = -2(k-1), c = 1$$

$$D = b^2 - 4ac$$

$$= [-2(k-1)]^2 - [4 \times (k+1) \times 1]$$

$$= 4(k^2 - 2k + 1) - 4(k+1)$$

$$= 4k^2 - 8k + 4 - 4k - 4$$

$$= 4k^2 - 12k$$

As the roots are real & equal, $\therefore D = 0$

ATQ,

$$4k^2 - 12k = 0$$

$$k^2 - 3k = 0$$

$$k(k-3) = 0$$

$$\boxed{k=0}$$

$$k-3=0$$

$$\boxed{k=3}$$

$$(iv) \quad x^2 + k(2x + k - 1) + 2 = 0$$

$$\cancel{a=1, b}$$

$$\Rightarrow x^2 + 2kx + k^2 - k + 2 = 0$$

$$\Rightarrow x^2 + 2kx + (k^2 - k + 2) = 0$$

$$a = 1, b = 2k, c = k^2 - k + 2$$

$$D = b^2 - 4ac$$

$$= (2k)^2 - [4 \times 1 \times (k^2 - k + 2)]$$

$$= 4k^2 - 4k^2 + 4k - 8$$

$$= 4k - 8$$

As the roots are real & equal, $\therefore D = 0$

ATQ,

$$4k - 8 = 0$$

$$4k = 8$$

$$k = \frac{8}{4}$$

$$k = 2$$

$$\boxed{k=2}$$

$$Q.5(iii) \quad x^2 - 4kx + k = 0$$

$$a = 1, b = -4k, c = k$$

$$D = b^2 - 4ac$$

$$= (-4k)^2 - (4 \times 1 \times k)$$

$$= 16k^2 - 4k$$

∵ the roots are real & equal, $\therefore D = 0$

ATQ,

$$16k^2 - 4k = 0$$

$$4k^2 - k = 0$$

$$k(4k - 1) = 0$$

$$k = 0$$

$$4k - 1 = 0$$

$$k = \frac{1}{4}$$

18/08/2020

classmate

Date _____

Page _____

ch-4
exercise 4.6

Q.5 (iv) $kx(x - 2\sqrt{5}) + 10 = 0$
 $kx^2 - 2\sqrt{5}kx + 10 = 0$

$$a = k, b = -2\sqrt{5}k, c = 10$$

$$\begin{aligned} D &= b^2 - 4ac \\ &= (-2\sqrt{5}k)^2 - (4 \times k \times 10) \\ &= 20k^2 - 40k \end{aligned}$$

As the roots are real & equal, $\therefore D = 0$

ATQ,

$$20k^2 - 40k = 0$$

$$k(k - 2) = 0$$

$$\boxed{k = 2}$$

$$\boxed{k = 0}$$

(v) $kx(x - 3) + 9 = 0$
 $kx^2 - 3kx + 9 = 0$

$$a = k, b = -3k, c = 9$$

$$\begin{aligned} D &= b^2 - 4ac \\ &= (-3k)^2 - (4 \times k \times 9) \\ &= 9k^2 - 36k \end{aligned}$$

As the roots are real & equal, $\therefore D = 0$

ATQ,

$$9k^2 - 36k = 0$$

$$k^2 - 4k = 0$$

$$k(k-4) = 0$$

$$\boxed{k=0}$$

$$k-4=0$$

$$\boxed{k=4}$$

ii) $4x^2 + kx + 3 = 0$

$$a = 4, b = k, c = 3$$

$$\begin{aligned} D &= b^2 - 4ac \\ &= (k)^2 - (4 \times 4 \times 3) \\ &= k^2 - 48 \end{aligned}$$

As the roots are real & equal, $\therefore D = 0$

ATQ,

$$k^2 - 48 = 0$$

$$k^2 = 48$$

$$k = \sqrt{48}$$

$$k = \sqrt{16 \times 3}$$

$$\boxed{k = \pm 4\sqrt{3}}$$

iii) $kx^2 + 2x + 1 = 0$

$$a = k, b = 2, c = 1$$

$$\begin{aligned} D &= b^2 - 4ac \\ &= (2)^2 - (4 \times k \times 1) \\ &= 4 - 4k \end{aligned}$$

