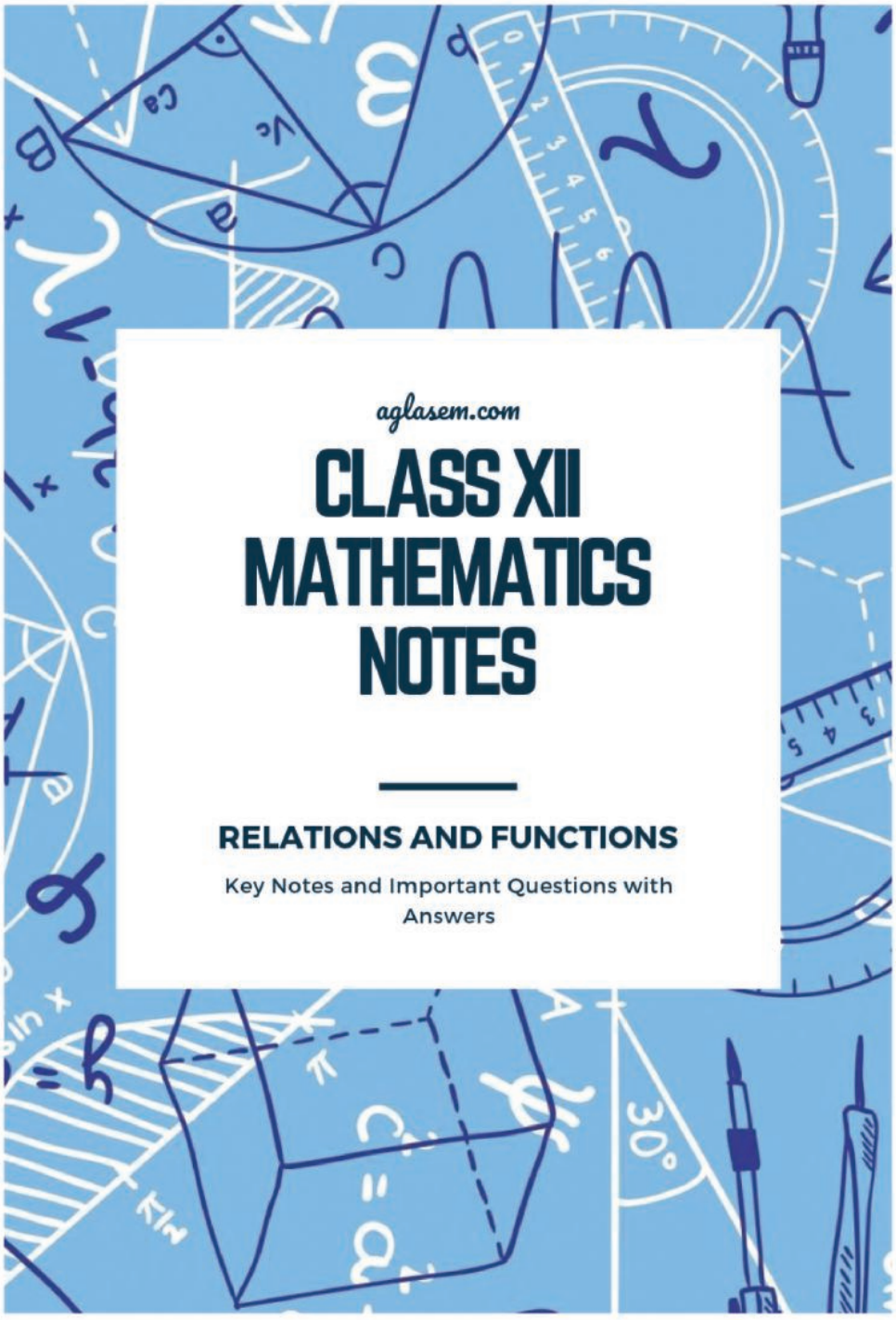

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CLASS XII
MATHEMATICS
NOTES
RELATIONS AND FUNCTIONS
Key Notes and Important Questions with Answers

