Chapter - 16

Understanding Shapes - II
(Quadrilaterals)

Exercise 16.1

Q.1 Define the following terms:

(i) Quadrilateral (ii) Convex Quadrilateral

Sol: Quadrilateral: A polygon of 4 sides is called a quadrilateral.

(ii) Convex Quadrilateral: A quadrilateral in which the line containing any side of the quadrilateral has the remaining vertices on the same side of it.

Q.2 In a quadrilateral, define each of the following

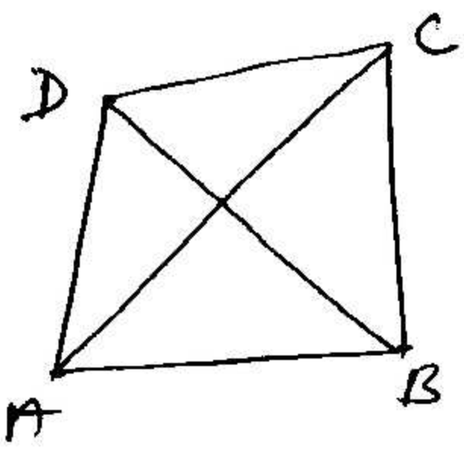
(i) Sides (ii) Vertices (iii) Angles (iv) Diagonals.

(v) Adjacent angles (vi) Adjacent sides.

(vii) opposite sides (viii) opposite angles (ix) Interior

(x) Exterior.

Sol: (i) Sides: In a quadrilateral ABCD, the four line segments AB, BC, CD and DA are called its sides.



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(II) Vertices: The points where the adjacent sides of quadrilateral meet each other.

here A, B, C and D are vertices.

(III) Angles: In quadrilateral ABCD, the angles ∠DAB, ∠ABC, ∠BCD and ∠CDA are called its angles.

(IV) Diagonals: The line segments joining the opposite vertices of quadrilateral are diagonals.

here AC and BD are diagonals.

(V) Adjacent angles: Two angles are said to be adjacent if they have a common side as an arm.

(∠A, ∠B) (∠B, ∠C) (∠C, ∠D) (∠D, ∠A)

(VI) Adjacent sides: Two sides of a quadrilateral are said to be adjacent if they have a common end point. (AB, BC) (BC, CD) (CD, DA) (AD, AB) are adjacent side

(VII) opposite sides: sides which have no common end point.

here AB, CD and AD, BC are opposite sides

(VIII) opposite angles: Two angles which are not adjacent angles are known as opposite angles.

(∠A and ∠C) or (∠B and ∠D) are opposite angles.

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(IX) Interior: The part of the plane made up by all such points as are enclosed by quadrilateral ABC This part of the plane is called Interior.

(X) Exterior: The part of the plane made up by all points which are not enclosed by quadrilateral is known as exterior.

Q3 Complete each of the following, so as to make a true statement:

(I) A quadrilateral has ---- sides.

(II) A quadrilateral has ---- angles.

(III) A quadrilateral has ----- vertices, no three of which are -----.

(IV) A quadrilateral has ---- diagonals.

(V) The number of pairs of adjacent angles of a quadrilateral is -----.

(VI) The number of pairs of opposite sides of a quadrilateral is -----.

(VII) The sum of the angles of quadrilateral is -----.

(VIII) A diagonal of a quadrilateral is a line segment that joins two ------ vertices of the quadrilateral.

(IX) The sum of the angles of quadrilateral is ----- right angles.

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(X) The measure of each angle of a convex quadrilateral is ---- 180°.

