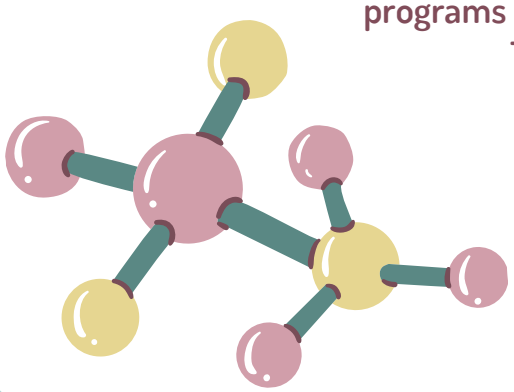
JOINT ADMISSION TEST FOR MASTERS

2024

IIT JAM 2024

Question Papers | Answer Key

The Joint Admission Test for Masters is a common admission test conducted every year for admission into Master of Science and other post-graduate science programs at Indian Institutes of Technology





aglasem.com

Special Instructions / Useful Data

$\mathbb{Z}_n = \{\bar{0}, \bar{1}, \ldots, \overline{n-1}\}$ denotes the additive group of integers modulo n

$\mathbb{R} =$ the set of all real numbers

$\mathbb{N} =$ the set of all positive integers

$\mathbb{Z} =$ the set of all integers

$\mathbb{C} =$ the set of all complex numbers

$\mathbb{Q} =$ the set of all rational numbers

$\gcd(r, n) =$ the greatest common divisor of the integers r and n

$S_n =$ the symmetric group of all permutations of $\{1,2, \ldots, n\}$

A_n = the group of all even permutations in S_n

$M_n(\mathbb{C}) =$ the set of all $n \times n$ matrices with entries from $\mathbb{C}$

$M_n(\mathbb{R}) =$ the set of all $n \times n$ matrices with entries from $\mathbb{R}$

$M^T =$ the transpose of the matrix M

$I_n =$ the $n \times n$ identity matrix

$P_n(x) =$ the real vector space of polynomials, in the variable x with real coefficients and having degree at most n, together with the zero polynomial. These polynomials are regarded as functions from $\mathbb{R}$ to $\mathbb{R}$

$\binom{n}{k} =$ the binomial coefficient defined as $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

$f \circ g =$ the composite function defined by $(f \circ g)(x) = f\big(g(x)\big)$

$A \setminus B$ = the complement of the set B in the set A, that is, $\{x \in A : x \notin B\}$

$\log x =$ the logarithm of x to the base e for a positive number x

$\mathbb{R}^n =$ the n-dimensional Euclidean space

$A \times B$ = the Cartesian product of the sets A and B

$M^{-1} =$ the inverse of an invertible matrix M

IIT Madras

Section A: Q.1 – Q.10 Carry ONE mark each.

Q.1 Let $y_c: \mathbb{R} \to (0, \infty)$ be the solution of the Bernoulli's equation

$$\frac{dy}{dx} - y + y^3 = 0, \qquad y(0) = c > 0.$$

Then, for every $c > 0$, which one of the following is true?

(A) $\lim_{x \to \infty} y_c(x) = 0$

(B) $\lim_{x \to \infty} y_c(x) = 1$

(C) $\lim_{x \to \infty} y_c(x) = e$

(D) $\lim_{x \to \infty} y_c(x)$ does not exist

IIT Madras

Q.2 For a twice continuously differentiable function $g: \mathbb{R} \to \mathbb{R}$, define

$$u_g(x,y) = \frac{1}{y}\int_{-y}^{y} g(x+t)dt \quad \text{for } (x,y) \in \mathbb{R}^2, \qquad y > 0.$$

Which one of the following holds for all such g?

(A) $\dfrac{\partial^2 u_g}{\partial x^2} = \dfrac{2}{y}\dfrac{\partial u_g}{\partial y} + \dfrac{\partial^2 u_g}{\partial y^2}$

(B) $\dfrac{\partial^2 u_g}{\partial x^2} = \dfrac{1}{y}\dfrac{\partial u_g}{\partial y} + \dfrac{\partial^2 u_g}{\partial y^2}$

(C) $\dfrac{\partial^2 u_g}{\partial x^2} = \dfrac{2}{y}\dfrac{\partial u_g}{\partial y} - \dfrac{\partial^2 u_g}{\partial y^2}$

(D) $\dfrac{\partial^2 u_g}{\partial x^2} = \dfrac{1}{y}\dfrac{\partial u_g}{\partial y} - \dfrac{\partial^2 u_g}{\partial y^2}$