CHAPTER 1

RELATIONS AND FUNCTIONS

IMPORTANT POINTS TO REMEMBER

- Relation R from a set A to a set B is subset of A × B and Relation R in set A is a subset of A × A.
- If $n(A) = r$, $n(B) = s$ from set A to set B then $n(A \times B) = rs$. and number of relations $=2^{rs}$
- $\emptyset$ is also a relation defined on set A, called the void (empty) relation.
- $R = A \times A$ is called universal relation.
- **Reflexive Relation:** Relation R defined on set A is said to be reflexive if $(a, a) \in R \ \forall \ a \in A$.
- **Symmetric Relation :** Relation R defined on set A is said to be symmetric iff $(a, b) \in R \Rightarrow (b, a) \in R \ \forall \ a, b, \in A$
- **Transitive Relation :** Relation R defined on set A is said to be transitive if $(a, b) \in R, (b, c) \in R \Rightarrow (a, c) \in R \ \forall \ a, b, c \in A$
- **Equivalence Relation:** A relation defined on set A is said to be equivalence relation if it is reflexive, symmetric and transitive.
- **Equivalence class of an element:** Let R be an equivalence relation of set A, then equivalence class of $a \in A$ is $[a] = \{ b \in A : (b, a) \in R\}$.
- **One-One Function :** $f : A \to B$ is said to be one-one if distinct elements in A have distinct images in B. *i.e.* $\forall x_1, x_2 \in A$ such that $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$.

OR

$\forall \ x_1, x_2 \in A$ such that $f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2$

One-one function is also called injective function.

- Onto function (surjective): A function $f: A \to B$ is said to be onto iff

  $R_f = B$ *i.e.* $\forall\ b \in B$, there exists $a \in A$ such that $f(a) = b$

- **Bijective Function :** A function which is both injective and surjective is called bijective function.

- **Composition of Two Functions :** If $f: A \to B$, $g: B \to C$ are two functions, then composition of f and g denoted by gof is a function from A to C given by, $(gof)(x) = g(f(x))\ \forall x \in A$

  Clearly gof is defined if Range of f is a subset of domain of g. Similarly fog can be defined.

- **Invertible Function:** A function $f: X \to Y$ is invertible iff it is bijective.

  If $f: X \to Y$ is bijective function, then function $g: Y \to X$ is said to be inverse of f iff $fog = I_y$ and $gof = I_x$

  when I_x, I_y are identity functions.

- Inverse of f is denoted by f^{-1}. [f^{-1} does not mean $\frac{1}{f}$]

- Let A and B are two nom empty set that $n(A) = p$ and $n(B) = q$

  Then

  a) Number of functions from A to B $= q^p$

  b) Number of one-one functions from A to B $= \begin{cases} {}^{q}P_{p} & , \ p \le q \\ 0 & , \ p > q \end{cases}$

  c) $Number\ of\ onto\ function\ from\ A\ to\ B \begin{cases} \sum_{r=1}^{q} (-1)^{q-r}\, {}^{q}C_{r}\, r^{p} & , \ p \ge q \\ 0 & , \ p < q. \end{cases}$

  d) Number of bijective functions from A to B $= \begin{cases} p! & , \ p = q \\ 0 & , \ p \ne q. \end{cases}$

- **Binary Operation:** A binary operation '*' defined on set A is a function from $A \times A \to A$.

  $*(a, b)$ is denoted by $a * b$.

- Number. of binary operation on set having n elements = $n^{(n^2)}$

- Binary operation * defined on set A is said to be commutative iff

  $a * b = b * a\ \forall\ a, b \in A$.

- Binary operation*defined on set A is called associative iff

  $a*(b*c) = (a*b)*c \ \forall\ a, b, c \in A$

- If * is Binary operation on A, then an element $e \in A$ (if exists) is said to be the identity element iff $a*e = e*a = a \ \forall a \in A$

- Identity element is unique.

- If * is Binary operation on set A, then an element $b \in A$(if exists)is said to be inverse of $a \in A$ iff $a*b = b*a = e$

- Inverse of an element, if it exists, is unique.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. If A is the set of students of a school then write, which of following relations are Universal, Empty or neither of the two.

   $R_1 = \{(a, b) : a, b$ are ages of students and $|a - b| > 0\}$

   $R_2 = \{(a, b) : a, b$ are weights of students, and $|a - b| < 0\}$

   $R_3 = \{(a, b) : a, b$ are students studying in same class$\}$

2. Is the relation R in the set $A = \{1, 2, 3, 4, 5\}$ defined as

   $R = \{(a, b) : b = a + 1\}$ reflexive?

3. If R, is a relation in set N given by

   $R = \{(a, b) : a = b - 3, b > 5\}$,

   then does element $(5, 7) \in R$?

4. If $f : \{1, 3\} \to \{1, 2, 5\}$ and $g : \{1, 2, 5\} \to \{1, 2, 3, 4\}$ be given by $f = \{(1, 2), (3, 5)\}$, $g = \{(1, 3), (2, 3), (5, 1)\}$,

   write g of.

5. Let $g, f: R \to R$ be defined by

$g(x) = \frac{x+2}{3}$, $f(x) = 3x - 2$. write fog (x)

6. If $f: R \to R$ defined by

$f(x) = \frac{2x-1}{5}$

be an invertible function, write $f^{-1}(x)$.

