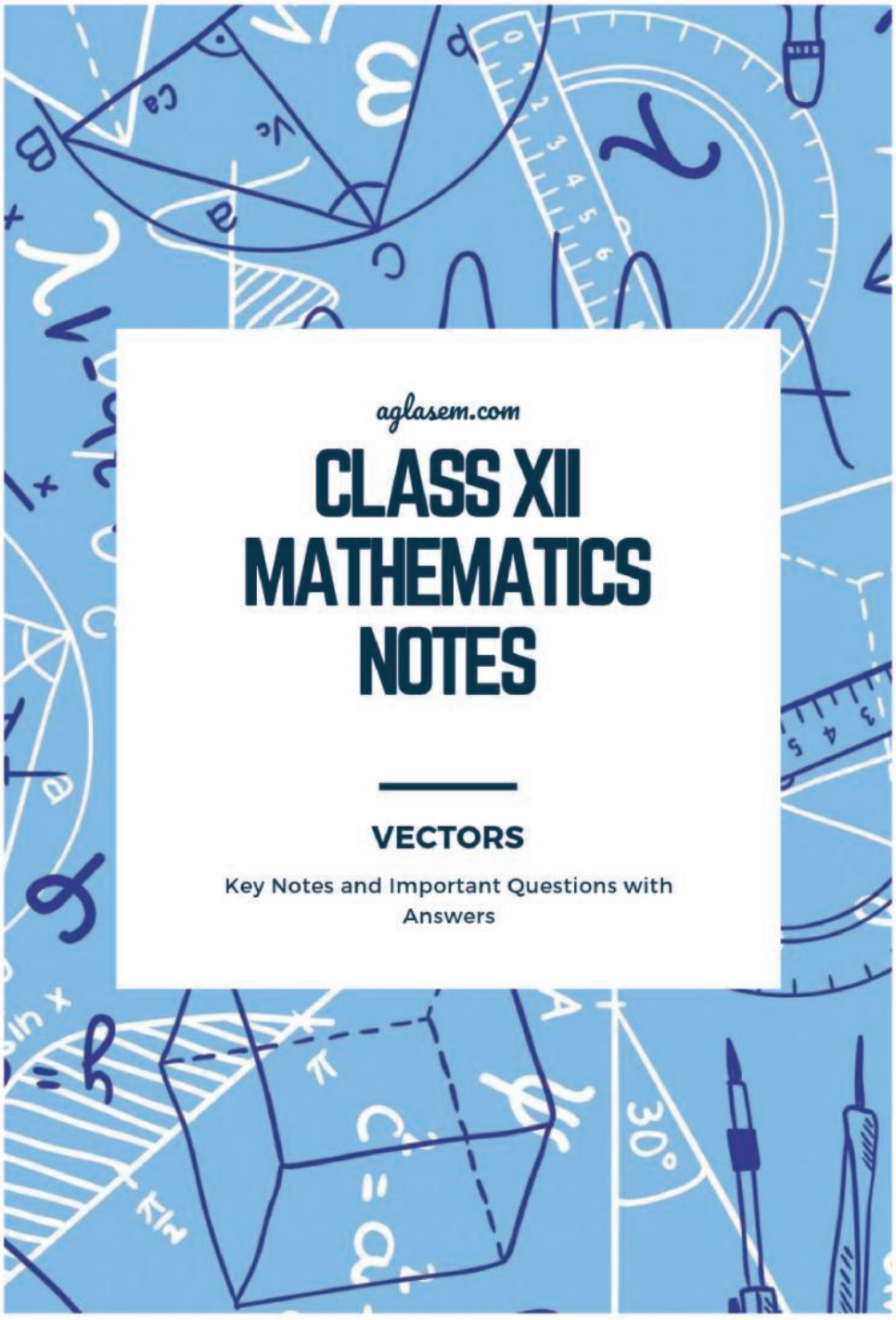

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CLASS XII
MATHEMATICS
NOTES
VECTORS
Key Notes and Important Questions with
Answers

