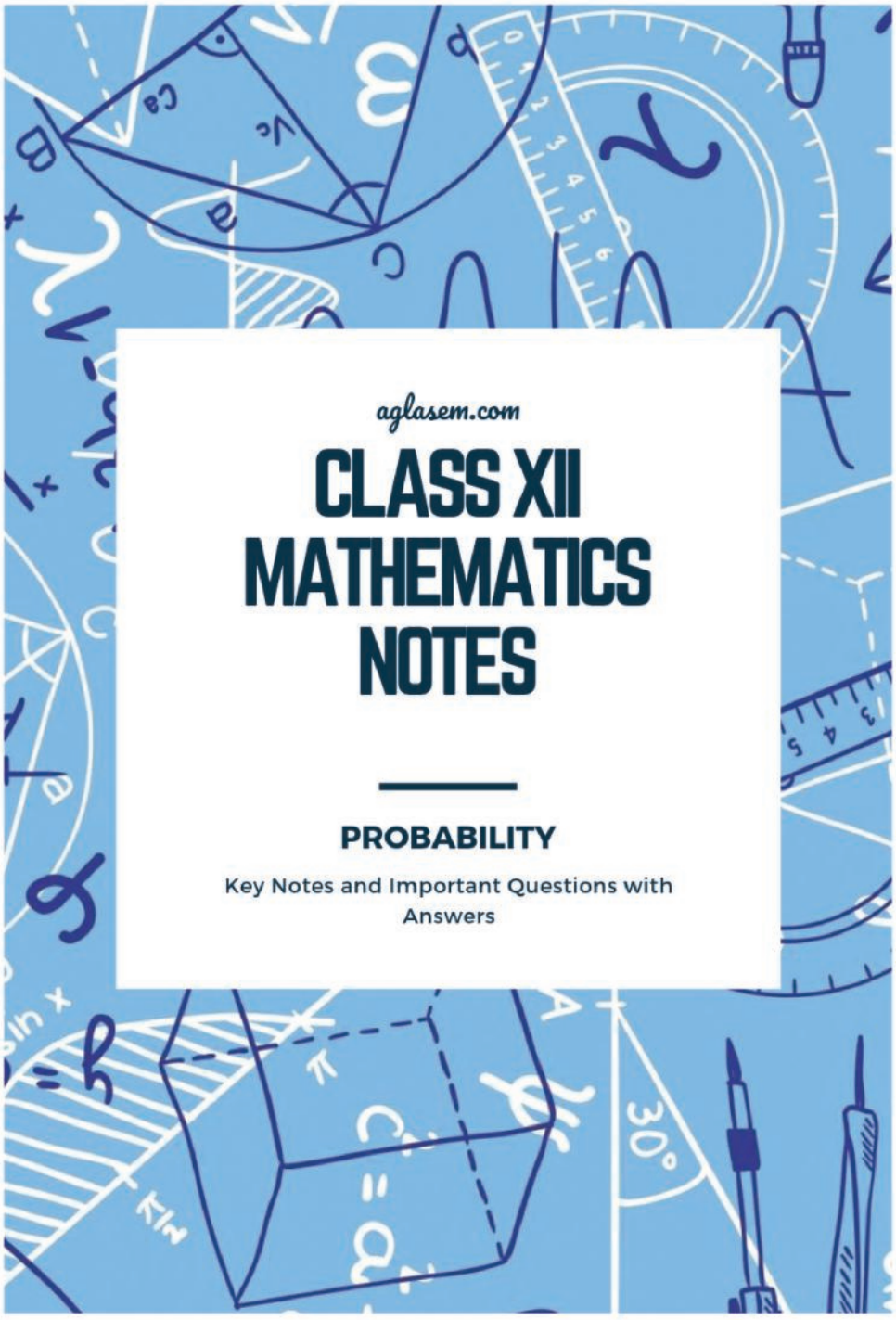

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CLASS XII
MATHEMATICS
NOTES
PROBABILITY
Key Notes and Important Questions with
Answers

CHAPTER 13

PROBABILITY

POINTS TO REMEMBER

- **Conditional Probability:** If A and B are two events associated with any random experiment, then P(A/B) represents the probability of occurrence of event A knowing that event B has already occurred.

$$P\ (A/B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0$$

P(B) ≠ 0, means that the event should not be impossible.