IIT Madras

Q.3 Let $y(x)$ be the solution of the differential equation

$$\frac{dy}{dx} = 1 + y \sec x \quad \text{for } x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

that satisfies $y(0) = 0$. Then, the value of $y\left(\frac{\pi}{6}\right)$ equals

(A) $\sqrt{3}\ \log\left(\frac{3}{2}\right)$

(B) $\left(\frac{\sqrt{3}}{2}\right)\log\left(\frac{3}{2}\right)$

(C) $\left(\frac{\sqrt{3}}{2}\right)\log 3$

(D) $\sqrt{3}\ \log 3$

IIT Madras

Q.4 Let $\mathcal{F}$ be the family of curves given by

$$x^2 + 2hxy + y^2 = 1, \quad -1 < h < 1 .$$

Then, the differential equation for the family of orthogonal trajectories to $\mathcal{F}$ is

(A) $(x^2y - y^3 + y)\dfrac{dy}{dx} - (xy^2 - x^3 + x) = 0$

(B) $(x^2y - y^3 + y)\dfrac{dy}{dx} + (xy^2 - x^3 + x) = 0$

(C) $(x^2y + y^3 + y)\dfrac{dy}{dx} - (xy^2 + x^3 + x) = 0$

(D) $(x^2y + y^3 + y)\dfrac{dy}{dx} + (xy^2 + x^3 + x) = 0$

Q.5 Let G be a group of order 39 such that it has exactly one subgroup of order 3 and exactly one subgroup of order 13. Then, which one of the following statements is TRUE?

(A) G is necessarily cyclic

(B) G is abelian but need not be cyclic

(C) G need not be abelian

(D) G has 13 elements of order 13

IIT Madras

Q.6 For a positive integer n, let $U(n) = \{\bar{r} \in \mathbb{Z}_n : \gcd(r, n) = 1\}$ be the group under multiplication modulo n. Then, which one of the following statements is TRUE?

(A) $U(5)$ is isomorphic to $U(8)$

(B) $U(10)$ is isomorphic to $U(12)$

(C) $U(8)$ is isomorphic to $U(10)$

(D) $U(8)$ is isomorphic to $U(12)$

Q.7 Which one of the following is TRUE for the symmetric group S_{13}?

(A) S_{13} has an element of order 42

(B) S_{13} has no element of order 35

(C) S_{13} has an element of order 27

(D) S_{13} has no element of order 60

IIT Madras

Q.8 Let G be a finite group containing a non-identity element which is conjugate to its inverse. Then, which one of the following is TRUE?

(A) The order of G is necessarily even

(B) The order of G is not necessarily even

(C) G is necessarily cyclic

(D) G is necessarily abelian but need not be cyclic

Q.9 Consider the following statements.

P: If a system of linear equations $Ax = b$ has a unique solution, where A is an $m \times n$ matrix and b is an $m \times 1$ matrix, then $m = n$.

Q: For a subspace W of a nonzero vector space V, whenever $u \in V \setminus W$ and $v \in V \setminus W$, then $u + v \in V \setminus W$.

Which one of the following holds?

(A) Both P and Q are true

(B) P is true but Q is false

(C) P is false but Q is true

(D) Both P and Q are false

IIT Madras

Q.10 Let $g: \mathbb{R} \to \mathbb{R}$ be a continuous function. Which one of the following is the solution of the differential equation

$$\frac{d^2y}{dx^2} + y = g(x) \quad \text{for} \quad x \in \mathbb{R},$$

satisfying the conditions $y(0) = 0,\ y'(0) = 1$?

(A) $y(x) = \sin x - \int_0^x \sin(x-t)\, g(t)dt$

(B) $y(x) = \sin x + \int_0^x \sin(x-t)\, g(t)dt$

(C) $y(x) = \sin x - \int_0^x \cos(x-t)\, g(t)dt$

(D) $y(x) = \sin x + \int_0^x \cos(x-t)\, g(t)dt$

IIT Madras

Section A: Q.11 – Q.30 Carry TWO marks each.

Q.11 Which one of the following groups has elements of order $1, 2, 3, 4, 5$ but does not have an element of order greater than or equal to 6 ?