As the roots are real & distinct, $\therefore D > 0$

ATQ,

$$4 - 4k > 0$$

$$1 - k > 0$$

$$\begin{array}{|l} 1 > k \\ k < 1 \end{array}$$

Q.7 $(4-k)x^2 + (2k+4)x + (8k+1) = 0$

$$a = 4-k, b = 2k+4, c = 8k+1$$

$$D = b^2 - 4ac$$

$$= (2k+4)^2 - [4 \times (4-k)(8k+1)]$$

$$= 4k^2 + 16k + 16 - 4(32k + 4 - 8k^2 - k)$$

$$= 4k^2 + 16k + 16 - 4(-8k^2 + 31k + 4)$$

$$= 4k^2 + 16k + 16 + 32k^2 - 124k - 16$$

$$= 36k^2 - 108k$$

As the eq. is a perfect square, therefore the roots are real & equal and $D = 0$

ATQ,

$$36k^2 - 108k = 0$$

$$k^2 - 3k = 0$$

$$k(k-3) = 0$$

$$\boxed{k=0}$$

$$k-3=0$$

$$\boxed{k=3}$$

Q.10 $(2p+1)x^2 - (7p+2)x + (7p-3) = 0$

$$a = 2p+1, b = -(7p+2), c = 7p-3$$

As the roots are equal, $\therefore D = 0$

$$\therefore D = 0$$

$$b^2 - 4ac = 0$$

$$[-(7p+2)]^2 - [4 \times (2p+1) \times (7p-3)] = 0$$

$$(7p+2)^2 - 4(14p^2 - 6p + 7p - 3) = 0$$

$$49p^2 + 28p + 4 - 56p^2 + 24p - 28p + 12 = 0$$

$$-7p^2 + 24p + 16 = 0$$

$$7p^2 - 24p - 16 = 0$$

$$7p^2 - 28p + 4p - 16 = 0$$

$$7p(p-4) + 4(p-4) = 0$$

$$(p-4)(7p+4) = 0$$

$$p - 4 = 0$$

$$\boxed{p = 4}$$

$$7p + 4 = 0$$

$$\boxed{p = -\frac{4}{7}}$$

As roots are equal,

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm 0}{2a}$$

$$x = \frac{7p+2}{2(2p+1)}$$

$$\underline{p = 4}$$

$$x = \frac{7(4)+2}{2[2(4)+1]}$$

$$x = \frac{28+2}{2(8+1)}$$

$$x = \frac{30}{18}$$

$$\boxed{x = \frac{5}{3}}$$

$$\underline{p = -\frac{4}{7}}$$

$$x = \frac{7\left(-\frac{4}{7}\right)+2}{2\left[2\left(-\frac{4}{7}\right)+1\right]}$$

