

[This question paper contains 5 printed pages.]

4346

Your Roll No.

B.A. Programme /II

G-I

MATHEMATICS – Paper II

Paper Code : B-155

(Geometry, Differential Equations and Algebra)

Time : 3 Hours

Maximum Marks : 75

*(Write your Roll No. on the top immediately
on receipt of this question paper.)*

Note :- The maximum marks printed on the question paper are applicable for the students of regular colleges (Cat 'A'). These marks will, however, be scaled up proportionately in respect of the students of SOL at the time of posting of awards for compilation of result.

All questions are compulsory.

*Attempt any **two parts** from each question.*

P.T.O.

1. (a) Find and sketch equation of the hyperbola with vertices $(0, \pm 8)$ and asymptotes

$$y = \pm \frac{4}{3}x.$$

Also state the reflection property of hyperbola. $(6\frac{1}{2})$

- (b) Describe the graph of the equation:

$$16x^2 + 9y^2 - 64x - 54y + 1 = 0 \quad (6\frac{1}{2})$$

- (c) Show that the locus of the middle point of the chords of the parabola $y^2 = 4ax$ through the vertex is $y^2 = 2ax$. $(6\frac{1}{2})$

2. (a) Find the equation of the sphere through the point $(1, -3, 4)$, $(1, -5, 2)$ and $(1, -3, 0)$ and whose center lies on the plane $x + y + z = 0$. (6)

- (b) Find the largest and the smallest distances between the point $P(1, 1, 1)$ and the sphere

$$x^2 + y^2 + z^2 - 2y + 6z - 6 = 0. \quad (6)$$

- (c) Find the area of the triangle with vertices $A(1, 0, 1)$, $B(0, 2, 3)$ and $C(2, 1, 0)$. Use the result to find the length of the altitude from the vertex C to the side $\overline{AB}$. (6)

3. (a) Solve

$$(x^2 + y^2 + x)dx + xy dy = 0 \quad (6\frac{1}{2})$$

(b) Solve the differential equation by the method of variation of parameter:

$$\frac{d^2 y}{dx^2} + 4y = 4\sec^2 2x \quad (6\frac{1}{2})$$

(c) Solve

$$x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = x^3 \quad (6\frac{1}{2})$$

4. (a) Find the complete integral of the partial differential equation:

$$z = px + qy + q^2 + p^2. \quad (6)$$

(b) Find whether the equation:

$$xyr - (x^2 - y^2)s - xyt + py - qx = 2(x^2 - y^2). \quad (6)$$

is elliptic, parabolic or hyperbolic.

(c) Find the general integral of partial differential equation

$$(2xy - 1)p + (z - 2x^2) q = 2(x - yz). \quad (6)$$

P.T.O.

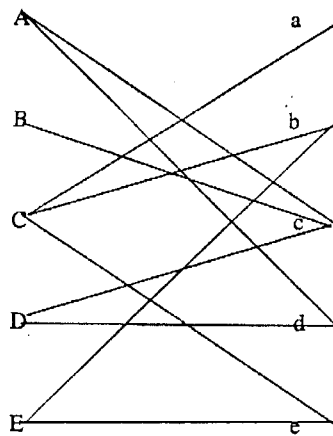
5. (a) Let G be a group and H a nonempty subset of G . Then H is a subgroup of G if and only if

$$ab^{-1} \in H \quad \forall a, b \in H \quad (6\frac{1}{2})$$

- (b) Let $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 3 & 10 & 7 & 8 & 2 & 1 & 4 & 4 & 9 & 5 \end{pmatrix}$ be a permutation in S_{10} . Write α as a product of disjoint cycles and as a product of transpositions. Find its inverse and order. $(6\frac{1}{2})$

- (c) Discuss the clockwise rotation of a square and find all the permutations obtained by the clockwise rotation. $(6\frac{1}{2})$

6. (a) Find the matching or explain why none exists for the following graph :



(6)

- (b) Following is the cost matrix for a travelling salesperson problem. Solve it to find the minimum cost.

	To	1	2	3	4
		1	2	3	4
From	1	-	3	9	7
	2	3	-	6	5
	3	5	6	-	6
	4	9	7	4	-

(6)

- (c) Show that the table for any finite group $(G, +)$ of order n is a Latin square of order n based on G . (6)