(A) The alternating group A_6

(B) The alternating group A_5

(C) S_6

(D) S_5

Q.12 Consider the group $G = \{A \in M_2(\mathbb{R}) : AA^T = I_2\}$ with respect to matrix multiplication. Let

$$Z(G) = \{A \in G : AB = BA, \text{ for all } \quad B \in G\}.$$

Then, the cardinality of $Z(G)$ is

(A) 1

(B) 2

(C) 4

(D) Infinite

IIT Madras

Q.13 Let V be a nonzero subspace of the complex vector space $M_7(\mathbb{C})$ such that every nonzero matrix in V is invertible. Then, the dimension of V over $\mathbb{C}$ is

(A) 1

(B) 2

(C) 7

(D) 49

IIT Madras

Q.14 For $n \in \mathbb{N}$, let

$$a_n = \frac{1}{(3n+2)(3n+4)} \quad \text{and} \quad b_n = \frac{n^3 + \cos(3^n)}{3^n + n^3}.$$

Then, which one of the following is TRUE?

(A) $\sum_{n=1}^{\infty} a_n$ is convergent but $\sum_{n=1}^{\infty} b_n$ is divergent

(B) $\sum_{n=1}^{\infty} a_n$ is divergent but $\sum_{n=1}^{\infty} b_n$ is convergent

(C) Both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are divergent

(D) Both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are convergent

IIT Madras

Q.15

Let $a = \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} \\ 0 \end{bmatrix}$. Consider the following two statements.

P: The matrix $I_4 - aa^T$ is invertible.

Q: The matrix $I_4 - 2aa^T$ is invertible.

Then, which one of the following holds?

(A) P is false but Q is true

(B) P is true but Q is false

(C) Both P and Q are true

(D) Both P and Q are false

IIT Madras

Q.16 Let A be a 6×5 matrix with entries in $\mathbb{R}$ and B be a 5×4 matrix with entries in $\mathbb{R}$. Consider the following two statements.

P: For all such nonzero matrices A and B, there is a nonzero matrix Z such that AZB is the 6×4 zero matrix.

Q: For all such nonzero matrices A and B, there is a nonzero matrix Y such that BYA is the 5×5 zero matrix.

Which one of the following holds?

(A) Both P and Q are true

(B) P is true but Q is false

(C) P is false but Q is true

(D) Both P and Q are false

IIT Madras

Q.17 Let $P_{11}(x)$ be the real vector space of polynomials, in the variable x with real coefficients and having degree at most 11, together with the zero polynomial. Let

$$E = \{s_0(x), s_1(x), \ldots, s_{11}(x)\}, \qquad F = \{r_0(x), r_1(x), \ldots, r_{11}(x)\}$$

be subsets of $P_{11}(x)$ having 12 elements each and satisfying

$$s_0(3) = s_1(3) = \cdots = s_{11}(3) = 0\,, \qquad r_0(4) = r_1(4) = \cdots = r_{11}(4) = 1\,.$$

Then, which one of the following is TRUE?

(A) Any such E is not necessarily linearly dependent and any such F is not necessarily linearly dependent

(B) Any such E is necessarily linearly dependent but any such F is not necessarily linearly dependent

(C) Any such E is not necessarily linearly dependent but any such F is necessarily linearly dependent

(D) Any such E is necessarily linearly dependent and any such F is necessarily linearly dependent

IIT Madras

Q.18 For the differential equation

$$y(8x - 9y)dx + 2x(x - 3y)dy = 0\,,$$

which one of the following statements is TRUE?

(A) The differential equation is not exact and has x^2 as an integrating factor

(B) The differential equation is exact and homogeneous

(C) The differential equation is not exact and does not have x^2 as an integrating factor

(D) The differential equation is not homogeneous and has x^2 as an integrating factor

IIT Madras

Q.19 For $x \in \mathbb{R}$, let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x.

For $x, y \in \mathbb{R}$, define

$$\min\{x, y\} = \begin{cases} x & \text{if } x \leq y, \\ y & \text{otherwise.} \end{cases}$$

Let $f: [-2\pi, 2\pi] \to \mathbb{R}$ be defined by

$$f(x) = \sin(\min\{x, x - \lfloor x \rfloor\}) \text{ for } x \in [-2\pi, 2\pi].$$

Consider the set $S = \{x \in [-2\pi, 2\pi]: f \text{ is discontinuous at } x\}$.

Which one of the following statements is TRUE?

(A) S has 13 elements

(B) S has 7 elements

(C) S is an infinite set

(D) S has 6 elements

IIT Madras

Q.20 Define the sequences $\{a_n\}_{n=3}^{\infty}$ and $\{b_n\}_{n=3}^{\infty}$ as

$$a_n = (\log n + \log\log n)^{\log n} \quad \text{and} \quad b_n = n^{\left(1+\frac{1}{\log n}\right)}.$$

Which one of the following is TRUE?