CHAPTER–10

VECTORS

POINTS TO REMEMBER

- A quantity that has magnitude as well as direction is called a vector. It is denoted by a directed line segment.
- Two or more vectors which are parallel to same line are called collinear vectors.
- Position vector of a point P(a, b, c) w.r.t. origin (0, 0, 0) is denoted by $\overrightarrow{OP}$ where $\overrightarrow{OP} = a\hat{i} + b\hat{j} + c\hat{k}$ and $|\overrightarrow{OP}| = \sqrt{a^2 + b^2 + c^2}$.
- If $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ be any two points in space, then

  $\overrightarrow{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$ and

  $|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z)^2}$
- Any vector $\vec{a}$ is called unit vector if $|\vec{a}| = 1$ It is denoted by $\hat{a}$
- If two vectors $\vec{a}$ and $\vec{b}$ are represented in magnitude and direction by the two sides of a triangle in order, then their sum $\vec{a} + \vec{b}$ is represented in magnitude and direction by third side of a triangle taken in opposite order. This is called triangle law of addition of vectors.
- If $\vec{a}$ is any vector and λ is a scalar, then $\lambda\,\vec{a}$ is vector collinear with $\vec{a}$ and $|\lambda\vec{a}| = |\lambda||\vec{a}|$.
- If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then $\vec{a} = \lambda\vec{b}$ where λ is some scalar.

- Any vector $\vec{a}$ can be written as $\vec{a} = |\vec{a}|\hat{a}$where $\hat{a}$ is a unit vector in the direction of $\vec{a}$.

- If $\vec{a}$ and $\vec{b}$ be the position vectors of points A and B, and C is any point which divides $\overrightarrow{AB}$ in ratio m:n internally then position vector $\vec{c}$ of point C is given as $\vec{c} = \frac{m\vec{b}+n\vec{a}}{m+n}$. If C divides $\overrightarrow{AB}$ in ratio m:n externally, then$\vec{c} = \frac{m\vec{b}-n\vec{a}}{m-n}$.

- The angles α, β *and* γ made by $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$with positive direction of x, y and z-axis are called angles and cosines of these angles are called direction cosines of$\vec{r}$usually denoted as $l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$

  Also $l=\frac{a}{|\vec{r}|}, m = \frac{b}{|\vec{r}|}, n = \frac{c}{|\vec{r}|}$ and $l^2 + m^2 + n^2 = 1$

- The numbers a, b, c proportional to l, m, n are called direction ratios.

- Scalar product or dot product of two vectors $\vec{a}$ and $\vec{b}$ is denoted as $\vec{a}.\vec{b}$ and is defined as $\vec{a}.\vec{b} = |\vec{a}||\vec{b}| \cos\theta$, θ is the angle between $\vec{a}$ *and* $\vec{b}$.$(0 \le \theta \le \pi)$.

- Dot product of two vectors is commutative i.e. $\vec{a}.\vec{b} = \vec{b}.\vec{a}$

- $\vec{a}.\vec{b} = 0 \Leftrightarrow \vec{a} = \vec{o}, \vec{b} = \vec{o}$ or $\vec{a} \perp \vec{b}$.

- $\vec{a}.\vec{a} = |\vec{a}|^2$, so $\hat{i}\ \hat{i} = \hat{j}\ \hat{j} = \hat{k}\ k = 1$

- If$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$,then

$$\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3.$$

- Projection of $\vec{a}$ on $\vec{b} = \left|\frac{\vec{a}.\vec{b}}{|\vec{b}|}\right|$ and

  Projection vector of $\vec{a}$ along $\vec{b} = \left(\frac{(\vec{a}.\vec{b})}{|\vec{b}|}\right)\hat{b}$.

- Cross product or vector product of two vectors $\vec{a}$ and $\vec{b}$ is denoted as $\vec{a} \times \vec{b}$ and is defined as $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta\, \hat{n}$. were θ is the angle between $\vec{a}$ and $\vec{b}$.$(0 \leq \theta \leq \pi)$. And $\hat{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ such that $\vec{a}$. $\vec{b}$ and $\hat{n}$ from a right handed system.

- Cross product of two vectors is not commutative i.e., $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$, but $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$.

- $\vec{a} \times \vec{b} = \vec{0} \Leftrightarrow \vec{a} = \vec{0}, \vec{b} = \vec{0}$ or $\vec{a} \| \vec{b}$.

- $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \vec{0}$.