7. If $f(x) = \log x$ and $g(x) = e^x$. Find fog and gof, $x > 0$.

8. Let * be a Binary operation defined on R, then if

(i) $a * b = a + b + ab$, write $3 * 2$

(ii) $a * b = \frac{(a+b)^2}{3}$, write $(2*3)*4$.

9. If $n(A) = n(B) = 3$, then how many bijective functions from A to B can be formed?

10. Is $f: N \to N$ given by $f(x) = x^2$, one-one? Give reason.

11. If $f: R \to A$, given by

$f(x) = x^2 - 2x + 2$ is onto function, find set A.

12. If $f: A \to B$ is bijective function such that $n(A) = 10$, then $n(B) = ?$

13. If $f: R \to R$ defined by $f(x) = \frac{x-1}{2}$, find $(fof)(x)$

14. $R = \{(a, b) : a, b \in N, a \neq b$ and a divides $b\}$. Is R reflexive? Give reason

15. Is $f: R \to R$, given by $f(x) = |x - 1|$ one-one? Give reason

16. $f: R \to B$ given by $f(x) = \sin x$ is onto function, then write set B.

17 If f(x) = $\log\left(\frac{1+x}{1-x}\right)$, show that f $\left(\frac{2x}{1+x^2}\right)$=2f (x).

18 If '*' is a binary operation on set *Q* of rational numbers given by *a* *b = $\frac{ab}{5}$then write the identity element in *Q*.

19 If * is Binary operation on *N* defined by *a* * *b* = *a* + *ab* ∀ *a*, *b* ∈ *N*, write the identity element in *N* if it exists.

SHORT ANSWER TYPE QUESTIONS (2 Marks)

20. Consider the binary operation * on the set {1, 2, 3, 4, 5,} defined by a * b = HCF of a and b write the operation table for the operation *
21. Check the following functions for one-one and onto
    (a) f : R → R, f(x) = $\frac{2x-3}{7}$
    (b) f : R → R, f(x) = |x + 1|
    (c) f : R – {2} → R, f(x) = $\frac{3x-1}{x-2}$
    (d) f : R – {– 1, 1}, f(x) = $\sin^2 x$.
22. If f, g : R → R be two functions defined by f(x) = |x| + x and g(x) = |x| – x, find go f and fog.
23. If f : [1, α) → [2, α) is defined by f(x) = x + $\frac{1}{x}$, find f^{-1}(x)
24. Let A = {1, 2, 3,} and define R = {(a, b) : a – b = 12 }. Show that R is empty relation on Set A.
25. Let A = {1, 2, 3,} and define R = {(a, b): a + b > o}. Show that R is a universal relation on set A.
26. Let A = {a, b, c}. How may relation can be defined in the set? How may of these are reflexive?
27. Let f: R → R be defined by f(x) = x^2 + 1, find the pre image of 17 and –3
28. If f: R → R, g : R → R, given by f (x) = [x], g (x) = |x|, then find fog $\left(-\frac{2}{3}\right)$ and gof $\left(-\frac{2}{3}\right)$.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

29. Let $f : R - \left\{\frac{-4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ be a function given by $f(x) = \frac{4x}{3x+4}$.
Show that f is invertible with $f^{-1}(x) = \frac{4x}{4-3x}$

30. Let R be the relation on set $A = \{x : x \in Z, 0 \leq x \leq 10\}$ given by $R = \{(a, b) : (a - b)$ is divisible by $4\}$. Show that R is an equivalence relation. Also, write all elements related to 4.

31. Show that function $f : A \to B$ defined as $f(x) = \frac{3x+4}{5x-7}$ where $A = R - \left\{\frac{7}{5}\right\}$, $B = R - \left\{\frac{3}{5}\right\}$ is invertible and hence find f^{-1}.

32. Let * be a binary operation on Q such that $a * b = a + b - ab$.

    (i) Prove that * is commutative and associative.

    (ii) Find identity element of * in Q (if it exists).

33. If * is a binary operation defined on $R - \{0\}$ defined by $a * b = \frac{2a}{b^2}$ then check * for commutativity and associativity.

34. If $A = N \times N$ and binary operation * is defined on A as

    $(a, b) * (c, d) = (ac, bd)$.

    (i) Check * for commutativity and associativity.

    (ii) Find the identity element for * in A (If it exists).

35. Show that the relation R defined by $(a, b)\ R(c, d) \Leftrightarrow a + d = b + c$ on the set $N \times N$ is an equivalence relation.

36. Let * be a binary operation on set Q defined by $a * b = \frac{ab}{4}$, *show that*

    (i) 4 is the identity element in Q.

    (ii) Every non zero element of Q is invertible with

    $a^{-1} = \frac{16}{a}, \forall a \in Q - \{0\}$.

37. Show that $f: R_+ \to R_+$ defined by $f(x) = \frac{1}{2x}$ is bijective, where R_+ is the set of all non-zero positive real numbers.

