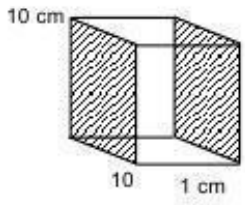
CHAPTER 28
HEAT TRANSFER

1. $t_1 = 90°C$, $t_2 = 10°C$



$l = 1\ cm = 1 \times 10^{-3}\ m$

$A = 10\ cm \times 10\ cm = 0.1 \times 0.1\ m^2 = 1 \times 10^{-2}\ m^2$

$K = 0.80\ w/m\text{-}°C$

$$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l} = \frac{8\times10^{-1}\times1\times10^{-2}\times80}{1\times10^{-2}} = 64\ J/s = 64 \times 60\ 3840\ J.$$

2. $t = 1\ cm = 0.01\ m$, $A = 0.8\ m^2$

$\theta_1 = 300$, $\theta_2 = 80$

$K = 0.025$,

$$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l} = \frac{0.025\times0.8\times(30030)}{0.01} = 440\ watt.$$

3. $K = 0.04\ J/m\text{-}5°C$, $A = 1.6\ m^2$

$t_1 = 97°F = 36.1°C$ $t_2 = 47°F = 8.33°C$

$l = 0.5\ cm = 0.005\ m$

$$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l} = \frac{4\times10^{-2}\times1.6\times27.78}{5\times10^{-3}} = 356\ J/s$$

4. $A = 25\ cm^2 = 25 \times 10^{-4}\ m^2$

$l = 1\ mm = 10^{-3}\ m$

$K = 50\ w/m\text{-}°C$

$\frac{Q}{t}$ = Rate of conversion of water into steam

$$= \frac{100\times10^{-3}\times2.26\times10^{6}}{1\ min} = \frac{10^{-1}\times2.26\times10^{6}}{60} = 0.376 \times 10^4$$

$$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l} \Rightarrow 0.376\times10^4 = \frac{50\times25\times10^{-4}\times(\theta - 100)}{10^{-3}}$$

$$\Rightarrow \theta = \frac{10^{-3}\times0.376\times10^4}{50\times25\times10^{-4}} = \frac{10^5\times0.376}{50\times25} = 30.1 \approx 30$$

5. $K = 46\ w/m\text{-}s°C$

$l = 1\ m$

$A = 0.04\ cm^2 = 4 \times 10^{-6}\ m^2$



$L_{fussion\ ice} = 3.36 \times 10^5\ j/Kg$

$$\frac{Q}{t} = \frac{46\times4\times10^{-6}\times100}{1} = 5.4 \times 10^{-8}\ kg \approx 5.4 \times 10^{-5}\ g.$$

6. $A = 2400\ cm^2 = 2400 \times 10^{-4}\ m^2$

$\ell = 2\ mm = 2 \times 10^{-3}\ m$

$K = 0.06\ w/m\text{-}°C$

$\theta_1 = 20°C$

$\theta_2 = 0°C$

$$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{\ell} = \frac{0.06\times2400\times10^{-4}\times20}{2\times10^{-3}} = 24 \times 6 \times 10^{-1} \times 10 = 24 \times 6 = 144\ J/sec$$

$$\text{Rate in which ice melts} = \frac{m}{t} = \frac{Q}{t\times L} = \frac{144}{3.4\times10^5}\ Kg/h = \frac{144\times3600}{3.4\times10^5}\ Kg/s = 1.52\ kg/s.$$

7. $\ell = 1\ mm = 10^{-3}\ m$ $m = 10\ kg$

$A = 200\ cm^2 = 2 \times 10^{-2}\ m^2$

$L_{vap} = 2.27 \times 10^6\ J/kg$

$K = 0.80\ J/m\text{-}s\text{-}°C$

$dQ = 2.27 \times 10^6 \times 10$,

$$\frac{dQ}{dt} = \frac{2.27\times10^7}{10^5} = 2.27 \times 10^2\ J/s$$

Again we know

$$\frac{dQ}{} = \frac{0.80\times2\times10^{-2}\times(42 - T)}{}$$

$\frac{}{dt} = \frac{}{1\times10^{-3}}$

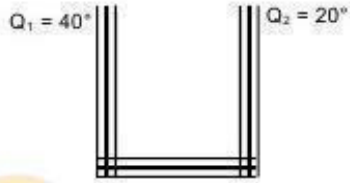
So, $\frac{8\times2\times10^{-3}(42-T)}{10^{-3}} = 2.27\times10^{2}$