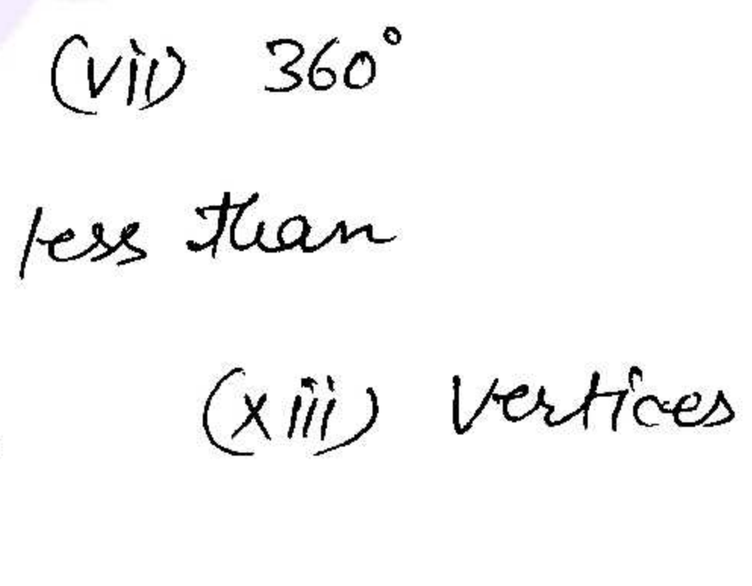
(XI) In a quadrilateral the point of intersection of the diagonals lies in ---- of the quadrilateral.

(XII) A point is in the interior of convex quadrilateral, if it is in the ..... of its two opposite angles.

(XIII) A quadrilateral is convex if for each side, the remaining ..... lie on the same side of the line containing the side.

Solution: (I) four (II) four (III) four, Collinear
(IV) Two (V) Four (VI) two (VII) 360°
(VIII) opposite (IX) four (X) less than
(XI) The interiors (XII) Interiors (XIII) Vertices.

Q.4 In fig. 16.19, ABCD is a quadrilateral.



(I) Name a pair of adjacent sides.

(II) Name a pair of opposite sides.

(III) How many pairs of adjacent sides are there.

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(iv) How many pairs of opposite sides are there.?

(v) Name a pair of adjacent angles.

(vi) Name a pair of opposite angles.

(vii) How many pairs of adjacent angles are there?

(viii) How many pairs of opposite angles are these?

Solution: (i) Pair of adjacent sides:

(AB, BC), (BC, CD), (CD, AD), (AD, AB)

(ii) Pair of opposite sides: (AB, CD) (BC, AD)

(iii) There are 4 pairs of adjacent sides.

(iv) There are 2 pairs of adjacent sides.

(v) Pair of adjacent angles: (∠A, ∠B) (∠B, ∠C) (∠C, ∠D)
(∠D, ∠A)

(vi) 2 Pair of opposite angles: (∠A, ∠C) (∠B, ∠D)

(vii) Total 4 Pair of adjacent angles.

(viii) Total 2 pair of opposite angles.

Q.5 The angles of quadrilateral are 110°, 72°, 55° and x°. Find the value of x.

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Solution: $\therefore \quad 110^\circ + 72^\circ + 55^\circ + x^\circ = 360^\circ$

(Angle sum property of quadrilateral)

$\Rightarrow \quad 237^\circ + x^\circ = 360^\circ$

$\Rightarrow \quad x = 360^\circ - 237^\circ$

$= 123^\circ$

Q: The three angles of a quadrilateral are respectively equal to 110°, 50° and 40°. Find its fourth angle.

Sol: Let fourth angle be x

then $110^\circ + 50^\circ + 40^\circ + x = 360^\circ$

$\Rightarrow \quad 200^\circ + x = 360^\circ$

$\Rightarrow x = 360^\circ - 200^\circ$

$= 160^\circ$

Q.7 A quadrilateral has three acute angles measures of each is 80°. What is the measure of fourth angle

Sol: Let Measure of fourth angle be x°

$80^\circ + 80^\circ + 80^\circ + x^\circ = 360^\circ$

$\Rightarrow \quad 240^\circ + x^\circ = 360^\circ$

$\Rightarrow x^\circ = 360^\circ - 240^\circ$

$\Rightarrow x^\circ = 120^\circ$.

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Q8 A quadrilateral has all its four angles of the same measure. What is the measure of each?