P(A ∩ B) = P(A and B) = P(B) × P(A/B)

- Similarly P(A ∩ B ∩ C) = = P(A) × P(B/A) × P(C/AB)

P (A/S) = P(A), P(A/A)= 1, P(S/A)= 1, P(A^1/B)= 1-P (A/B)

- **Multiplication Theorem on Probability:** If the event A and B are associated with any random experiment and the occurrence of one depends on the other, then

P(A ∩ B) = P(A) × P(B/A) where P(A) ≠ 0

- When the occurrence of one does not depend on the other then these event are said to be independent events.

Here P(A/B) = P(A) and P(B/A) = P(B)

P(A ∩ B) = P(A) × P(B)

- Theorem on total probability: If $E_1, E_2, E_3 \ldots., E_n$ be a partition of sample space and $E_1, E_2 \ldots., E_n$ all have non-zero probability. A be any event associated with sample space S, which occurs with $E_1, or\ E_2, \ldots., or\ E_n$, then

P(A) = P(E_1) . P(A/E_1) + P(E_2) . P(A/E_2) + ... + P(E_n) . P(A/E_n)

If A & B are independent then (i) A ∩ B^c, (ii) A^c ∩ B& (iii) A^c ∩ b^c are also independent.

- **Bayes' theorem :** Let S be the sample space and $E_1, E_2 \ldots, E_n$ be n mutually exclusive and exhaustive events associated with a random experiment. If A is any event which occurs with $E_1, or\ E_{2\ or}\ \ldots, E_n$, then

$$P(E_i/A) = \frac{P(E_i)P(A/E_i)}{\sum_{i=0}^{n} P(E_i)P(A/E_i)}$$

- **Random variable :** It is real valued function whose domain is the sample space of random experiment.

- **Probability distribution :**It is a system of number of random variable (X), such that

X:	X_1	X_2	$X_{3\ldots}$	X_n
P(X):	$P(X_{1)}$	$P(X_2)$	$P(X_{3)\ldots}$	$P(X_{n)}$

Where P(x_i)> 0 and$\sum_{i=0}^{n} P(x_i) = 1$

- Mean or expectation of a random variables (X) is donated by E(X)

$$E(X) = \mu = \sum_{i=0}^{n} x_{i\ P(x_i)}$$

- Variance of X denoted by var(X) or σ_{x^2} and

$$Var(X) = \sigma_{x^2} = \sum_{i=0}^{n} (x_i - \mu)^2\, P(X_i) = \sum_{i=0}^{n} x_i^2 P(x_i) - \mu^2$$

- The non-negative number $\sigma_x = \sqrt{var(X)}$ is called standard deviation of random variable X.

- **Bernoulli Trials:** Trials of random experiment are called Bernoulli trails if:

(xi) Number of trials is finite.

(xii) Trials are independent.

(xiii) Each trial has exactly two outcomes-either success or failure.

(xiv) Probability of success remain same in each trail.

- **Binomial distribution:**

P(X = r) = n_{c_r} , $q^{n-r}p^r$, where r= 0, 1, 2, …n

P = Probability of Success

q = Probability of Failure

n = total number of trails

r = value of random variables.

Very Short Answer Type Question (1 Mark)

1. Find P (A/B) if P(A)= 0.4, P(B)= 0.8 and P (B/A)= 0.6
2. Find P (A ∩ B) if A and B are two events such that P(A) = 0.5, P(B) = 0.6 and P (A ∪ B) = 0.8
3. A soldier fires three bullets on enemy: The probability that the enemy will be killed by one bullet is 0.7. What is the probability that the enemy is still alive?
4. If P(A)= $\frac{1}{2}$, P(B)= $\frac{7}{12}$ and P (not A or not B)=$\frac{1}{4}$. State whether A and B are independent.
5. Three coins are tossed once. Find the probability of getting at least one head.
6. The probability that a student is not a swimmer is $\frac{1}{5}$. Find the probability that out of 5 students, 4 are swimmers.
7. Find P(A/B), if P(B) = 0.5 and P(A ∩ B) = 0.32