$\Rightarrow 16\times42 - 16T = 227 \Rightarrow T = 27.8 \approx 28°C$

8. K = 45 w/m-°C

$\ell = 60\ cm = 60\times10^{-2}\ m$

$A = 0.2\ cm^2 = 0.2\times10^{-4}\ m^2$



Rate of heat flow,

$= \frac{KA(\theta_1-\theta_2)}{\ell} = \frac{45\times0.2\times10^{-4}\times20}{60\times10^{-2}} = 30\times10^{-3}\ 0.03\ w$

9. $A = 10\ cm^2$, $h = 10\ cm$

$\frac{\Delta Q}{\Delta t} = \frac{KA(\theta_1-\theta_2)}{\ell} = \frac{200\times10^{-3}\times30}{1\times10^{-3}} = 6000$

Since heat goes out from both surfaces. Hence net heat coming out.

$= \frac{\Delta Q}{\Delta t} = 6000\times2 = 12000, \quad \frac{\Delta Q}{\Delta t} = MS\frac{\Delta\theta}{\Delta t}$

$\Rightarrow 6000\times2 = 10^{-3}\times10^{-1}\times1000\times4200\times\frac{\Delta\theta}{\Delta t}$

$\Rightarrow \frac{\Delta\theta}{\Delta t} = \frac{72000}{420} = 28.57$

So, in 1 Sec. 28.57°C is dropped

Hence for drop of 1°C $\frac{1}{28.57}$ sec. = 0.035 sec. is required

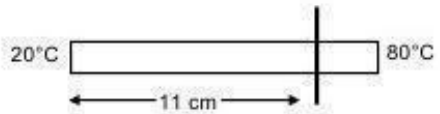
10. $\ell = 20\ cm = 20\times10^{-2}\ m$

$A = 0.2\ cm^2 = 0.2\times10^{-4}\ m^2$

$\theta_1 = 80°C, \quad \theta_2 = 20°C, \quad K = 385$

(a) $\frac{Q}{t} = \frac{KA(\theta_1-\theta_2)}{\ell} = \frac{385\times0.2\times10^{-4}(80-20)}{20\times10^{-2}} = 385\times6\times10^{-4}\times10 = 2310\times10^{-3} = 2.31$

(b) Let the temp of the 11 cm point be θ



$\frac{\Delta\theta}{\Delta l} = \frac{Q}{tKA}$

$\Rightarrow \frac{\Delta\theta}{\Delta l} = \frac{2.31}{385\times0.2\times10^{-4}}$

$\Rightarrow \frac{\theta-20}{11\times10^{-2}} = \frac{2.31}{385\times0.2\times10^{-4}}$

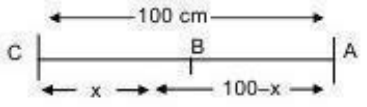
$\Rightarrow \theta - 20 = \frac{2.31\times10^{4}}{385\times0.2}\times11\times10^{-2} = 33$

$\Rightarrow \theta = 33 + 20 = 53$

11. Let the point to be touched be 'B'

No heat will flow when, the temp at that point is also 25°C

i.e. $Q_{AB} = Q_{BC}$



So, $\frac{KA(100-25)}{100-x} = \frac{KA(25-0)}{x}$

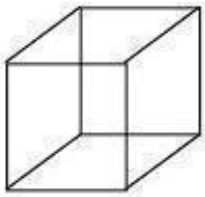
$\Rightarrow 75x = 2500 - 25x \Rightarrow 100x = 2500 \Rightarrow x = 25$ cm from the end with 0°C