- $\hat{i} \times \hat{j} = \hat{k}, \hat{j} \times \hat{k} = \hat{i}, \hat{k} \times \hat{i} = \hat{j}$ and $\hat{j} \times \hat{i} = -\hat{k}, \hat{k} \times \hat{j} = -\hat{i}, \hat{i} \times \hat{k} = -\hat{j}$

- If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

- Unit vector perpendicular to both $\vec{a}$ and $\vec{b} = \pm\left(\frac{(\vec{a} \times \vec{b})}{|\vec{a} \times \vec{b}|}\right)$.

- $|\vec{a} \times \vec{b}|$ is the area of parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$

- $\frac{1}{2}|\vec{a} \times \vec{b}|$ is the area of parallelogram where diagonals are $\vec{a}$ and $\vec{b}$.

- If $\vec{a}, \vec{b}$ and $\vec{c}$ form a triangle, then area of the triangle

- $= \frac{1}{2}|\vec{a} \times \vec{b}| = \frac{1}{2}|\vec{b} \times \vec{c}| = \frac{1}{2}|\vec{c} \times \vec{a}|$.
- Scalar triple product of three vector $\vec{a}, \vec{b}$ and $\vec{c}$ is defined as $\vec{a}.(\vec{b} \times \vec{c})$ and is denotes as $[\vec{a}\vec{b}\vec{c}]$
- Geometrically, absolute value of scalar triple product $[\vec{a}\vec{b}\vec{c}]$ represents volume of a parallelopiped whose coterminous edges are $\vec{a}\vec{b}$ and $\vec{c}$.
- $\vec{a}\ \vec{b}\ \vec{c}$ are coplanar $\Leftrightarrow [\vec{a}\ \vec{b}\ \vec{c}] = 0$
- $[\vec{a}\ \vec{b}\ \vec{c}] = [\vec{b}\ \vec{c}\ \vec{a}] = [\vec{c}\ \vec{a}\ \vec{b}]$
- If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ then

$$[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

- Then scalar triple product of three vectors is zero if any two of them are same or collinear.

Very Short Answer Type Questions (1 MARK)

1. If $\overrightarrow{AB} = 3\hat{i} + 2\hat{j} - \hat{k}$ and the coordinate of A are (4,1,1), then find the coordinates of B.
2. Let $\vec{a} = -2\hat{i} + \hat{j}$, $\vec{b} = \hat{i} + 2\hat{j}$ and $\vec{c} = 4\hat{i} + 3\hat{j}$. Find the values of x and y such that $\vec{c} = x\vec{a} + y\vec{b}$.
3. Find a unit vector in the direction of the resultant of the vectors $\hat{i} - \hat{j} + 3\hat{k}$, $2\hat{i} + \hat{j} - 2\hat{k}$ and $\hat{i} + 2\hat{j} - 2\hat{k}$.

4. Find a vector of magnitude of 5 units parallel to the resultant of vector $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ and $\vec{b} = (\hat{\imath} - 2\hat{\jmath} - \hat{k})$

5. For what value of λ are the vector $\vec{a}\ and\ \vec{b}$ perpendicular to each other? Where $\vec{a} = \lambda\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{b} = 5\hat{\imath} - 9\hat{\jmath} + 2\hat{k}$

6. Write the value of p for which $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 9\hat{k}$ and $\vec{b} = \hat{\imath} + p\hat{\jmath} + 3\hat{k}$ are parallel vectors.

7. For any two vectors $\vec{a}\ and\ \vec{b}$ write when $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ holds.

8. Find the value of p if $(2\hat{\imath} + 6\hat{\jmath} + 27\hat{k}) \times (\hat{\imath} + 3\hat{\jmath} + p\hat{k}) = \vec{0}$

9. Evaluate: $\hat{\imath}.(\hat{\jmath} \times \hat{k}) + (\hat{\imath} \times \hat{k}).\hat{\jmath}$

10. If $\vec{a} = 2\hat{\imath} - 3\hat{\jmath}$, $\vec{b} = \hat{\imath} + \hat{\jmath} - \hat{k}, \vec{c} = 3\hat{\imath} - \hat{k}$, find $[\vec{a}\vec{b}\vec{c}]$

11. If $\vec{a} = 5\hat{\imath} - 4\hat{\jmath} + \hat{k}$, $\vec{b} = -4\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$ and $\vec{c} = \hat{\imath} - 2\hat{\jmath} - 2\hat{k}$, then evaluate $\vec{c}.(\vec{a} \times \vec{b})$

12. Show that vector $\hat{\imath} + 3\hat{\jmath} + \hat{k}$, $2\hat{\imath} - \hat{\jmath} - \hat{k}$, $7\hat{\jmath} + 3\hat{k}$ are parallel to same plane.

