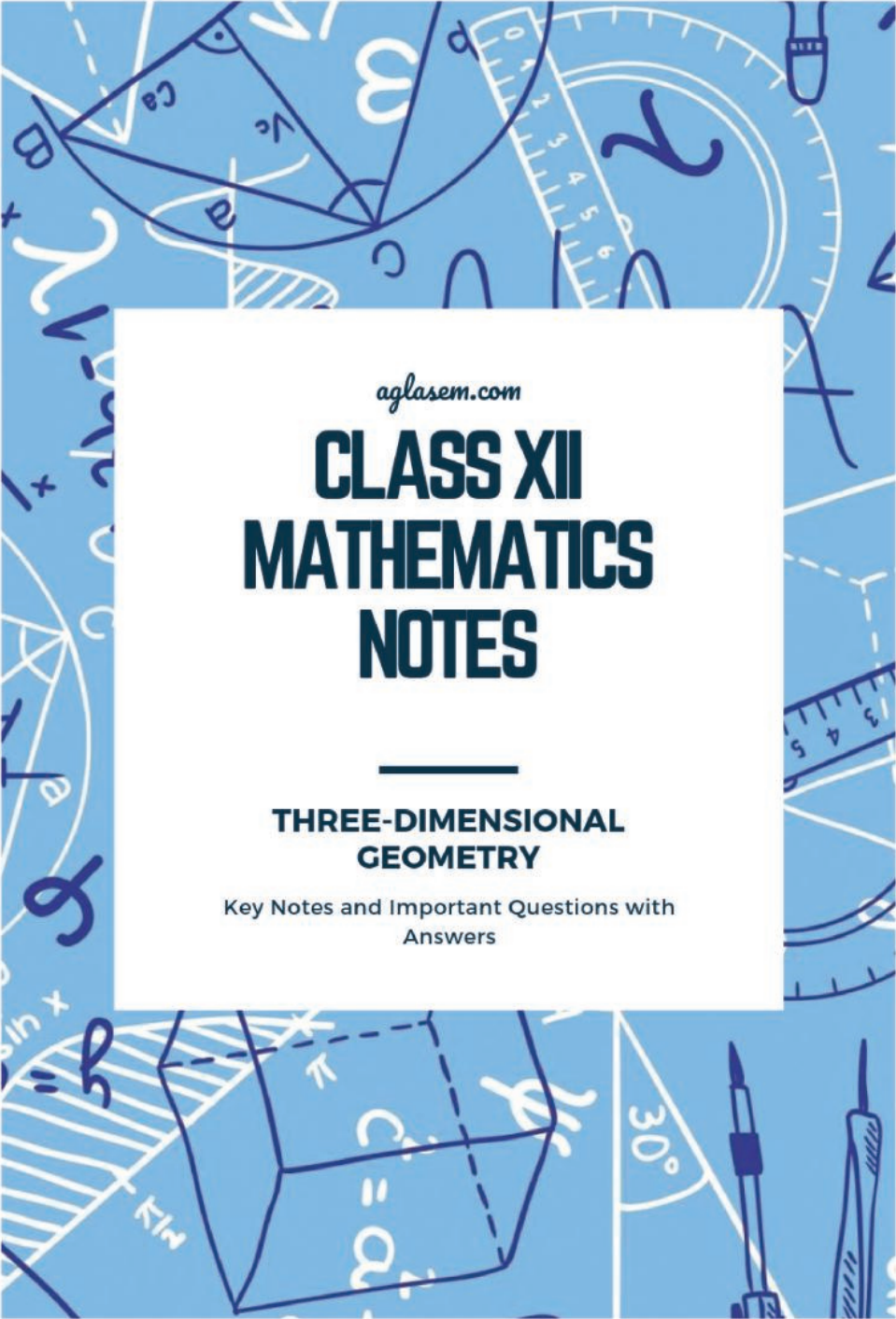


The background is a light blue field filled with various white mathematical sketches. These include geometric shapes like triangles with labeled sides (a, b, c), circles with radii, a 3D cube with dashed lines, a 30-degree angle, a ruler with markings, and various mathematical symbols like pi, infinity, and plus signs.