Solution: Let measure of each angle be x°.

then $x^\circ + x^\circ + x^\circ + x^\circ = 360^\circ$

$\Rightarrow 4x^\circ = 360^\circ$

$\Rightarrow x^\circ = \frac{360^\circ}{4} = 90^\circ$.

$\therefore$ Measure of each angle $= 90^\circ$.

Q.9 Two angles of the quadrilateral are of measure 65° each and the other two angles are equal. What is the Measure of each of these two angles.

Sol: Let measure of each of two angles $= x^\circ$.

$\therefore 65^\circ + 65^\circ + x + x = 360^\circ$

$\Rightarrow 130 + 2x = 360^\circ$

$\Rightarrow 2x = 360 - 130^\circ$

$2x = 230^\circ$

$\Rightarrow x = \frac{230}{2} = 115^\circ$.

Q.10 Three angles of quadrilateral are equal. Fourth angle is of measure 150°. What is the measure of equal angles.

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Sol: Let Measure of three equal angles be x°

fourth angle $= 150^\circ$.

$\therefore \quad x + x + x + 150 = 360^\circ$

$\Rightarrow \quad 3x + 150 = 360^\circ$

$\Rightarrow 3x = 360 - 150^\circ$

$3x = 210^\circ$

$\Rightarrow x = \frac{210}{3} = 70^\circ$

Q.11 The four angles of a quadrilateral are as 3:5:7:9, find the angles.

Solution: Let the angles are $3x, 5x, 7x$ and $9x$

$\therefore \quad 3x + 5x + 7x + 9x = 360$

$\Rightarrow \quad 24x = 360^\circ$

$\Rightarrow x = \frac{360}{24} = 15$

$\therefore$ angles are $3x = 3 \times 15 = 45^\circ$

$4x = 4 \times 15 = 60^\circ$

$5x = 5 \times 15 = 75^\circ$

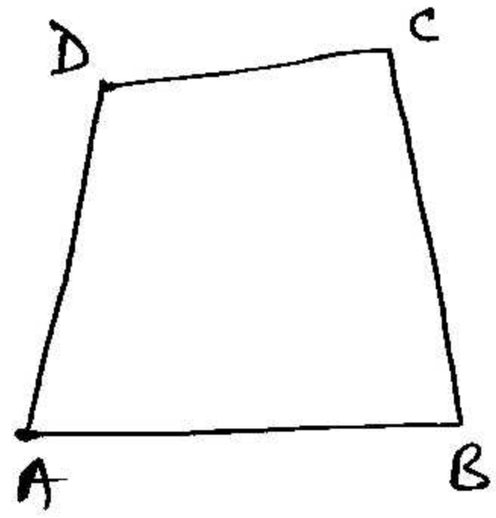
$6x = 6 \times 15^\circ = 90^\circ$.

Q.12 If the sum of the two angles of a quadrilateral is 180°. what is the sum of the remaining two angles?

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Sol: Let Sum of $\angle A + \angle B = 180^\circ$.



$\because \angle A + \angle B + \angle C + \angle D = 360^\circ$

$\Rightarrow 180^\circ + (\angle C + \angle D) = 360^\circ$

$\Rightarrow \angle C + \angle D = 360 - 180^\circ$

$= 180^\circ$

Q.13 In fig. 16.20, find the Measure of $\angle MPN$.