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12. $V = 216\ cm^3$

$a = 6\ cm, \quad$ Surface area $= 6a^2 = 6\times36\ m^2$

$t = 0.1\ cm \quad \frac{Q}{t} = 100\ W,$



$\frac{Q}{t} = \frac{KA(\theta_1-\theta_2)}{\ell}$

$\Rightarrow 100 = \frac{K\times6\times36\times10^{-4}\times5}{0.1\times10^{-2}}$

$\Rightarrow K = \frac{100}{6\times36\times5\times10^{-1}} = 0.9259\ W/m°C \approx 0.92\ W/m°C$

13. Given $\theta_1 = 1°C, \quad \theta_2 = 0°C$

$K = 0.50$ w/m-°C, $d = 2$ mm $= 2 \times 10^{-3}$ m

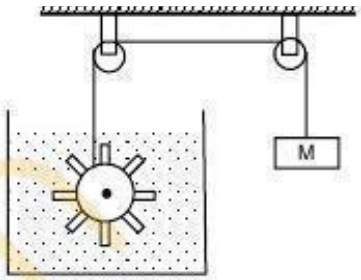
$A = 5 \times 10^{-2}$ m^2, $v = 10$ cm/s $= 0.1$ m/s

Power = Force × Velocity = Mg × v

Again Power $= \frac{dQ}{dt} = \frac{KA(\theta_1 - \theta_2)}{d}$

So, $Mgv = \frac{KA(\theta_1 - \theta_2)}{d}$

$\Rightarrow M = \frac{KA(\theta_1 - \theta_2)}{dvg} = \frac{5 \times 10^{-1} \times 5 \times^{-2} \times 1}{2 \times 10^{-3} \times 10^{-1} \times 10} = 12.5$ kg.



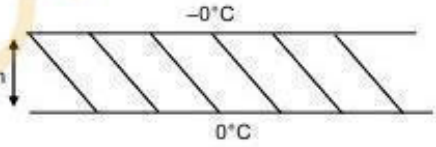
14. $K = 1.7$ W/m-°C $\quad f_w = 1000$ Kg/m^3

$L_{ice} = 3.36 \times 10^5$ J/kg $\quad T = 10$ cm $= 10 \times 10^{-2}$ m

(a) $\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{\ell} \Rightarrow \frac{\ell}{t} = \frac{KA(\theta_1 - \theta_2)}{Q} = \frac{KA(\theta_1 - \theta_2)}{mL}$

$= \frac{KA(\theta_1 - \theta_2)}{Atf_wL} = \frac{1.7 \times [0 - (-10)]}{10 \times 10^{-2} \times 1000 \times 3.36 \times 10^5}$

$= \frac{17}{3.36} \times 10^{-7} = 5.059 \times 10^{-7} \approx 5 \times 10^{-7}$ m/sec

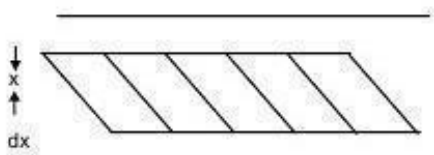


(b) let us assume that x length of ice has become formed to form a small strip of ice of length dx, dt time is required.

$\frac{dQ}{dt} = \frac{KA(\Delta\theta)}{x} \Rightarrow \frac{dmL}{dt} = \frac{KA(\Delta\theta)}{x} \Rightarrow \frac{Adxf\omega L}{dt} = \frac{KA(\Delta\theta)}{x}$

$\Rightarrow \frac{dxf\omega L}{dt} = \frac{K(\Delta\theta)}{x} \Rightarrow dt = \frac{xdxf\omega L}{K(\Delta\theta)}$

$\Rightarrow \int_0^t dt = \frac{f\omega L}{K(\Delta\theta)} \int_0^t x dx \qquad \Rightarrow t = \frac{f\omega L}{K(\Delta\theta)} \left[\frac{x^2}{2}\right]_0^l = \frac{f\omega L}{K\Delta\theta} \frac{l^2}{2}$

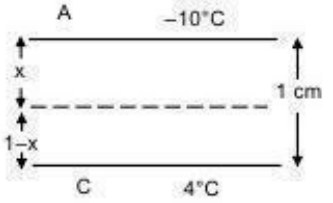


Putting values

$\Rightarrow t = \frac{1000 \times 3.36 \times 10^5 \times \left(10 \times 10^{-2}\right)^2}{1.7 \times 10 \times 2} = \frac{3.36}{2 \times 17} \times 10^6$ sec. $= \frac{3.36 \times 10^6}{2 \times 17 \times 3600}$ hrs = 27.45 hrs ≈ 27.5 hrs.