$$x = \frac{-4+2}{2\left[-\frac{8}{7}+1\right]}$$

$$x = \frac{-2}{2\left(-\frac{1}{7}\right)}$$

$$x = \frac{-2}{-\frac{2}{7}}$$

$$x = 7 \Rightarrow \boxed{x = 7}$$

Q.11

$$2x^2 + px - 15 = 0$$

$$x = -5 \text{ (given)}$$

$$2(-5)^2 + p(-5) - 15 = 0$$

$$2(25) - 5p - 15 = 0$$

$$50 - 15 - 5p = 0$$

$$-5p + 35 = 0$$

$$5p = 35$$

$$p = \frac{35}{5}$$

$$\boxed{p = 7}$$

$$p(x^2 + x) + k = 0$$

Putting value of p

$$7(x^2 + x) + k = 0$$

$$7x^2 + 7x + k = 0$$

$$a = 7, b = 7, c = k$$

$$D = b^2 - 4ac$$

$$= (7)^2 - (4 \times 7 \times k)$$

$$= 49 - 28k$$

As the roots are real & equal, $\therefore D = 0$

ATQ,

$$49 - 28k = 0$$

$$28k = 49$$

$$k = \frac{49}{28}$$

$$\frac{7}{4}$$

$$\boxed{k = \frac{7}{4}}$$

ex - 4Exercise 4.6

$$3x^2 + px - 8 = 0$$

$$x = 2 \text{ (given)}$$

$$3(2)^2 + p(2) - 8 = 0$$

$$12 + 2p - 8 = 0$$

$$4 + 2p = 0$$

$$2p = -4$$

$$p = \frac{-4}{2}$$

$$\boxed{p = -2}$$

$$4x^2 - 2px + k = 0$$

Putting value of p

$$4x^2 - 2(-2)x + k = 0$$

$$4x^2 + 4x + k = 0$$

$$a = 4, b = 4, c = k$$

$$D = b^2 - 4ac$$

$$= (4)^2 - (4 \times 4 \times k)$$

$$= 16 - 16k$$

As the roots are real & equal, $\therefore D = 0$

ATQ,

$$16 - 16k = 0$$

$$16k = 16$$

$$k = \frac{16}{16} \Rightarrow \boxed{k = 1}$$

Q.14

$$(\beta+1)x^2 - 6(\beta+1)x + 3(\beta+9) = 0, \beta \neq 1$$

$$a = \beta+1, b = -6(\beta+1), c = 3(\beta+9)$$

$$D = b^2 - 4ac$$

$$= [-6(\beta+1)]^2 - [4 \times (\beta+1) \times 3(\beta+9)]$$

$$= 36(\beta+1)^2 - 12(\beta+1)(\beta+9)$$

As the roots are real & equal, $\therefore D = 0$

ATQ,

$$36(\beta+1)^2 - 12(\beta+1)(\beta+9) = 0$$

$$36(\beta+1)^2 = 12(\beta+1)(\beta+9)$$

$$3 \cancel{36} (\beta+1)^{\cancel{2}} = \beta+9$$

$$\cancel{12} (\beta+1)$$

$$3(\beta+1) = \beta+9$$

$$3\beta+3 = \beta+9$$

$$3\beta - \beta = 9 - 3$$

$$2\beta = 6$$

$$\beta = \frac{6}{2}$$

$$\boxed{\beta = 3}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{6(\beta+1) \pm \sqrt{0}}{2(\beta+1)}$$

$$x = \frac{3 \cancel{6} (\beta+1)}{\cancel{2} (\beta+1)}$$

$$\boxed{x = 3}, \boxed{x = 3}$$

5i) $9a^2b^2x^2 - 24abcdx + 16c^2d^2 = 0$, $a \neq 0, b \neq 0$
 $a = 9a^2b^2x^2$, $b = -24abcdx$, $c = 16c^2d^2$

$$D = b^2 - 4ac$$

$$= (-24abcdx)^2 - (4 \times 9a^2b^2x^2 \times 16c^2d^2)$$

$$= \frac{576a^2b^2c^2d^2x^2}{x^2} - \frac{576a^2b^2c^2d^2x^2}{x^2}$$

$$= \boxed{0}$$

$\therefore$ The roots are real & equal.

iv) $(b+c)x^2 - (a+b+c)x + a = 0$

$$a = b+c, b = -(a+b+c), c = a$$

$$D = b^2 - 4ac$$

$$= [-(a+b+c)]^2 - [4 \times (b+c) \times a]$$

$$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca - 4ab - 4ca$$

$$= a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$$

$$= \boxed{(-a+b+c)^2}$$

As $D > 0$,

$\therefore$ The roots are real & distinct.

vi) $2x^2 + kx + 2 = 0$

$$a = 2, b = k, c = 2$$

$$D = b^2 - 4ac$$

$$= (k)^2 - (4 \times 2 \times 2)$$

$$= k^2 - 16$$

As the roots are real,

$$\therefore D \geq 0$$

$$k^2 - 16 > 0$$

$$k^2 > 16$$

$$k > \sqrt{16}$$

$$k > 4, k < -4$$

$$(iv) \quad 2x^2 + kx - 4 = 0$$

$$a = 2, b = k, c = -4$$

$$D = b^2 - 4ac$$

$$= (k)^2 - [4 \times 2 \times (-4)]$$

$$= k^2 + 32$$

As the roots are real,

$$\therefore D \geq 0$$

ATQ,

$$k^2 + 32 \geq 0$$

$$k^2 \geq -32$$

$$k^2 \geq \sqrt{-32}$$

$\therefore k$ is not a real number.