Short Answer Type Questions (2 Marks)

8. If A and B are two events such that P(A) $\neq$ 0, then find P (B/A) if (i) A is a subset of B (ii) A $\cap$ B = ϕ

9. A random variable X has the following probability distribution find K.

X	0	1	2	3	4	5
P(X)	$\frac{1}{15}$	K	$\frac{15K-2}{15}$	K	$\frac{15K-1}{15}$	$\frac{1}{15}$

10. If P(A) = $\frac{1}{2}$, P(A $\cup$ B)= $\frac{3}{5}$, and P(B) =q find the value of q if A and B are (i) Mutually exclusive (ii) independent events.

11. If P(A) = $\frac{3}{10}$, P(B) = $\frac{2}{5}$ and P(A $\cup$ B) = $\frac{3}{5}$, then find P(B/A) + P(A/B)

12. A die is rolled if the out come is an even number. What is the probability that it is a prime?

13. If A and B are two-events such that P(A) = $\frac{1}{4}$, P(B) = $\frac{1}{2}$ and P(A $\cap$ B) = $\frac{1}{8}$ Find P (not A and not B).

14. A pair of die is rolled six times. Find the probability that a third sum of 7 is observed in sixth throw.

15. Ten eggs are drawn successively with replacement from a lot containing 10% defective eggs. Find the probability that there is at least one defective egg.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

16. A problem in mathematics is given to three students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. What is the probability that the problem is solved?

17. Two aeroplanes X and Y bomb a target in succession. There probabilities to hit correctly are 0.3 and 0.2 respectively. The second plane will bomb only if first miss the target. Find the probability that target is hit by Y plane.

18. A can hit a target 4 times in 5 shots B three times in 4 shots and C twice in 3 shots. They fire a volley. What is the probability that at least two shots hit?

19. Two dice are thrown once. Find the probability of getting an even number on the first die or a total of 8.

20. A and B throw a die alternatively till one of them throws a '6' and wins the game. Find their respective probabilities of winning, if A starts the game.

21. A man takes a step forward with probability 0.4 and backward with probability 0.6. Find the probability that at the end of eleven steps he is one step away from the starting point.

22. Two cards are drawn from a pack of well shuffled 52 cards one by one with replacement. Getting an ace or a spade is considered a success. Find the probability distribution for the number of successes.

23. In a game, a man wins a rupee for a six and looses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins/looses.

24. Suppose that 10% of men and 5% of women have grey hair. A grey haired person is selected at random. What is the probability that the selected person is male assuming that there are 60% males and 40% females?

25. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn. What is the probability that they both are diamonds?

26. If A and B are two independent events such that $P(\overline{A} \cap B) = \frac{2}{15}$ and $P(A \cap \overline{B}) = \frac{1}{6}$ then find P(A) and P(B).

Long Answer Type Questions (6 Marks)

27. In a hurdle race, a player has to cross 8hurdles. The probability that he will clear a hurdle is $\frac{4}{5}$, what is the probability that he will knock down in fewer than 2 hurdles?

28. Bag A contains 4 red, 3 white and 2 black balls. Bag B contains 3 res, 2 white and 3 black balls. One ball is transferred from bag A to bag B and then a ball is drawn from bag B. The ball so drawn is found to be red. Find the probability that the transferred ball is black.

29. If a fair coin is tossed 10 times, find the probability of getting.

    (i) Exactly six heads,

    (ii) at least six heads,

    (iii) at most six heads.

30. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter by other means of transport are respectively $\frac{3}{10}, \frac{1}{5}, \frac{1}{10} and \frac{2}{5}$.The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3} and \frac{1}{12}$ if he comes by train, bus and scooter respectively but if comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that he comes by train?

31. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is six. Find the probability that it is actually a six.

32. An insurance company insured 2000 scoter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident is 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

33. Three cards from a pack of 52 cards are lost. One card is drawn from the remaining cards. If drawn card is heart, find the probability that the lost cards were all hearts.

34. A box X contains 2 white and 3 red balls and a bag Y contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag Y.

35. In answering a question on a multiple choice, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be incorrect with probability $\frac{1}{4}$. What is the probability that the student knows the answer, given that he answered correctly?