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CLASS XII MATHEMATICS NOTES

THREE-DIMENSIONAL GEOMETRY

Key Notes and Important Questions with
Answers

CHAPTER-11

THREE-DIMENSIONAL GEOMETRY

POINTS TO REMEMBER

- **Distance Formula:** Distance (d) between two points (x_1, y_1, z_1) and (x_2, y_2, z_2)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

- **Section Formula:** line segment AB is divided by P (x, y, z) in ratio m:n

(a) Internally	(b) Externally
$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n} \right)$	$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n} \right)$

- **Direction ratio** of a line through (x_1, y_1, z_1) and (x_2, y_2, z_2) are $x_2 - x_1, y_2 - y_1, z_2 - z_1$
- **Direction cosines** of a line having direction ratios as a, b, c are:

$$l = \pm \frac{a}{\sqrt{a^2+b^2+c^2}}, \quad m = \pm \frac{b}{\sqrt{a^2+b^2+c^2}}, \quad n = \pm \frac{c}{\sqrt{a^2+b^2+c^2}}$$

- **Equation of line in space:**

Vector form	Cartesian form
(i) Passing through point $\vec{a}$ and parallel to vector $\vec{b}$; $\vec{r} = \vec{a} + \lambda \vec{b}$	(i) Passing through point (x_1, y_1, z_1) and having direction ratios a, b, c;

	$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$
(ii) Passing through two points $\vec{a}$ and $\vec{b}$; $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$	(ii) Passing through two points (x_1, y_1, z_1) and (x_2, y_2, z_2) ; $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

• **Angle between two lines:**

Vector form	Cartesian form
(i) For lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ $\cos \theta = \frac{ \vec{b}_1 \cdot \vec{b}_2 }{ \vec{b}_1 \vec{b}_2 }$	(ii) For lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ $\cos \theta = \frac{ a_1 a_2 + b_1 b_2 + c_1 c_2 }{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$
(iii) Lines are perpendicular if $\vec{b}_1 \cdot \vec{b}_2 = 0$	(ii) Lines are perpendicular if $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$
(iv) Lines are parallel if $\vec{b}_1 = k \vec{b}_2$; $k \neq 0$	(i) Lines are parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

• **Equation of plane:**

If p is length of perpendicular from origin to plane and $\hat{n}$ is unit vector normal to plane $\vec{r} \cdot \hat{n} = p$	If p is length of perpendicular from origin to plane and l, m, n are d.c.s of normal to plane $lx + my + nz = p$
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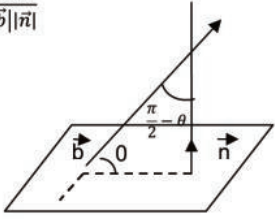
Passing through $\vec{a}$ and $\vec{n}$ is normal to plane : $(\vec{r} - \vec{a}) \cdot \vec{n} = 0$	Passing through (x_1, y_1, z_1) and a, b, c are d.r.s of normal to plane: $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$
Passing through three non collinear points $\vec{a}, \vec{b}, \vec{c}$: $(\vec{r} - \vec{a}) \cdot [(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})] = 0$	Passing through three non collinear points $(x_1, y_1, z_1)(x_2, y_2, z_2)(x_3, y_3, z_3)$: $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$
If a, b, c are intercepts on co-ordinate axes $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$	If x_1, y_1, z_1 are intercepts on coordinate axes $\frac{x}{x_1} + \frac{y}{y_1} + \frac{z}{z_1} = 1$
Plane passing through line of intersection of planes $\vec{r} \cdot \vec{n}_1 = d_1$ and $\vec{r} \cdot \vec{n}_2 = d_2$ is $\vec{r} \cdot (\vec{n}_1 + \lambda \vec{n}_2) = d_1 + \lambda d_2$ ($\lambda = \text{real no.}$)	Plane passing through the line of intersection of planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is $(a_1x + b_1y + c_1z + d_1) + \lambda(a_2x + b_2y + c_2z + d_2) = 0$