Sol: In quad. in MONP

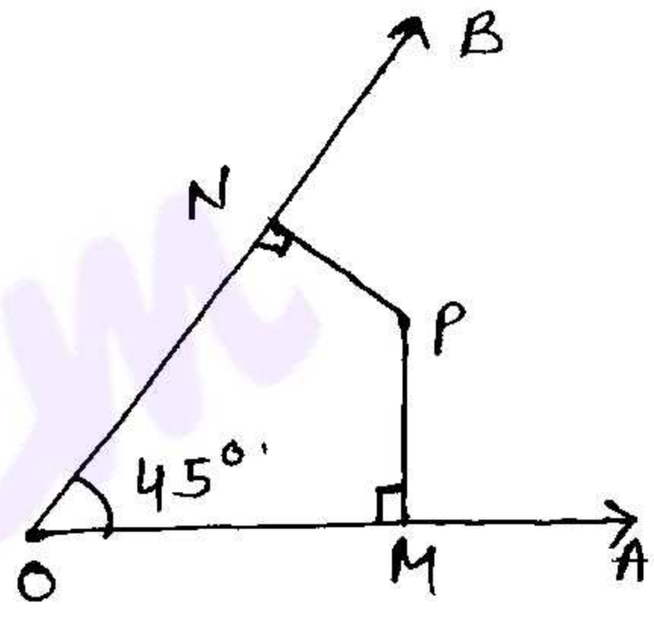
$\angle M + \angle N + \angle MPN + \angle MON = 360^\circ$

$\Rightarrow 90^\circ + 90^\circ + 45^\circ + \angle MPN = 360^\circ$

$\Rightarrow 225^\circ + \angle MPN = 360^\circ$

$\Rightarrow \angle MPN = 360^\circ - 225^\circ$.

$= 135^\circ$



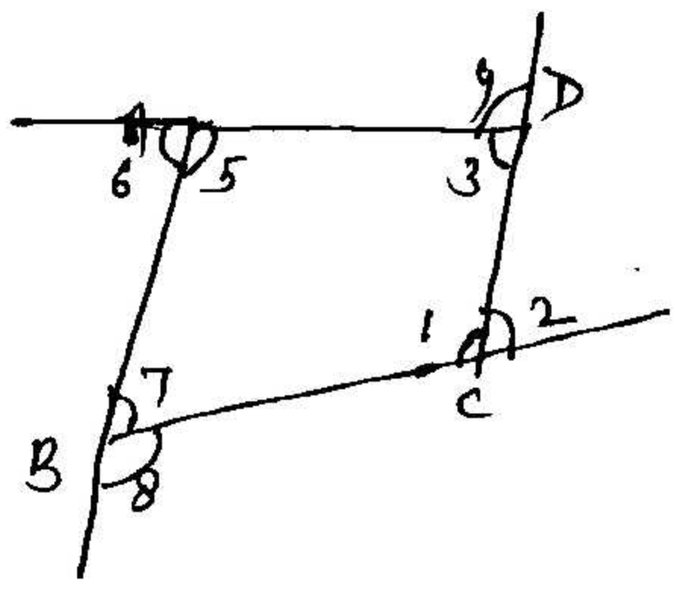
Q.14 The sides of a quadrilateral are produced in order. What is the sum of the four exterior angles.

Sol: $\because \angle 1 + \angle 2 = 180^\circ$.

$\angle 3 + \angle 4 = 180^\circ$

$\angle 5 + \angle 6 = 180^\circ$

$\angle 7 + \angle 8 = 180^\circ$



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$\because$ Adding

$$\angle 1+\angle 2+\angle 3+\angle 4+\angle 5+\angle 6+\angle 7+\angle 8 = 180+180+180+180$$

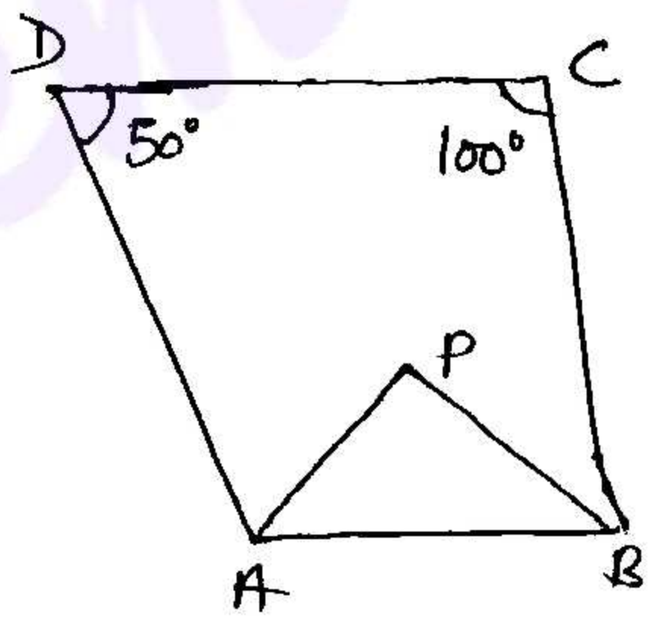
$$\Rightarrow (\angle 1+\angle 3+\angle 5+\angle 7)+(\angle 2+\angle 4+\angle 6+\angle 8) = 720^\circ$$