38. Let $A = \{1, 2, 3, ..., 12\}$ and R be a relation in A × A defined by (a, b) R (c, d) if ad = bc$_\forall$ (a, b), (c, d) ∈ A × A. Prove that R is an equivalence relation. Also obtain the equivalence class [(3, 4)].

39. If '*' is a binary operation on R defined by $a * b = a + b + ab$. Prove that * is commutative and associative. Find the identify element. Also show that every element of R is invertible except –1.

LONG ANSWER TYPE QUESTIONS (6 MARKS)

41 Let N denote the set of all natural numbers and R be the relation on N × N defined by (a, b) R (c, d) if ad (b + c) = bc (a + d). Show that R is an equivalence relation.

42 Let f : N→R be a function defined as $f(x) = 4x^2 + 12x + 15$.

Show that f : N → S, where S is the range of f, is invertible. Also find the inverse of f. Hence find f^{-1} (31).

43 If the function f : R→ R be defined by f (x) = 2x – 3 and g : R→R by g(x) = $x^3 + 5$, then show that fog is invertible. Also find $(\text{fog})^{-1}$ (x), hence find $(\text{fog})^{-1}$ (9).

44 Let A = $Q \times Q$, where Q is the set of rational number, and * be a binary operation on A defined by (a, b) * (c, d) = (ac, b + ad) ∀ (a, b), (c, d) ∈ A.

i. Is * commutative?

ii. Is * Associative?

iii. Find identity element of * in A.

iv. Find invertible element of A and hence write the inverse of (1,2) and $(\frac{1}{3}, -5)$

45. Test whether relation R defined in R as R = {(a, b) : $a^2 - 4ab + 3b^2 = 0$, a, b ∈R} is reflexive, summative and transitive

ANSWERS

1. R_1 : is universal relation.

   R_2 : is empty relation.

   R_3 : is neither universal nor empty.

2. No, R is not reflexive.

3. $(5, 7) \notin R$

4. $gof = \{(1, 3), (3, 1)\}$

5. $(fog)(x) = x \ \forall x \in R$

6. $f^{-1}(x) = \frac{5x+1}{2}$

7. $gof(x) = x$, $fog, (x) = x$

8. (i) $3 * 2 = 11$ (ii) $\frac{1369}{27}$

9. 6

10. Yes, f is one-one $\because \forall x_1, x_2 \in N \Rightarrow x_1^2 = x_2^2$.

11. $A = [1, \infty)$ because $R_f = [1, \infty)$

12. $n(B) = 10$

13. $(fof)(x) = \frac{x-3}{4}$

14. No, R is not reflexive $\because (a, a) \notin R \ \forall a \in N$

15. f is not one-one function

    $\because f(3) = f(-1) = 2$

    $3 \neq -1$ *i.e.* distinct elements have same images.

16. $B = [-1, 1]$

18. $e = 5$

19. Identity element does not exists.

20.

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

21. (a) Bijective (one-one, onto)

(b) Neither one-one nor onto

(c) One-one but not onto

(d) Neither one-one nor onto

22. $\text{go}f(x) = 0 \nrightarrow x \in R$

$$\text{fog}(x) = \begin{cases} 0, & x \geq 0 \\ -4x & x > 0 \end{cases}$$

23. $f^{-1}(x) = \dfrac{x + \sqrt{x^2 - 4}}{2}$

26. 512, 64

27. ± 4, pre image of – 3 does not exist.

28. $fog\left(-\dfrac{2}{3}\right) = 0, gof\left(-\dfrac{2}{3}\right) = 1$

30. Elements related to 4 are 0, 4, 8.

31. $f^{-1}(x)=\frac{7x+4}{5x-3}$

32. 0 is the identity element.

33. Neither commutative nor associative.

34. (i) Commutative and associative.

(ii) (1, 1) is identity in $N \times N$

38. $[(3, 4)]=\{(3, 4), (6, 8), (9,12)\}$

39. *0* is the identity element.

40. $(fog)(x) = x^2 + x$

$(gof)(x) = x^2 - x + 1$

Clearly, they are unequal.

42. $f^{-1}(y)=\frac{\sqrt{y-6}-3}{2}$, $f^{-1}(31) = 1$

43. $(fog)^{-1}(x)=\left(\frac{x+7}{2}\right)^{1/3}$, $(fog)^{-1}(9) = 2$

44. I. Not commutative

II. Associative

III. (1,0)

IV. Inverse of (a, b) = $\left(\frac{1}{a}, \frac{-b}{a}\right)$, Inverse of (1, 2) = (1,-2) and Inverse of $\left(\frac{1}{3}, -5\right)$ = (3, 15)

45. Reflexive, not symmetric, not transitive