13. Find a vector of magnitude 6 which is perpendicular to both the vectors $2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $4\hat{\imath} - \hat{\jmath} + 3\hat{k}$.

14. If $\vec{a}\,.\vec{b} = 0$, then what can you say about $\vec{a}\ and\ \vec{b}$?

15. If $\vec{a}\ and\ \vec{b}$ are two vectors such that $|\vec{a} \times \vec{b}| = \vec{a}.\vec{b}$, then what is the angle between $\vec{a}\ and\ \vec{b}$?

16. Find the area of a parallelogram having diagonals $3\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and$\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$.

17. If $\hat{\imath}, \hat{\jmath}\ and\ \hat{k}$ are three mutually perpendicular vectors, then find the value of $\hat{\jmath}.(\hat{k} \times \hat{\imath})$.

18. P and Q are two points with position vectors $3\vec{a} - 2\vec{b}$ and $\vec{a} + \vec{b}$ respectively. Write the position vector of a point R which divides the segment PQ in the ratio 2:1 externally.

19. Find λ when scalar projection of $\vec{a} = \lambda\hat{\imath} + \hat{\jmath} + 4\hat{k}$ on $\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$ is 4 units.

20. Find "a" so that the vectors $\vec{p} = 3\hat{\imath} - 2\hat{\jmath}$ and $\vec{q} = 2\hat{\imath} + a\hat{\jmath}$ be orthogonal.

21. If $\vec{a} = \hat{\imath} - \hat{\jmath} + \hat{k}$, $\vec{b} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$ and $\vec{c} = \lambda\hat{\imath} - \hat{\jmath} + \lambda\hat{k}$ are coplanar, find the value of λ.

22. What is the point of trisection of PQ nearer to P if positions of P and Q are $3\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$ and $9\hat{\imath} + 8\hat{\jmath} - 10\hat{k}$ respectively?

23. What is the angle between $\vec{a}$and $\vec{b}$, if $\vec{a}.\vec{b} = 3$ and $|\vec{a} \times \vec{b}| = 3\sqrt{3}$.

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

Q.1. A vector $\vec{r}$ is inclined to x – axis at 45° and y-axis at 60° if | $\vec{r}$ | = 8 units. find $\vec{r}$.

Q.2. if $|\vec{a}+\vec{b}|$ = 60, $|\vec{a}-\vec{b}|$ = 40 and $\vec{b}$ = 46 find $|\vec{a}|$

Q.3. Write the projection of $\vec{b}+\vec{c}$ on $\vec{a}$ where

$\vec{a}=2\hat{i}-2\hat{j}+\hat{k},\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$

Q.4. If the points (–1, –1, 2), (2, m, 5) and (3, 11, 6) are collinear, find the value of *m*.

Q.5. For any three vectors $\vec{a},\vec{b}$ and $\vec{c}$ write value of the following.

$\vec{a}\times(\vec{b}+\vec{c})+\vec{b}\times(\vec{c}+\vec{a})+\vec{c}\times(\vec{a}+\vec{b})$

Q.6. If $(\vec{a}+\vec{b})^2+(\vec{a}.\vec{b})^2$ = 144 and $|\vec{a}|$ = 4. Find the value of $|\vec{b}|$.

Q.7. If for any two vectors $\vec{a}$ and $\vec{b}$,

$(\vec{a}+\vec{b})^2+(\vec{a}-\vec{b})^2=\lambda\left[(\vec{a})^2+(\vec{b})^2\right]$ then write the value of λ.

Q.8. if $\vec{a},\vec{b}$ are two vectors such that $|(\vec{a}+\vec{b})|$ = $|\vec{a}|$ then prove that $2\vec{a}+\vec{b}$ is perpendicular to $\vec{b}$.