$$\Rightarrow 360^\circ + \angle 2+\angle 4+\angle 8+\angle 8 = 720^\circ$$

$$\Rightarrow \angle 2+\angle 4+\angle 6+\angle 8 = 720-360$$

$$= 360^\circ$$

Q.15 In fig. 16.21, the bisectors of $\angle A$ and $\angle B$ meet at a point P. If $\angle C = 100^\circ$ and $\angle D = 50^\circ$. Find Measure of $\angle APB$.



Sol:

$\because \angle A+\angle B+\angle C+\angle D = 360^\circ$

$\Rightarrow (\angle A+\angle B)+100^\circ+50^\circ = 360^\circ$

$\Rightarrow \angle A+\angle B = 360-150 = 210^\circ$

Dividing by 2 both sides.

$\Rightarrow \frac{1}{2}\angle A+\frac{1}{2}\angle B = \frac{1}{2}\times 210^\circ$

$\Rightarrow \angle PAB+\angle PBA = 105^\circ$

Now In $\triangle APB$:

$\angle APB+\angle PAB+\angle PBA = 180^\circ$

$\Rightarrow \angle APB+105^\circ = 180^\circ$

$\Rightarrow \angle APB = 180-105 = 75^\circ$.

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Q.16 In a quadrilateral ABCD, the angles A, B, C, D are in the ratio 1 : 2 : 4 : 5. Find the measure of each angle of the quadrilateral.

Sol: Let angles are $x, 2x, 4x, 5x$.

$$\therefore \quad x + 2x + 4x + 5x = 360^\circ$$

$$\Rightarrow \quad 12x = 360^\circ$$

$$\Rightarrow \quad x = \frac{360}{12} = 30$$

$\therefore$ The angles are $x = 30^\circ$

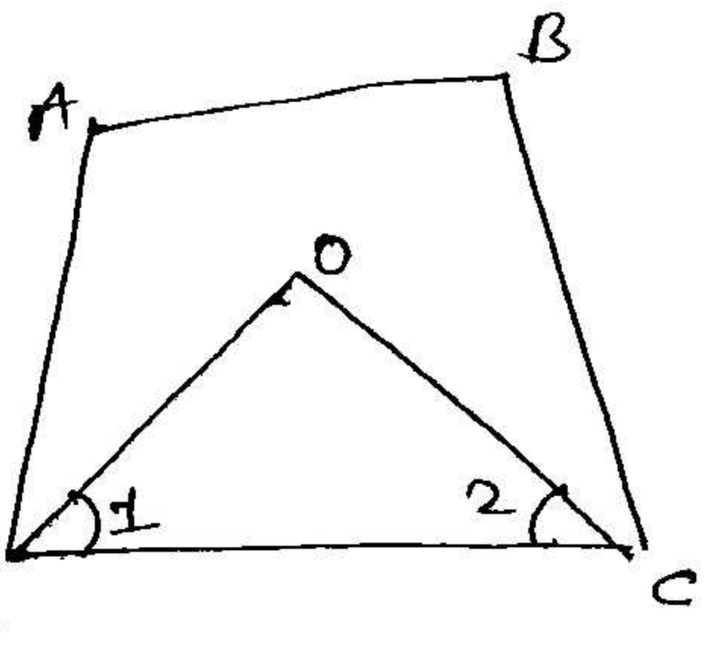
$$2x = 2 \times 30 = 60^\circ$$

$$4x = 4 \times 30^\circ = 120^\circ$$

$$5x = 5 \times 30^\circ = 150^\circ$$

Q.17 In a quadrilateral ABCD, CO and DO are the bisectors of $\angle C$ and $\angle D$ respectively. Prove that