• **Angle between planes:**

Angle θ between planes $\vec{r} \cdot \vec{n}_1 = d_1$ and $\vec{r} \cdot \vec{n}_2 = d_2$ is $\cos \theta = \frac{ \vec{n}_1 \cdot \vec{n}_2 }{ \vec{n}_1 \vec{n}_2 }$	Angle θ between planes $a_1x + b_1y + c_1z = d_1$ and $a_2x + b_2y + c_2z = d_2$ is $\cos \theta = \frac{ a_1a_2 + b_1b_2 + c_1c_2 }{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$
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Planes are perpendicular iff $\vec{n}_1 \cdot \vec{n}_2 = 0$	Planes are perpendicular iff $a_1a_2 + b_1b_2 + c_1c_2 = 0$
Planes are parallel iff $\vec{n}_1 = \lambda \vec{n}_2$; $\lambda \neq 0$	Planes are parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

• **Angle between line and plane:**

Angle θ between line $\vec{r} = \vec{a} + \lambda \vec{b}$ and plane $\vec{r} \cdot \vec{n} = d$ is $\sin \theta = \cos(90^\circ - \theta)$ $= \frac{\vec{b} \cdot \vec{n}}{ \vec{b} \vec{n} }$ 	Angle θ between line $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and plane $a_2x + b_2y + c_2z = d$ is $\sin \theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$
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• **Distance of a point from a plane**

The perpendicular distance p from the point P with position vector $\vec{a}$ to the plane $\vec{r} \cdot \vec{n} = d$ is given by $p = \frac{ \vec{a} \cdot \vec{n} - d }{ \vec{n} }$	The perpendicular distance p from the point $P(x_1, y_1, z_1)$ to the plane $Ax + By + Cz + D = 0$ is given by $p = \frac{ Ax_1 + By_1 + Cz_1 + D }{\sqrt{A^2 + B^2 + C^2}}$
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- **Coplanarity**

Two lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ are coplanar iff $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 0$	Two lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ are coplanar iff $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$
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- **Shortest distance between two skew lines**

The shortest distance between lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ is $d = \frac{ (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) }{ \vec{b}_1 \times \vec{b}_2 }$	The shortest distance between $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ is $d = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{D}}$ Where $D = \{(a_1 b_2 - a_2 b_1)^2 + (b_1 c_2 - b_2 c_1)^2 + (c_1 a_2 - c_2 a_1)^2\}$
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Very Short Answer Type Questions (1 Mark)

1. What is the distance of point (a, b, c) from x-axis?
2. What is the angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$?
3. Write the equation of a line passing through (2, -3, 5) and parallel to line $\frac{x-1}{3} = \frac{y-2}{4} = \frac{z+1}{-1}$.
4. Write the equation of a line through (1, 2, 3) and parallel to $\vec{r} \cdot (\hat{i} - \hat{j} + 3\hat{k}) = 5$.
5. What is the value of λ for which the lines $\frac{x-1}{2} = \frac{y-3}{5} = \frac{z-1}{\lambda}$ and $\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z}{2}$ are perpendicular to each other?
6. Write line $\vec{r} = (\hat{i} - \hat{j}) + \lambda (2\hat{j} - \hat{k})$ into Cartesian form.
7. If the direction ratios of a line are 1, -2, 2 then what are the direction cosines of the line?
8. Find the angle between the planes $2x - 3y + 6z = 9$ and xy - plane.
9. Write equation of a line passing through (0, 1, 2) and equally inclined to co-ordinate axes.
10. What is the perpendicular distance of plane $2x - y + 3z = 10$ from origin?

11. What is the y-intercept of the plane $x - 5y + 7z = 10$?
12. What is the distance between the planes $2x + 2y - z + 2 = 0$ and $4x + 4y - 2z + 5 = 0$.
13. What is the equation of the plane which cuts off equal intercepts of unit length on the coordinate axes?
14. Are the planes $x + y - 2z + 4 = 0$ and $3x + 3y - 6z + 5 = 0$ intersecting?
15. What is the equation of the plane through the point $(1, 4, -2)$ and parallel to the plane $-2x + y - 3z = 7$?
16. Write the vector equation of the plane which is at a distance of 8 units from the origin and is normal to the vector $(2\hat{i} + \hat{j} + 2\hat{k})$.
17. What is equation of the plane if the foot of perpendicular from origin to this plane is $(2, 3, 4)$?
18. Find the angles between the planes $\vec{r} \cdot (\hat{i} - 2\hat{j} - 2\hat{k}) = 1$ and $\vec{r} \cdot (3\hat{i} - 6\hat{j} + 2\hat{k}) = 0$.
19. If O is origin $OP = 3$ with direction ratios proportional to $-1, 2, -2$ then what are the coordinates of P?
20. What is the distance between the line $\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(\hat{i} + \hat{j} + 4\hat{k})$ from the plane $\vec{r} \cdot (-\hat{i} + 5\hat{j} - \hat{k}) + 5 = 0$.
21. Write the line $2x = 3y = 4z$ in vector form.