$$Q.17 \quad (b-x)x^2 + (c-a)x + (a-b) = 0$$

$$a = b-x, b = c-a, c = a-b$$

$$D = b^2 - 4ac$$

$$= (c-a)^2 - [4 \times (b-x) \times (a-b)]$$

As the roots are real & equal,

$$\therefore D = 0$$

ATQ,

$$(c-a)^2 - 4(b-x)(a-b) = 0$$

$$c^2 + a^2 - 2ca - 4(ab - b^2 - xa + bc) = 0$$

$$c^2 + a^2 - 2ac - 4ab + 4b^2 + 4ca - 4bc = 0$$

$$a^2 + 4b^2 + c^2 - 4ab - 4bc + 2ca = 0$$

$$(a - 2b + c)^2 = 0$$

$$a - 2b + c = 0$$

$$2b = a + c$$

Hence, proved.

$$(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$$

$$a = a^2 + b^2, c = -2(ac + bd), c = c^2 + d^2$$

$$D = b^2 - 4ac$$

$$= [-2(ac + bd)]^2 - [4 \times (a^2 + b^2) \times (c^2 + d^2)]$$

$$= 4(ac + bd)^2 - 4[a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2]$$

$$= 4(a^2c^2 + b^2d^2 + 2abcd) - 4[a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2]$$

$$= \cancel{4a^2c^2} + \cancel{4b^2d^2} + 8abcd - \cancel{4a^2c^2} - 4a^2d^2 - 4b^2c^2 - \cancel{4b^2d^2}$$

$$= 8abcd - 4a^2d^2 - 4b^2c^2$$

As the roots are real & equal,

$$\therefore D = 0$$

ATQ,

$$8abcd - 4a^2d^2 - 4b^2c^2 = 0$$

$$a^2d^2 + b^2c^2 - 2abcd = 0$$

$$(ad - bc)^2 = 0$$

$$ad - bc = 0$$

$$ad = bc$$

$$\boxed{\frac{a}{b} = \frac{c}{d}}$$

Hence, proved.

Q.19 $ax^2 + 2bx + c = 0$ ——— ①
 $bx^2 - 2\sqrt{ac}x + b = 0$ ——— ②

$$D_1 = (2b)^2 - (4 \times a \times c) = 4b^2 - 4ac$$

$$D_2 = (-2\sqrt{ac})^2 - (4 \times b \times b) = 4ac - 4b^2$$

As they have real roots, $\therefore D \geq 0$

ATQ,

$$D_1 \geq 0$$

$$4b^2 - 4ac \geq 0$$

$$4b^2 \geq 4ac$$

$$b^2 \geq ac \text{ ——— ③}$$

$$D_2 \geq 0$$

$$4ac - 4b^2 \geq 0$$

$$4ac \geq 4b^2$$

$$ac \geq b^2 \text{ ——— ④}$$

From ③ & ④,

$$\cancel{b^2} \geq ac \geq \cancel{b^2}$$

$$ac \geq b^2 \geq ac$$

$$\boxed{b^2 = ac}$$

Hence, proved

Q.21 $(x^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$
 $a = x^2 - ab$, $b = -2(a^2 - bc)$, $c = b^2 - ac$

$$D = b^2 - 4ac$$

$$= [-2(a^2 - bc)]^2 - [4 \times (x^2 - ab) \times (b^2 - ac)]$$

$$\begin{aligned}
 &= 4(a^4 + b^2c^2 - 2a^2bc) - 4[b^2c^2 - ac^3 - ab^3 + a^2bc] \\
 &= 4a^4 + 4b^2c^2 - 8a^2bc - 4b^2c^2 + 4ac^3 + 4ab^3 - 4a^2bc \\
 &= 4a^4 + 4ab^3 + 4ac^3 - 12a^2bc \\
 &= 4a(a^3 + b^3 + c^3 - 3abc)
 \end{aligned}$$

As the roots are real & equal, $\therefore D = 0$

At Q,

$$4a(a^3 + b^3 + c^3 - 3abc) = 0$$

$$4a = 0$$

$$a = \frac{0}{4}$$

$$\boxed{a = 0}$$

$$a^3 + b^3 + c^3 - 3abc = 0$$

$$\boxed{a^3 + b^3 + c^3 = 3abc}$$

Hence, proved