36. Suppose a girl throws a die. If she gets 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4 she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head. What is the probability that she throws 1, 2, 3 or 4 with the die?

37. In a bolt factory machines, A, B and C manufacture bolts in the ratio 6:3:1. 2%, 5% and 10% of the bolts produced by them respectively are defective. A bolt is picked up at random from the product and is found to be defective. What is the probability that it has been manufactured by machine A?

38. Two urns A and B contain 6 black and 4 white, 4 black and 6 white balls respectively. Two balls are drawn from one of the urns. If both the balls drawn are white, find the probability that the balls are drawn from urn B.

39. Two cards are drawn from a well shuffled pack of 52 cards. Find the mean and variance for the number of face cards obtained.

40. A letter is known to have come from TATA NAGAR or from CALCUTTA on the envelope first two consecutive letters 'TA' are visible. What is the probability that the letter come from TATA NAGAR?

41. Two groups are competing for the position on the Board of Directors of a corporation. The probabilities that first and the second group will win are 0.6 and 0.4 respectively. Further if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product introduced was by the second group.

42. Two numbers are selected at random (without replacement) from positive integers 2, 3, 4, 5, 6, 7. Let X denotes the larger of the two numbers obtained. Find the mean and variance of the probability distribution of X.

43. An urn contains five balls. Two balls are drawn and are found to be white. What is the probability that all the balls are white?

44. Find the probability distribution of the number of doublets in four throws of a pair of dice. Also find the mean and S.D. of his distribution.

45. Three critics review a book. Odds in favour of the book are 5:2, 4:3 and 3:4 respectively for the three critics. Find the probability that the majority are in favour of the book.

46. A box contains 2 Black, 4 White and 3 Red balls. One by one all balls are drawn without replacement and arranged in sequence of drawing. Find the probability that the drawn balls are in sequence of BBWWWRRR.

47. A bag contains 3 White, 3Black and 2 Red balls. 3 balls are successively drawn without replacement. Find the probability that third ball is red.

Answers

1. 0.3
2. $\frac{3}{10}$
3. $(0.3)^3$
4. No
5. $\frac{7}{8}$
6. $\left(\frac{4}{5}\right)^4$
7. $\frac{16}{25}$
8. (i) 1 (ii) 0
9. K = $\frac{4}{15}$
10. (i) $\frac{1}{10}$ (ii) $\frac{1}{5}$
11. $7/12$
12. $1/3$
13. $3/8$
14. $1250 \times \left(\frac{1}{6}\right)^6$

15. $1-\left(\frac{9}{10}\right)^{10}$

16. $\frac{3}{4}$

17. $\frac{7}{22}$

18. $\frac{5}{6}$

19. $\frac{5}{9}$

20. $\frac{6}{11}, \frac{5}{11}$

21. 0.3678 or $11_{C_5}(0.4)^5(0.6)^5$

22.

X	0	1	2
P(X)	81/169	72/169	16/169

23. $-\frac{91}{54}$

24. $\frac{3}{4}$

25. $\frac{11}{50}$

26. $P(A)=\frac{1}{5}, P(B)=\frac{1}{6}$ or $P(A)=5/6$, P(B)= 4/5

27. $\frac{12}{5}\left(\frac{4}{5}\right)^7$

28. $\frac{6}{31}$

29. $(i)\frac{105}{512}$ (ii)$\frac{193}{512}$ (iii)$\frac{53}{64}$

30. $\frac{1}{2}$

31. $\frac{3}{8}$

33. $\frac{10}{49}$

34. $\frac{25}{52}$

35. $\frac{12}{13}$

36. $\frac{8}{11}$

37. $\frac{12}{37}$

38. $\frac{5}{7}$

39. Mean $=\frac{6}{13}$ Variance $=\frac{60}{169}$

40. $\frac{7}{11}$

41. $\frac{2}{9}$

42. Mean $=\frac{17}{3}$ Variance$=\frac{14}{9}$

43. $\frac{1}{2}$

44. Mean = 2/3 S.D.= $\frac{\sqrt{5}}{3}$

45. $\frac{209}{343}$

46. $\frac{1}{1260}$

47. $\frac{1}{4}$