22. The line $\frac{x-4}{1} = \frac{2y-4}{2} = \frac{k-z}{-2}$ lies exactly in the plane $2x - 4y + z = 7$.
Find the value of k.

SHORT ANSWER TYPE QUESTIONS (2 MARKS EACH)

- Q.23. What is the angle between the line $\frac{x+1}{3} = \frac{2y-1}{4} = \frac{2-z}{-4}$ and the plane $2x + y - 2z + 4 = 0$
- Q.24. Find the equation of a line passing through $(2, 0, 5)$ and which is parallel to line $6x - 2 = 3y + 1 = 2z - 2$
- Q.25. Find the equation of the plane passing through the points $(2, 3, -4)$ and $(1, -1, 3)$ and parallel to the x - axis.
- Q.26. Find the distance between the planes $2x + 3y - 4z + 5 = 0$ and $\hat{r} \cdot (4\hat{i} + 6\hat{j} - 8\hat{k}) = 11$
- Q.27. The equation of a line are $5x - 3 = 15y + 7 = 3 - 10z$. Write the direction cosines of the line
- Q.28. If a line makes angle α, β, γ with Co-ordinate axis then what is the value of $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$
- Q.29. Find the equation of a line passing through the point $(2, 0, 1)$ and parallel to the line whose equation is $\vec{r} = (2\lambda + 3)\hat{i} + (7\lambda - 1)\hat{j} + (-3\lambda + 2)\hat{k}$
- Q.30. The plane $2x - 3y + 6z - 11 = 0$ makes an angle $\sin^{-1} \alpha$ with x - axis. Find the value of α .
- Q.31. If $4x + 4y - cz = 0$ is the equation of the plane passing through the origin that contains the line $\frac{x+5}{2} = \frac{y}{3} = \frac{z-7}{4}$, then find the value of c.

- Q.32. Find the equation of the plane passing through the point $(-2, 1, -3)$ and making equal intercept on the coordinate axes.
- Q.33. Write the sum of intercepts cut off by the plane $\hat{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) - 5 = 0$ on the three axis.

Short Answer Type Questions (4 Marks)

34. Find the equation of a plane containing the points $(0, -1, -1)$, $(-4, 4, 4)$ and $(4, 5, 1)$. Also show that $(3, 9, 4)$ lies on that plane.
35. Find the equation of the plane which is perpendicular to the plane $\bar{r} \cdot (5\hat{i} + 3\hat{j} + 6\hat{k}) + 8 = 0$ and which is containing the line of intersection of the planes $\bar{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) = 4$ and $\bar{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$.
36. Find the distance of the point $(3, 4, 5)$ from the plane $x + y + z = 2$ measured parallel to the line $2x = y = z$.
37. Find the equation of the plane passing through the intersection of two planes $x + 2y + 3z - 5 = 0$ and $3x - 2y - z + 1 = 0$ and cutting equal intercepts on x-axis and z-axis.
38. Find vector and Cartesian equation of a line passing through a point with position vector $2\hat{i} - \hat{j} + \hat{k}$ and which is parallel to the line joining the points with position vectors $-\hat{i} + 4\hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 2\hat{k}$.
39. Find the equation of the plane passing through the point $(3, 4, 2)$ and $(7, 0, 6)$ and is perpendicular to the plane $2x - 5y = 15$.