(A) $\sum_{n=3}^{\infty} \frac{1}{a_n}$ is convergent but $\sum_{n=3}^{\infty} \frac{1}{b_n}$ is divergent

(B) $\sum_{n=3}^{\infty} \frac{1}{a_n}$ is divergent but $\sum_{n=3}^{\infty} \frac{1}{b_n}$ is convergent

(C) Both $\sum_{n=3}^{\infty} \frac{1}{a_n}$ and $\sum_{n=3}^{\infty} \frac{1}{b_n}$ are divergent

(D) Both $\sum_{n=3}^{\infty} \frac{1}{a_n}$ and $\sum_{n=3}^{\infty} \frac{1}{b_n}$ are convergent

IIT Madras

Q.21 For $p, q, r \in \mathbb{R}$, $r \neq 0$ and $n \in \mathbb{N}$, let

$$a_n = p^n\, n^q \left(\frac{n}{n+2}\right)^{n^2} \text{ and } b_n = \frac{n^n}{n!\; r^n}\left(\sqrt{\frac{n+2}{n}}\right).$$

Then, which one of the following statements is TRUE?

(A) If $1 < p < e^2$ and $q > 1$, then $\sum_{n=1}^{\infty} a_n$ is convergent

(B) If $e^2 < p < e^4$ and $q > 1$, then $\sum_{n=1}^{\infty} a_n$ is convergent

(C) If $1 < r < e$, then $\sum_{n=1}^{\infty} b_n$ is convergent

(D) If $\frac{1}{e} < r < 1$, then $\sum_{n=1}^{\infty} b_n$ is convergent

IIT Madras

Q.22 Let $P_7(x)$ be the real vector space of polynomials, in the variable x with real coefficients and having degree at most 7, together with the zero polynomial. Let T: $P_7(x) \to P_7(x)$ be the linear transformation defined by

$$T\big(f(x)\big) = f(x) + \frac{df(x)}{dx}\ .$$

Then, which one of the following is TRUE?

(A) T is not a surjective linear transformation

(B) There exists $k \in \mathbb{N}$ such that T^k is the zero linear transformation

(C) 1 and 2 are the eigenvalues of T

(D) There exists $r \in \mathbb{N}$ such that $(T - I)^r$ is the zero linear transformation, where I is the identity map on $P_7(x)$

IIT Madras

Q.23 For $\alpha \in \mathbb{R}$, let $y_\alpha(x)$ be the solution of the differential equation

$$\frac{dy}{dx} + 2y = \frac{1}{1+x^2} \quad \text{for } x \in \mathbb{R}$$

satisfying $y(0) = \alpha$. Then, which one of the following is TRUE?

(A) $\lim_{x\to\infty} y_\alpha(x) = 0$ for every $\alpha \in \mathbb{R}$

(B) $\lim_{x\to\infty} y_\alpha(x) = 1$ for every $\alpha \in \mathbb{R}$

(C) There exists an $\alpha \in \mathbb{R}$ such that $\lim_{x\to\infty} y_\alpha(x)$ exists but its value is different from 0 and 1

(D) There is an $\alpha \in \mathbb{R}$ for which $\lim_{x\to\infty} y_\alpha(x)$ does not exist

IIT Madras

Q.24 Consider the following two statements.

P: There exist functions $f: \mathbb{R} \to \mathbb{R}$, $g: \mathbb{R} \to \mathbb{R}$ such that f is continuous at $x = 1$ and g is discontinuous at $x = 1$ but $g \circ f$ is continuous at $x = 1$.

Q: There exist functions $f: \mathbb{R} \to \mathbb{R}$, $g: \mathbb{R} \to \mathbb{R}$ such that both f and g are discontinuous at $x = 1$ but $g \circ f$ is continuous at $x = 1$.

Which one of the following holds?

(A) Both P and Q are true

(B) Both P and Q are false

(C) P is true but Q is false

(D) P is false but Q is true

IIT Madras

Q.25 Let $f: \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \frac{(x^2+1)^2}{x^4+x^2+1} \quad \text{for } x \in \mathbb{R}.$$

Then, which one of the following is TRUE?

(A) f has exactly two points of local maxima and exactly three points of local minima

(B) f has exactly three points of local maxima and exactly two points of local minima

(C) f has exactly one point of local maximum and exactly two points of local minima

(D) f has exactly two points of local maxima and exactly one point of local minimum

IIT Madras

Q.26 Let $f: \mathbb{R} \to \mathbb{R}$ be a solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\,\frac{dy}{dx} + y = 2e^x \text{ for } x \in \mathbb{R}.$$

Consider the following statements.