Q.9. Show that vectors $\vec{a}=3\hat{i}-2\hat{j}+\hat{k}$

$\vec{b}=\hat{i}-3\hat{j}+5\hat{k},\vec{c}=2\hat{i}+\hat{j}-4\hat{k}$ form a right angle triangle.

Q.10. If $\vec{a},\vec{b},\vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}$ = 0 and $|\vec{a}|$ = 5, $|\vec{b}|$ = 12, $|\vec{c}|$ = 13, then find $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$

Q.11. The two vectors $\hat{i}+\hat{j}$ and $3\hat{i}-\hat{j}+4\hat{k}$ represents the two sides AB and AC respectively of Δ ABC, find the length of median through A.

Short Answer Type Questions (4 Marks)

1. The points A,B and C with position vectors $3\hat{\imath} - y\hat{\jmath} + 2\hat{k}$, $5\hat{\imath} - \hat{\jmath} + \hat{k}$ and $3x\hat{\imath} + 3\hat{\jmath} - \hat{k}$ are collinear. Find the values of x and y and also the ratio in which the point B divides AC.

2. If sum of two unit vectors is a unit vector, prove that the magnitude of their difference is $\sqrt{3}$.

3. Let $\vec{a} = 4\hat{\imath} + 5\hat{\jmath} - \hat{k}$, $\vec{b} = \hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ and $\vec{c} = 3\hat{\imath} + \hat{\jmath} - \hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and satisfying $\vec{d}.\vec{c} = 21$

4. If $\hat{a}$ and $\hat{b}$ are unit vectors inclined at an angleθ then proved that

   (i) $\cos\frac{\theta}{2} = \frac{1}{2}|\hat{a} + \hat{b}|$

   (ii) $\tan\frac{\theta}{2} = \left|\frac{\hat{a} - \hat{b}}{\hat{a} - \hat{b}}\right|$

5. If $\vec{a},\vec{b},\vec{c}$ are three mutually perpendicular vectors of equal magnitude. Prove that $\vec{a}+\vec{b}+\vec{c}$is equally in clined with vectors $\vec{a},\vec{b}$ and $\vec{c}$. Also find angles.

6. For any vector $\vec{a}$ prove that $|\vec{a} \times \hat{\imath}|^2 + |\vec{a} \times \hat{\jmath}|^2 + |\vec{a} \times \hat{k}|^2 = 2|\vec{a}|^2$

7. Show that $(\vec{a} \times \vec{b})^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a}.\vec{b})^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} \\ \vec{a}.\vec{b} & \vec{b}.\vec{b} \end{vmatrix}$

8. If $\vec{a}, \vec{b}$ and $\vec{c}$ are the position vectors of vertices A,B,C of a Δ ABC, show that the area of triangle ABC is $\frac{1}{2}|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$. Deduce the condition for points $\vec{a}, \vec{b}$ and $\vec{c}$ to be collinear.

9. Let $\vec{a}, \vec{b}$ *and* $\vec{c}$ be unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and the angle between b and c is $\pi/6$, prove that $\vec{a} = \pm 2(\vec{b} \times \vec{c})$.

10. If $\vec{a}, \vec{b}$ *and* $\vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then prove that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$.

11. If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{c} = \hat{\jmath} - \hat{k}$ are given vectors, then find a vector $\vec{b}$ satisfying the equations $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a}.\vec{b} = 3$.

12. Let $\vec{a}, \vec{b}$ *and* $\vec{c}$ be three non zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ *and* $\vec{b}$. if the angle between $\vec{a}$ *and* $\vec{b}$ is $\pi/6$, prove that $[\vec{a}\vec{b}\vec{c}]^2 = \frac{1}{4}|\vec{a}|^2|\vec{b}|^2$

13. If the vectors $\vec{\alpha} = a\hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{\beta} = \hat{\imath} + b\hat{\jmath} + \hat{k}$ and $\vec{\gamma} = \hat{\imath} + \hat{\jmath} + c\hat{k}$ are coplanar, then prove that $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$ where a$\neq 1, b \neq 1$ *and* $c \neq 1$

14. Find the altitude of a parallelepiped determined by the vectors $\vec{a}, \vec{b}$ *and* $\vec{c}$ if the base is taken as parallelogram determined by $\vec{a}$ *and* $\vec{b}$ and if $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{b} = 2\hat{\imath} + 4\hat{\jmath} - \hat{k}$ and $\vec{c} = \hat{\imath} + \hat{\jmath} + 3\hat{k}$.