$$\angle COD = \frac{1}{2}(\angle A + \angle B)$$



Sol: $\because$ CO and DO are bisectors of $\angle C$ and $\angle D$

$$\therefore \quad \angle 2 = \frac{1}{2}\angle C, \quad \angle 1 = \frac{1}{2}\angle D$$

Now $\angle A + \angle B + \angle C + \angle D = 360^\circ$

$$\Rightarrow \quad \angle A + \angle B + 2\angle 2 + 2\angle 1 = 360^\circ$$

$$\Rightarrow \quad 2(\angle 1 + \angle 2) = 360^\circ - (\angle A + \angle B)$$

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$$\angle 1+\angle 2 = \frac{360^\circ - (\angle A+\angle B)}{2}$$

$$= 180^\circ - \frac{1}{2}(\angle A+\angle B)$$

Now In $\triangle COD$

$$\angle 1+\angle 2+\angle COD = 180^\circ$$

$$\Rightarrow 180^\circ - \frac{1}{2}(\angle A+\angle B) + \angle COD = 180^\circ$$

$$\Rightarrow \angle COD = 180^\circ - 180^\circ + \frac{1}{2}(\angle A+\angle B)$$

$$\Rightarrow \angle COD = \frac{1}{2}(\angle A+\angle B)$$

Q.18 Find the number of sides of a regular polygon, when each of its angles has a measure of.

(i) 160° (ii) 135° (iii) 175° (iv) 162°

(v) 150°.

Solution : $\because$ No. of sides $= \frac{360^\circ}{\text{each exterior angle}}$

also Interior angle + exterior angle $= 180^\circ$.

(i) Each Interior angle $= 160^\circ$

$\therefore$ exterior angle $= 180 - 160 = 20^\circ$

hence, no. of sides $= \frac{360}{20} = 18$.

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(ii) Interior angle $= 135^\circ$

exterior angle $= 180 - 135^\circ$

$= 45^\circ$

$\therefore$ no. of sides $= \frac{360}{45} = 8$ sides.

(iii) Interior angle $= 175^\circ$

exterior angle $= 180 - 175^\circ$

$= 5^\circ$.

$\therefore$ no. of sides $= \frac{360}{5} = 72$ sides.

(iv) Interior angle $= 162^\circ$

exterior angle $= 180 - 162^\circ$

$= 18^\circ$

$\therefore$ no. of sides $= \frac{360}{18} = 20$ sides

(v) Interior angle $= 150^\circ$

exterior angle $= 180 - 150$

$= 30^\circ$

$\therefore$ exterior angle $= \frac{360}{30} = 12$ sides.

Q.19 Find the number of degrees in each exterior angle of regular pentagon.

Sol: $\because$ Sum of all exterior angle $= 360^\circ$

no. of sides $= 5$

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$\therefore$ Measure of each ext. $\angle s = \frac{360}{5} = 72°$.

Q.20 The measure of angles of a hexagon are $x°, (x-5)°, (x-5)°, (2x-5)°, (2x-5)°, (2x+20)°$. Find the value of x.