P: If $f(x) > 0$ for all $x \in \mathbb{R}$, then $f'(x) > 0$ for all $x \in \mathbb{R}$.

Q: If $f'(x) > 0$ for all $x \in \mathbb{R}$, then $f(x) > 0$ for all $x \in \mathbb{R}$.

Then, which one of the following holds?

(A) P is true but Q is false

(B) P is false but Q is true

(C) Both P and Q are true

(D) Both P and Q are false

IIT Madras

Q.27 For $a > b > 0$, consider

$$D = \left\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq a^2 \text{ and } x^2 + y^2 \geq b^2\right\}.$$

Then, the surface area of the boundary of the solid D is

(A) $4\pi(a+b)\sqrt{a^2-b^2}$

(B) $4\pi\left(a^2 - b\sqrt{a^2-b^2}\right)$

(C) $4\pi(a-b)\sqrt{a^2-b^2}$

(D) $4\pi\left(a^2 + b\sqrt{a^2-b^2}\right)$

IIT Madras

Q.28 For $n \geq 3$, let a regular n-sided polygon P_n be circumscribed by a circle of radius R_n and let r_n be the radius of the circle inscribed in P_n. Then

$$\lim_{n\to\infty}\left(\frac{R_n}{r_n}\right)^{n^2}$$

equals

(A) $e^{(\pi^2)}$

(B) $e^{\left(\frac{\pi^2}{2}\right)}$

(C) $e^{\left(\frac{\pi^2}{3}\right)}$

(D) $e^{(2\pi^2)}$

IIT Madras

Q.29 Let L_1 denote the line $y = 3x + 2$ and L_2 denote the line $y = 4x + 3$. Suppose that $f: \mathbb{R} \to \mathbb{R}$ is a four times continuously differentiable function such that the line L_1 intersects the curve $y = f(x)$ at exactly three distinct points and the line L_2 intersects the curve $y = f(x)$ at exactly four distinct points. Then, which one of the following is TRUE?

(A) $\frac{df}{dx}$ does not attain the value 3 on $\mathbb{R}$

(B) $\frac{d^2f}{dx^2}$ vanishes at most once on $\mathbb{R}$

(C) $\frac{d^3f}{dx^3}$ vanishes at least once on $\mathbb{R}$

(D) $\frac{df}{dx}$ does not attain the value $\frac{7}{2}$ on $\mathbb{R}$

IIT Madras

Q.30 Define the function $f: \mathbb{R}^2 \to \mathbb{R}$ by

$$f(x, y) = 12xy\, e^{-(2x+3y-2)}.$$

If (a, b) is the point of local maximum of f, then $f(a, b)$ equals

(A) 2

(B) 6

(C) 12

(D) 0

IIT Madras

Section B: Q.31 – Q.40 Carry TWO marks each.

Q.31 Let $\{a_n\}_{n=1}^{\infty}$ be a sequence of real numbers.

Then, which of the following statements is/are always TRUE?

(A) If $\sum_{n=1}^{\infty} a_n$ converges absolutely, then $\sum_{n=1}^{\infty} a_n^2$ converges absolutely

(B) If $\sum_{n=1}^{\infty} a_n$ converges absolutely, then $\sum_{n=1}^{\infty} a_n^3$ converges absolutely

(C) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} a_n^2$ converges

(D) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} a_n^3$ converges

IIT Madras

Q.32 Which of the following statements is/are TRUE?

(A) $\sum_{n=1}^{\infty} n \log\left(1+\frac{1}{n^3}\right)$ is convergent

(B) $\sum_{n=1}^{\infty} \left(1-\cos\left(\frac{1}{n}\right)\right) \log n$ is convergent

(C) $\sum_{n=1}^{\infty} n^2 \log\left(1+\frac{1}{n^3}\right)$ is convergent

(D) $\sum_{n=1}^{\infty} \left(1-\cos\left(\frac{1}{\sqrt{n}}\right)\right) \log n$ is convergent

IIT Madras

Q.33 Which of the following statements is/are TRUE?

(A) The additive group of real numbers is isomorphic to the multiplicative group of positive real numbers

(B) The multiplicative group of nonzero real numbers is isomorphic to the multiplicative group of nonzero complex numbers

(C) The additive group of real numbers is isomorphic to the multiplicative group of nonzero complex numbers

(D) The additive group of real numbers is isomorphic to the additive group of rational numbers

Q.34 Let $f:(1,\infty)\to(0,\infty)$ be a continuous function such that for every $n\in\mathbb{N}$, $f(n)$ is the smallest prime factor of n. Then, which of the following options is/are CORRECT?