15. Prove that the four points $(4\hat{\mathrm{i}} + 5\hat{\mathrm{j}} + \hat{\mathrm{k}}), -(\hat{\mathrm{j}} + \hat{\mathrm{k}}), (3\hat{\mathrm{i}} + 9\hat{\mathrm{j}} + 4\hat{\mathrm{k}})$ and $4(-\hat{\mathrm{i}} + \hat{\mathrm{j}} + \hat{\mathrm{k}})$ are coplanar.

16. If $|\vec{a}| = 3$, $|\vec{b}| = 4$ *and* $|\vec{c}| = 5$ such that each is perpendicular to sum of the other two, find $|\vec{a} + \vec{b} + \vec{c}|$

17. Decompose the vector $6\hat{\imath} - 3\hat{\jmath} - 6\hat{k}$ into vectors which are parallel and perpendicular to the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$.

18. If $\vec{a}, \vec{b}$ *and* $\vec{c}$ are vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c}$, $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, $a \neq 0$, then show that $\vec{b} = \vec{c}$.

19. If $\vec{a}, \vec{b}$ *and* $\vec{c}$ are three non zero vectors such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{b} \times \vec{c} = \vec{a}$. Prove that $\vec{a}, \vec{b}$ *and* $\vec{c}$ are mutually at right angles and $|\vec{b}| = 1$ and $|\vec{c}| = |\vec{a}|$

20. Simplify $[\vec{a} - \vec{b},\ \vec{b} - \vec{c},\ \vec{c} - \vec{a}]$

21. If $[\vec{a}\vec{b}\vec{c}] = 2$, find the volume of the parallelepiped whose co-terminus edges are $2\vec{a} + \vec{b},\ 2\vec{b} + \vec{c},\ 2\vec{c} + \vec{a}$.

22. If $\vec{a}, \vec{b}$ *and* $\vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 7$, find the angle between $\vec{a}$ *and* $\vec{b}$.

23. The magnitude of the vector product of the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$ with a unit vector along the sum of the vector $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ is equal to $\sqrt{2}$. Find the value of λ.

24. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, prove that $(\vec{a} - \vec{d})$ is parallel to $(\vec{b} - \vec{c})$, where $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$.

25. Find a vector of magnitude $\sqrt{171}$ which is perpendicular to both of the vectors $\vec{a} = \hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b} = 3\hat{i}-\hat{j}+2\hat{k}$.

26. If a is a nonzero real number, prove that the vectors
$\vec{\alpha} = a\hat{i}+2a\hat{j}-3a\hat{k}, \vec{\beta} = (2a+1)\hat{i}+(2a+3)\hat{j}+(a+1)\hat{k}$
and $\vec{\gamma} = (3a+5)\hat{i}+(a+5)\hat{j}+(a+2)\hat{k}$ are never coplanar.

27. If $\vec{\alpha} = 3\hat{i} - \hat{j}$ and $\vec{\beta} = 2\hat{i} + \hat{j} + 3\hat{k}$ then express $\vec{\beta}$ in the form of $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.

28. Find a unit vector perpendicular to plane ABC, when position vectors of A,B,C are $\widehat{3\imath} - \hat{\jmath} + 2\hat{k}$, $\hat{\imath} - \hat{\jmath} - 3\hat{k}$ and $4\hat{\imath} - \widehat{3\jmath} + \hat{k}$ respectively.

29. Find a unit vector in XY plane which makes an angle 45° with the vector $\hat{\imath} + \hat{\jmath}$ at angle of 60° with the vector $3\hat{\imath} - 4\hat{\jmath}$.

30. Suppose $\vec{a} = \lambda\hat{\imath} - 7\hat{\jmath} + 3\hat{k}$, $\vec{b} = \lambda\hat{\imath} + \hat{\jmath} + 2\lambda\hat{k}$. If the angle between $\vec{a}$ and $\vec{b}$ is greater than 90°, then prove that λ satisfies the inequality $-7 < \lambda < 1$.