Sol: Sum of angles of hexagon $= 720°$

$\therefore x+(x-5)+(x-5)+(2x-5)+(2x-5)+(2x+20) = 720°$

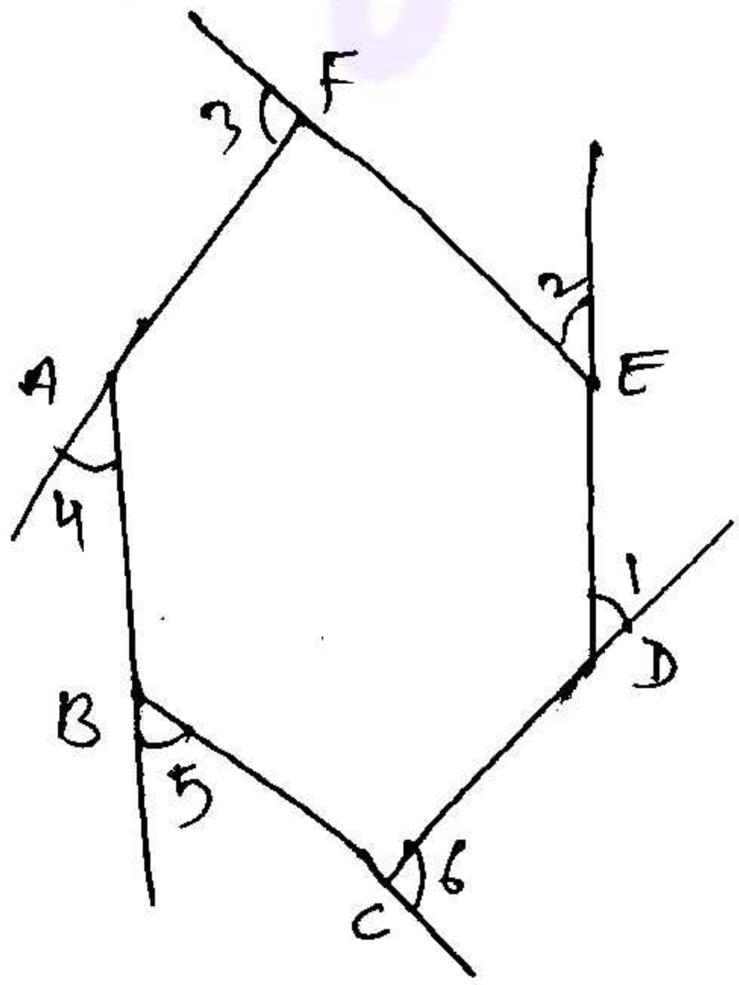
$\Rightarrow \quad 9x-20+20 = 720°$

$9x = 720°$

$x = \frac{720}{9} = 80$

Q.21 In a Conven hexagon, Prove that the sum of all interior angle is equal to twice the sum of its exterior angles formed by producing the sides in the same order.

Sol:



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∴ Sum of all interior angles of regular polygon $= (n-2) \times 180^\circ$

$= (6-2) \times 180$

$= 4 \times 180^\circ$

$= 720^\circ$

Also, sum of all exterior of n-sided polygon $= 360^\circ$

∴ Sum of Interior angles = 2x Sum of exterior angles.

Q.22 The sum of the interior angles of a polygon is three times the sum of its exterior angle. Determine sides of polygon.

Sol: As. sum of all ext. $\angle s = 360^\circ$

∴ Sum of Int. $\angle s = 3 \times 360$

$(n-2) \times 180 = 1080^\circ$

$\Rightarrow 180n - 360 = 1080$

$180n = 1080 + 360$

$180n = 1440$

$n = \frac{1440}{180}$

$n = 8$

∴ No. of sides = 8

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Q.23 Determine the number of sides of polygon whose exterior and interior angles are in the ratio 1:5.

Sol: Let exterior angle = x

Interior angle = 5x

∴ exterior angle + Interior angle = 180

$x + 5x = 180$

$\Rightarrow 6x = 180^\circ$

$\Rightarrow x = \frac{180}{6}$

$\Rightarrow x = 30^\circ$.

Then exterior angle $= 30^\circ$

Interior angle $= 180 - 30 = 150^\circ$.

Q.24 PQRSTU is a Regular hexagon. Determine each angle PQT.

Sol: ∵ Figure of this question is not given.

∴ Can't Solve.

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