(A) $\lim_{x\to\infty} f(x)$ exists

(B) $\lim_{x\to\infty} f(x)$ does not exist

(C) The set of solutions to the equation $f(x)=2024$ is finite

(D) The set of solutions to the equation $f(x)=2024$ is infinite

IIT Madras

Q.35 Let

$$S = \{(x, y) \in \mathbb{R}^2 : x > 0, y > 0\},$$

and $f : S \to \mathbb{R}$ be given by

$$f(x, y) = 2x^2 + 3y^2 - \log x - \frac{1}{6}\log y.$$

Then, which of the following statements is/are TRUE?

(A) There is a unique point in S at which $f(x, y)$ attains a local maximum

(B) There is a unique point in S at which $f(x, y)$ attains a local minimum

(C) For each point $(x_0, y_0) \in S$, the set $\{(x, y) \in S : f(x, y) = f(x_0, y_0)\}$ is bounded

(D) For each point $(x_0, y_0) \in S$, the set $\{(x, y) \in S : f(x, y) = f(x_0, y_0)\}$ is unbounded

IIT Madras

Q.36 The center $Z(G)$ of a group G is defined as

$$Z(G) = \{x \in G : xg = gx \text{ for all } g \in G\}.$$

Let $|G|$ denote the order of G. Then, which of the following statements is/are TRUE for any group G?

(A) If G is non-abelian and $Z(G)$ contains more than one element, then the center of the quotient group $G/Z(G)$ contains only one element

(B) If $|G| \geq 2$, then there exists a non-trivial homomorphism from $\mathbb{Z}$ to G

(C) If $|G| \geq 2$ and G is non-abelian, then there exists a non-identity isomorphism from G to itself

(D) If $|G| = p^3$, where p is a prime number, then G is necessarily abelian

IIT Madras

Q.37 For a matrix M, let Rowspace(M) denote the linear span of the rows of M and Colspace(M) denote the linear span of the columns of M. Which of the following hold(s) for all $A, B, C \in M_{10}(\mathbb{R})$ satisfying $A = BC$?

(A) Rowspace(A) $\subseteq$ Rowspace(B)

(B) Rowspace(A) $\subseteq$ Rowspace(C)

(C) Colspace(A) $\subseteq$ Colspace(B)

(D) Colspace(A) $\subseteq$ Colspace(C)

IIT Madras

Q.38 Define $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ as follows

$$f(x) = \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{2^{2m}\ (m!)^2} \text{ and } g(x) = \frac{x}{2} \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{2^{2m}\ (m+1)!\ m!} \text{ for } x \in \mathbb{R}.$$

Let $x_1, x_2, x_3, x_4 \in \mathbb{R}$ be such that $0 < x_1 < x_2$, $0 < x_3 < x_4$,

$$f(x_1) = f(x_2) = 0, \qquad f(x) \neq 0 \text{ when } x_1 < x < x_2 ,$$

$$g(x_3) = g(x_4) = 0 \quad \text{and } g(x) \neq 0 \text{ when } x_3 < x < x_4.$$

Then, which of the following statements is/are TRUE?

(A) The function f does not vanish anywhere in the interval (x_3, x_4)

(B) The function f vanishes exactly once in the interval (x_3, x_4)

(C) The function g does not vanish anywhere in the interval (x_1, x_2)

(D) The function g vanishes exactly once in the interval (x_1, x_2)

IIT Madras

Q.39 For $0 < \alpha < 4$, define the sequence $\{x_n\}_{n=1}^{\infty}$ of real numbers as follows:

$$x_1 = \alpha \text{ and } x_{n+1} + 2 = -x_n(x_n - 4) \text{ for } n \in \mathbb{N}.$$

Which of the following is/are TRUE?

(A) $\{x_n\}_{n=1}^{\infty}$ converges for at least three distinct values of $\alpha \in (0,1)$

(B) $\{x_n\}_{n=1}^{\infty}$ converges for at least three distinct values of $\alpha \in (1,2)$

(C) $\{x_n\}_{n=1}^{\infty}$ converges for at least three distinct values of $\alpha \in (2,3)$

(D) $\{x_n\}_{n=1}^{\infty}$ converges for at least three distinct values of $\alpha \in (3,4)$

Q.40 Consider

$$G = \{m + n\sqrt{2} : m, n \in \mathbb{Z}\}$$

as a subgroup of the additive group $\mathbb{R}$.

Which of the following statements is/are TRUE?