31. Let $\vec{v} = \widehat{2\imath} + \hat{\jmath} - \hat{k}$ and $\vec{w} = \hat{\imath} + 3\hat{k}$. If u is a unit vector, then find the maximum value of he scalar triple products $\vec{u}, \vec{v}, \vec{w}$.

32. If $\vec{a} = \hat{\imath} - \hat{k}$, $\vec{b} = x\hat{\imath} + \hat{\jmath} + (1-x)\hat{k}$ and $\vec{c} = y\hat{\imath} + x\hat{\jmath} + (1+x-y)\hat{k}$ then prove that $[\vec{a}\vec{b}\vec{c}]$ depends upon neither x nor y.

33. A, b and c are distinct non negative numbers, if the vectors $a\hat{\imath} + a\hat{\jmath} + c\hat{k}$, $\hat{\imath} + \hat{k}$ and $c\hat{\imath} + c\hat{\jmath} + b\hat{k}$ lie in a plane, then prove that c is the geometric mean of a and b.

34. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and vectors $(1, a, a^2), (1, b, b^2)$ and $(1, c, c^2)$ are non-coplanar, then find the value of abc. (Ans. = –1)

Answers

Very Short Answer

1. (7, 3, 0)

2. $x = -1,\ y = 2$

3. $\frac{1}{\sqrt{21}}\left(4\hat{i} + 2\hat{j} - \hat{k}\right)$

4. $\sqrt{\frac{5}{2}}(3\hat{i}+\hat{j})$

5. $\lambda=\frac{16}{5}$

6. $\frac{2}{3}$

7. $\vec{a}$ *and* $\vec{b}$ *are per perdicular*

8. $\frac{27}{2}$

9. 0

10. 4

11. –5

12. .....

13. $-2\hat{i}+4\hat{j}+4\hat{k}$

14. Either $\vec{a}$ = 0 or $\vec{b}$ = 0 or $\vec{a} \perp \vec{b}$

15. 45°

16. $5\sqrt{3}$ sq. Units

17. 1

18. $-\vec{a} + 4\vec{b}$

19. $\lambda = 5$

20. a = 3

21. $\lambda = 1$

22. $\left(5, \frac{14}{3}, -6\right)$

23. $\pi/3$

SHORT ANSWER TYPE [2 MARKS]

1. $4(\sqrt{2}\hat{i} + \hat{j} + \hat{k})$
2. 22
3. 2
4. $m = 8$
5. 0
6. 3
7. $\lambda = 2$
8. —
9. —

10. –169

11. $2\sqrt{2}$

Short Answer Type Answer (4 marks)

1. $x = 3,\ y = 3,\ 1:2$

3. $\vec{d} = 7\hat{\imath} - 7\hat{\jmath} - 7\hat{k}$

5. $\cos^{-1}\frac{1}{\sqrt{3}}$

11. $\vec{b} = \frac{5}{3}\hat{\imath} + \frac{2}{3}\hat{\jmath} + \frac{2}{3}\hat{k}$

14. $\frac{4}{\sqrt{38}}$ units

16. $5\sqrt{2}$

17. $(-\hat{\imath} - \hat{\jmath} - \hat{k}) + (7\hat{\imath} - 2\hat{\jmath} - 5\hat{k})$

20. 0

21. 18 cu. Units

22. 60°

23. $\lambda = 1$

25. $\hat{\mathrm{i}} - 11\hat{\mathrm{j}} - 7\hat{\mathrm{k}}$

27. $\vec{\beta} = \left(\frac{3}{2}\hat{\imath} - \frac{1}{2}\hat{\jmath}\right) + \left(\frac{1}{2}\hat{\imath} + \frac{3}{2}\hat{\jmath} + 3\hat{k}\right)$

28. $\frac{-1}{\sqrt{165}}\left(10\hat{\imath} + 7\hat{\jmath} - 4\hat{k}\right)$

29. $\frac{13}{\sqrt{170}}\hat{i}+\frac{1}{\sqrt{170}}\hat{j}$

31. $\sqrt{59}$

34. -1