40. Find equation of plane through line of intersection of planes $\vec{r} \cdot (2\hat{i} + 6\hat{j}) + 12 = 0$ and $\vec{r} \cdot (3\hat{i} - \hat{j} + 4\hat{k}) = 0$ which is at a unit distance from origin.
41. Find the image of point $(3, -2, 1)$ in the plane $3x - y + 4z = 2$.
42. Find image (reflection) of the point $(7, 4, -3)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.
43. Find equation of a plane passing through the points $(2, -1, 0)$ and $(3, -4, 5)$ and parallel to the line $2x = 3y = 4z$.
44. Find the distance of the point $(-1, -5, -10)$ from the point of intersection of line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2}$ and the plane $x - y + z = 5$.
45. Find the distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$, measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$.
46. Find the equation of the plane passing through the intersection of two plane $3x - 4y + 5z = 10$, $2x + 2y - 3z = 4$ and parallel to the line $x = 2y = 3z$.
47. Find the equation of the planes parallel to the plane $x - 2y + 2z - 3 = 0$ whose perpendicular distance from the point $(1, 2, 3)$ is 1 unit.
48. Show that the lines line $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and line $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect each other. Find the point of intersection.

49. Find the shortest distance between the lines:

$$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \mu(2\hat{i} + 3\hat{j} + 4\hat{k}) \text{ and}$$

$$\vec{r} = (2\hat{i} + 4\hat{j} + 5\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 5\hat{k}).$$

50. Find the distance of the point $(-2, 3, -4)$ from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured parallel to the plane $4x + 12y - 3z + 1 = 0$.

51. Find the equation of plane passing through the point $(-1, -1, 2)$ and perpendicular to each of the plane

$$\vec{r} \cdot (2\hat{i} + 3\hat{j} - 3\hat{k}) = 2 \text{ and } \vec{r} \cdot (5\hat{i} - 4\hat{j} + \hat{k}) = 6$$

52. Find the equation of a plane passing through $(-1, 3, 2)$ and parallel to each of the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$

53. Show that the plane $\vec{r} \cdot (\hat{i} - 3\hat{j} + 5\hat{k}) = 7$ contains the line

$$\vec{r} = (\hat{i} + 3\hat{j} + 3\hat{k}) + \lambda(3\hat{i} + \hat{j}).$$

Long Answer Type Questions (6 Marks)

54. Check the co planarity of lines

$$\vec{r} = (-3\hat{i} + \hat{j} + 5\hat{k}) + \lambda(-3\hat{i} + \hat{j} + 5\hat{k}).$$

$$\vec{r} = (-\hat{i} + 2\hat{j} + 5\hat{k}) + \mu(-\hat{i} + 2\hat{j} + 5\hat{k})$$

If they are coplanar, find equation of the plane containing the lines.

55. Find shortest distance between the lines:

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \text{ and } \frac{x-3}{1} = \frac{5-y}{2} = \frac{z-7}{1}$$

56. Find the shortest distance between the lines:

$$\vec{r} = (1 - \lambda)\hat{i} + (\lambda - 2)\hat{j} + (3 - 2\lambda)\hat{k}$$

$$\vec{r} = (\mu + 1)\hat{i} + (2\mu - 1)\hat{j} - (2\mu + 1)\hat{k}$$

57. A variable plane is at a constant distance 3 p from the origin and meets the coordinates axes in A, B and C. If the centroid of ΔABC is (α, β, γ) , then show that $\alpha^{-2} + \beta^{-2} + \gamma^{-2} = p^{-2}$
58. A vector $\vec{n}$ of magnitude 8 units is inclined to x-axis at 45° , y axis at 60° and an acute angle with z-axis. If a plane passes through a point $(\sqrt{2}, -1, 1)$ and is normal to $\vec{n}$, find its equation in vector form.

59. Find the foot of perpendicular from the point $2\hat{i} - \hat{j} + 5\hat{k}$ on the line $\vec{r} = (11\hat{i} - 2\hat{j} - 8\hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 11\hat{k})$. Also find the length of the perpendicular.
60. A line makes angles $\alpha, \beta, \gamma, \delta$ with the four diagonal of a cube. Prove that $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta = \frac{4}{3}$
61. Find the equation of the plane passing through the intersection of planes $2x + 3y - z = -1$ and $x + y - 2z + 3 = 0$ and perpendicular to the plane $3x - y - 2z = 4$. Also find the inclination of this plane with xy-plane.
62. Find the length and the equations of the line of shortest distance between the lines $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.
63. Show that $\frac{x-1}{2} = \frac{y+1}{3} = z$ and $\frac{x+1}{5} = \frac{y-2}{2}, z = 2$. do not intersect each other.

