[This question paper contains 5 printed pages.]
4371
Your Roll No. $\qquad$
B.A. Programme /II

G-II

## MATHEMATICS - Paper II

Paper Code' : B-155
(Geometry, Differential Equations and Algebra)

Time : 3 Hours
Maximum Marks : 75
(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :- The maximum marks printed on the question paper are applicable for the students of (Category ' $A$ '). These marks will, however, be scaled up proportionately in respect of the students of SOL at the time of posting of awards for compilation of results.

All questions are compulsory.

Attempt any two parts from each question.
P.T.O.

1. (a) Find an equation for the ellipse that passes through $(-3,2)$ and ends of whose major axis are $(0, \pm 6) .\left(6 \frac{1}{2}\right)$
(b) Describe the graph of the equation:

$$
\begin{equation*}
y^{2}-8 x-6 y-23=0 . \tag{1/2}
\end{equation*}
$$

(c) Suppose the axes of an $x y$-coordinate system are rotated through an angle of $\theta=45^{\circ}$ to obtain an $x^{\prime} y^{\prime}$ coordinates system. Find the equation of the curve $x^{2}-x y+y^{2}-6$ $=0$ in $x^{\prime} y^{\prime}$ coordinates and also sketch it.
2. (a) Find the area of the triangle that is determined by $P_{1}(2,2,0), P_{2}(-1,0,2)$ and $P_{3}(0,4,3)$
(b) Find k so that the vector from the point $\mathrm{A}(1,-1,3)$ to the point $\mathrm{B}(3,0,5)$ is orthogonal to the vector from A to the point $\mathrm{P}(\mathrm{k}, \mathrm{k}, \mathrm{k})$.
(c) Find the angle of intersection of the two planes:

$$
\begin{equation*}
2 x-4 y+4 z=6 \quad \text { and } \quad 6 x+2 y-3 z=4 \tag{6}
\end{equation*}
$$

3. (a) Show that $e^{x}$ and $x e^{x}$ are linearly independent solution of the differential equation

$$
\begin{equation*}
\frac{d^{2} y}{d x^{2}}-2 \frac{d y}{d x}+y=0 \tag{1/2}
\end{equation*}
$$

(b) Solve the differential equation by the method of variation of parameters.

$$
\begin{equation*}
\frac{d^{2} y}{d x^{2}}+4 y=4 \tan x \tag{1/2}
\end{equation*}
$$

(c) A body cools from $60^{\circ} \mathrm{C}$ to $50^{\circ} \mathrm{C}$ in 15 minutes in air which is maintained at $30^{\circ} \mathrm{C}$. How long will it take this body to cool from $100^{\circ} \mathrm{C}$ to $80^{\circ} \mathrm{C}$ in air that is maintained at $50^{\circ} \mathrm{C}$ ?
4. (a) Find the complete integral of the partial differential equation:

$$
\begin{equation*}
q^{2}=z^{2} p^{2}\left(1-p^{2}\right) . \tag{6}
\end{equation*}
$$

(b) Find whether the equation:

$$
x^{2} r-2 x s+t+q=0 .
$$

is elliptic, parabolic or hyperbolic.
P.T.O.
(c) Find the partial differential equation by eliminating arbitrary function from the equation

$$
\begin{equation*}
f\left(x^{2}+y^{2}+z^{2}, z^{2}-2 x y\right)=0 \tag{6}
\end{equation*}
$$

5. (a) Let $G$ be a group and $H$ be a subgroup of $G$. Prove that the order of the subgroup $H$ of $G$ divides the order of the group $G$.
(b) Write $f=\left(\begin{array}{rrrrrrrrrr}1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 3 & 4 & 10 & 5 & 7 & 8 & 2 & 6 & 9 & 1\end{array}\right)$ as a product of transposition. Find its inverse and order.
(c) Prove that the set I of all integer is a group with respect to the binary operation * defined by $\mathrm{a}^{*} \mathrm{~b}=\mathrm{a}+\mathrm{b}+1 \quad \forall$ $a, b \in I$.
6. (a) Find the matching or explain why none exists for the following graph :

(6)
(b) Solve the travelling salesperson problem for the given cost matrix.

| To | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- |
| 1 | - | 7 | 2 | 6 |
| 2 | 1 | - | 8 | 9 |
| 3 | 2 | 1 | - | 3 |
| 4 | 4 | 2 | 6 | - |

(c) For the following graph find all sets of 2 vertices whose removal disconnects the graph of remaining vertices.