(A) G is a cyclic subgroup of $\mathbb{R}$ under addition

(B) $G \cap I$ is non-empty for every non-empty open interval $I \subseteq \mathbb{R}$

(C) G is a closed subset of $\mathbb{R}$

(D) G is isomorphic to the group $\mathbb{Z} \times \mathbb{Z}$, where the group operation in $\mathbb{Z} \times \mathbb{Z}$ is defined by $(m_1, n_1) + (m_2, n_2) = (m_1 + m_2, n_1 + n_2)$

IIT Madras

Section C: Q.41 – Q.50 Carry ONE mark each.

Q.41 The area of the region

$$R = \left\{ (x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1, 0 \leq y \leq 1 \text{ and } \frac{1}{4} \leq xy \leq \frac{1}{2} \right\}$$

is _____ (*rounded off to two decimal places*).

Q.42 Let $y: \mathbb{R} \to \mathbb{R}$ be the solution to the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 1$$

satisfying $y(0) = 0$ and $y'(0) = 1$.

Then, $\lim_{x \to \infty} y(x)$ equals _____ (*rounded off to two decimal places*).

Q.43 For $\alpha > 0$, let $y_\alpha(x)$ be the solution to the differential equation

$$2\frac{d^2y}{dx^2} - \frac{dy}{dx} - y = 0$$

satisfying the conditions

$$y(0) = 1, \quad y'(0) = \alpha.$$

Then, the smallest value of α for which $y_\alpha(x)$ has no critical points in $\mathbb{R}$ equals _____ (*rounded off to the nearest integer*).

IIT Madras

Q.44 Consider the 4×4 matrix

$$M = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 3 & 2 & 1 & 0 \end{pmatrix}.$$

If $a_{i,j}$ denotes the $(i,j)^{\text{th}}$ entry of M^{-1}, then $a_{4,1}$ equals _____ (*rounded off to two decimal places*).

Q.45 Let $P_{12}(x)$ be the real vector space of polynomials in the variable x with real coefficients and having degree at most 12, together with the zero polynomial. Define

$$V = \left\{ f \in P_{12}(x) : f(-x) = f(x) \text{ for all } x \in \mathbb{R} \text{ and } f(2024) = 0 \right\}.$$

Then, the dimension of V is _____

Q.46 Let

$$S = \left\{ f : \mathbb{R} \to \mathbb{R} : f \text{ is a polynomial and } f\big(f(x)\big) = \big(f(x)\big)^{2024} \text{ for } x \in \mathbb{R} \right\}.$$

Then, the number of elements in S is _____

IIT Madras

Q.47 Let $a_1 = 1, b_1 = 2$ and $c_1 = 3$. Consider the convergent sequences

$$\{a_n\}_{n=1}^{\infty}\ ,\ \{b_n\}_{n=1}^{\infty} \text{ and } \{c_n\}_{n=1}^{\infty}$$

defined as follows:

$$a_{n+1} = \frac{a_n + b_n}{2}, \qquad b_{n+1} = \frac{b_n + c_n}{2} \text{ and } c_{n+1} = \frac{c_n + a_n}{2} \text{ for } n \geq 1.$$

Then,

$$\sum_{n=1}^{\infty} b_n c_n (a_{n+1} - a_n) + \sum_{n=1}^{\infty} (b_{n+1} c_{n+1} - b_n c_n) a_{n+1}$$

equals _____ (*rounded off to two decimal places*)

Q.48 Let

$$S = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 4, (x-1)^2 + y^2 \leq 1, z \geq 0\}.$$

Then, the surface area of S equals _____ (*rounded off to two decimal places*).

IIT Madras

Q.49 Let $P_7(x)$ be the real vector space of polynomials in x with degree at most 7, together with the zero polynomial. For $r = 1, 2, \ldots, 7$, define

$$s_r(x) = x(x-1)\cdots\big(x-(r-1)\big) \text{ and } s_0(x) = 1.$$

Consider the fact that $B = \{s_0(x), s_1(x), \ldots, s_7(x)\}$ is a basis of $P_7(x)$.

If

$$x^5 = \sum_{k=0}^{7} \alpha_{5,k}\, s_k(x)\,,$$

where $\alpha_{5,k} \in \mathbb{R}$, then $\alpha_{5,2}$ equals _____ (*rounded off to two decimal places*)

Q.50 Let

$$M = \begin{pmatrix} 0 & 0 & 0 & 0 & -1 \\ 2 & 0 & 0 & 0 & -4 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 3 \\ 0 & 0 & 0 & 2 & 2 \end{pmatrix}.$$

If $p(x)$ is the characteristic polynomial of M, then $p(2) - 1$ equals _____

IIT Madras

Section C: Q.51 – Q.60 Carry TWO marks each.