ANSWERS

3 DIMENSIONAL GEOMETRY 2 MARKS EACH

- Q.23. 0° (line is parallel to plane) Q.26. $\frac{21}{2\sqrt{29}}$ units
- Q.24. $\frac{x-2}{1} = \frac{y}{2} = \frac{z-5}{3}$ Q.27. $\frac{6}{7}, \frac{2}{7}, \frac{-3}{7}$
- Q.25. $7y + 4z = 5$

Q.28. 2

Q.31. $C = 5$

Q.29. $\vec{r} = (2\hat{i} + \hat{k}) + \lambda(2\hat{i} + 7\hat{j} - \hat{k})$

Q.32. $x + y + z = -4$

Q.30. $\alpha = \frac{2}{7}$

Q.33. $\frac{5}{2}$

Answers

1. $\sqrt{b^2 + c^2}$

11. -2

2. 90°

12. $\frac{1}{6}$

3. $\frac{x-2}{3} = \frac{y+3}{4} = \frac{z-5}{-1}$

13. $x + y + z = 1$

4. $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + 3\hat{k})$

14. No

5. $\lambda = 2$

15. $-2x + y - 3z = 8$

6. $\frac{x-1}{0} = \frac{y+1}{2} = \frac{z}{-1}$

16. $\vec{r} \cdot (2\hat{i} + \hat{j} + 2\hat{k}) = 24$

7. $\pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{2}{3}$

17. $2x + 3y + 4z = 29$

8. $\cos^{-1}(6/7)$

18. $\cos^{-1}\left(\frac{11}{21}\right)$

9. $\frac{x}{a} = \frac{y-1}{a} = \frac{z-2}{a},$
 $a \in \mathbb{R} - \{0\}$

19. $(-1, 2, -2)$

10. $\frac{10}{\sqrt{14}}$

20. $\frac{10}{3\sqrt{3}}$

21. $\vec{r} = \vec{0} + \lambda(6\hat{i} + 4\hat{j} + 3\hat{k}).$

22. $k = 7$
34. $5x - 7y + 11z + 4 = 0$
35. $\vec{r} \cdot (-51\hat{i} - 15\hat{j} + 50\hat{k}) = 173$
36. 6 units
37. $5x + 2y + 5z - 9 = 0$
38. $\vec{r} = (2\hat{i} - \hat{j} + \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$
and $\frac{x-2}{2} = \frac{y+1}{-2} = \frac{z-1}{1}$
39. $5x + 2y - 3z - 17 = 0$
40. $\vec{r} \cdot (2\hat{i} + \hat{j} + 2\hat{k}) + 3 = 0$ or
 $\vec{r} \cdot (-\hat{i} + 2\hat{j} - 2\hat{k}) + 3 = 0$
41. $(0, -1, -3)$
42. $\left(-\frac{51}{7}, -\frac{18}{7}, \frac{43}{7}\right)$
43. $29x - 27y - 22z = 85$
44. 13
45. 1 unit
46. $x - 20y + 27z = 14$
47. $x - 2y + 2z = 0$ and $x - 2y + 2z = 6$
48. $\left(\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\right)$
49. $\frac{1}{\sqrt{6}}$
50. $\frac{17}{2}$ units
51. $\vec{r} \cdot (9\hat{i} + 17\hat{j} + 23\hat{k}) = 20$
52. $2x - 7y + 4z + 15 = 0$
54. $x - 2y + z = 0$
55. $2\sqrt{29}$ units
56. $\frac{8}{\sqrt{29}}$
58. $\vec{r} \cdot (\sqrt{2}\hat{i} + \hat{j} + \hat{k}) = 2$
59. $(1, 2, 3), \sqrt{14}$
61. $7x + 13y + 4z = 9, \cos^{-1}\left(\frac{4}{\sqrt{234}}\right)$
62. $SD = 14$ units,
 $\frac{x-5}{2} = \frac{y-7}{3} = \frac{z-3}{6}$