Q.51 For $\alpha \in (-2\pi, 0)$, consider the differential equation

$$x^2 \frac{d^2y}{dx^2} + \alpha x \frac{dy}{dx} + y = 0 \quad \text{for} \quad x > 0.$$

Let D be the set of all $\alpha \in (-2\pi, 0)$ for which all corresponding real solutions to the above differential equation approach zero as $x \to 0^+$. Then, the number of elements in $D \cap \mathbb{Z}$ equals _____

Q.52 The value of

$$\lim_{t\to\infty} \left(\left(\log\left(t^2 + \frac{1}{t^2} \right) \right)^{-1} \int_1^{\pi t} \frac{\sin^2 5x}{x} dx \right)$$

equals _____ (*rounded off to two decimal places*).

IIT Madras

Q.53 Let T be the planar region enclosed by the square with vertices at the points $(0,1), (1,0), (0,-1)$ and $(-1,0)$. Then, the value of

$$\iint_T \Big(\cos(\pi(x-y)) - \cos(\pi(x+y))\Big)^2 \, dx\, dy$$

equals _____ (*rounded off to two decimal places*).

Q.54 Let

$$S = \{(x,y,z) \in \mathbb{R}^3 : \; x^2 + y^2 + z^2 < 1\}.$$

Then, the value of

$$\frac{1}{\pi} \iiint_S \Big((x-2y+z)^2 + \; (2x-y-z)^2 + (x-y+2z)^2 \Big) dxdydz$$

equals _____ (*rounded off to two decimal places*).

Q.55 For $n \in \mathbb{N}$, if

$$a_n = \frac{1}{n^3+1} + \frac{2^2}{n^3+2} + \cdots + \frac{n^2}{n^3+n}$$

then the sequence $\{a_n\}_{n=1}^{\infty}$ converges to _____ (*rounded off to two decimal places*)

IIT Madras

Q.56 Consider the function $f: \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^3 - 4x^2 + 4x - 6$.

For $c \in \mathbb{R}$, let

$$S(c) = \left\{ x \in \mathbb{R} : f(x) = c \right\}$$

and $|S(c)|$ denote the number of elements in $S(c)$. Then, the value of

$$|S(-7)| + |S(-5)| + |S(3)|$$

equals _____

Q.57 Let $c > 0$ be such that

$$\int_0^c e^{s^2}\, ds = 3$$

Then, the value of

$$\int_0^c \left(\int_x^c e^{x^2+y^2}\, dy \right) dx$$

equals _____(*rounded off to one decimal place*).

IIT Madras

Q.58 For $k \in \mathbb{N}$, let $0 = t_0 < t_1 < \cdots < t_k < t_{k+1} = 1$. A function $f: [0,1] \to \mathbb{R}$ is said to be piecewise linear with nodes $t_1, \ldots, t_k$, if for each $j = 1, 2, \ldots, k+1$, there exist $a_j \in \mathbb{R}$, $b_j \in \mathbb{R}$ such that

$$f(t) = a_j + b_j t \text{ for } t_{j-1} < t < t_j .$$

Let V be the real vector space of all real valued continuous piecewise linear functions on $[0,1]$ with nodes $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}$. Then, the dimension of V equals _____

Q.59 For $n \in \mathbb{N}$, let

$$a_n = \frac{1}{n^{n-1}} \sum_{k=0}^{n} \frac{n!}{k!\,(n-k)!} \frac{n^k}{k+1}$$

and $\beta = \lim_{n \to \infty} a_n$. Then, the value of $\log \beta$ equals _____ (*rounded off to two decimal places*).

Q.60 Define the function $f: (-1,1) \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ by

$$f(x) = \sin^{-1} x$$

Let a_6 denote the coefficient of x^6 in the Taylor series of $\left(f(x)\right)^2$ about $x = 0$. Then, the value of $9a_6$ equals _____ (*rounded off to two decimal places*).

IIT Madras

Entrance Exams

- Agricultural Entrance Exams
- Architecture Entrance Exam
- Arts and Humanities Entrance Exams
- Commerce Entrance Examinations
- Common Entrance Examinations
- Computer Application Entrance Exams
- Design Entrance Exams
- Education Entrance Exams
- Engineering Entrance Exams
- Hotel Management Entrance Exams
- Law Entrance Exams
- MBA Entrance Exams
- Media & Journalism Entrance Exams
- Medical Entrance Exams
- Nursing Entrance Exams
- Pharmacy Entrance Exams
- Science Entrance Exams
- Diploma & Polytechnic
- Lateral Entry