15. let 'B' be the maximum level upto which ice is formed. Hence the heat conducted at that point from both the levels is the same.

Let AB = x



i.e. $\frac{Q}{t}$ ice $= \frac{Q}{t}$ water $\Rightarrow \frac{K_{ice} \times A \times 10}{x} = \frac{K_{water} \times A \times 4}{(1-x)}$

$\Rightarrow \frac{1.7 \times 10}{x} = \frac{5 \times 10^{-1} \times 4}{1-x} \Rightarrow \frac{17}{x} = \frac{2}{1-x}$

$\Rightarrow 17 - 17x = 2x \Rightarrow 19x = 17 \Rightarrow x = \frac{17}{19} = 0.894 \approx 89$ cm

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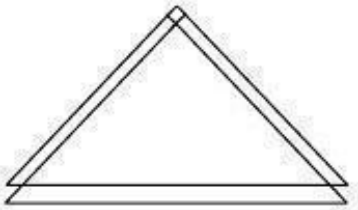
16. $K_{AB} = 50$ j/m-s-°c $\quad \theta_A = 40°C$

$K_{BC} = 200$ j/m-s-°c $\quad \theta_B = 80°C$

$K_{AC} = 400$ j/m-s-°c $\quad \theta_C = 80°C$

Length $= 20$ cm $= 20 \times 10^{-2}$ m

$A = 1$ cm$^2 = 1 \times 10^{-4}$ m^2



(a) $\frac{Q_{AB}}{t} = \frac{K_{AB} \times A(\theta_B - \theta_A)}{l} = \frac{50 \times 1 \times 10^{-4} \times 40}{20 \times 10^{-2}} = 1$ W.

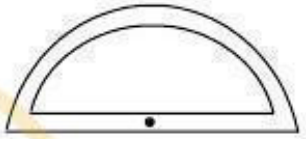
(b) $\frac{Q_{AC}}{t} = \frac{K_{AC} \times A(\theta_C - \theta_A)}{l} = \frac{400 \times 1 \times 10^{-4} \times 40}{20 \times 10^{-2}} = 800 \times 10^{-2} = 8$

(c) $\frac{Q_{BC}}{t} = \frac{K_{BC} \times A(\theta_B - \theta_C)}{l} = \frac{200 \times 1 \times 10^{-4} \times 0}{20 \times 10^{-2}} = 0$

17. We know $Q = \frac{KA(\theta_1 - \theta_2)}{d}$

$Q_1 = \frac{KA(\theta_1 - \theta_2)}{d_1}, \qquad Q_2 = \frac{KA(\theta_1 - \theta_2)}{d_2}$

$KA(\theta_1 - \theta_1)$



$$\frac{Q_1}{Q_2} = \frac{\frac{\pi r}{KA(\theta_1-\theta_1)}}{\frac{2r}{}} = \frac{2r}{\pi r} = \frac{2}{\pi} \qquad [d_1 = \pi r, \quad d_2 = 2r]$$

18. The rate of heat flow per sec.

$$= \frac{dQ_A}{dt} = KA\frac{d\theta}{dt}$$

The rate of heat flow per sec.

$$= \frac{dQ_B}{dt} = KA\frac{d\theta_B}{dt}$$

This part of heat is absorbed by the red.

$$\frac{Q}{t} = \frac{ms\Delta\theta}{dt} \qquad \text{where } \frac{d\theta}{dt} = \text{Rate of net temp. variation}$$

$$\Rightarrow \frac{msd\theta}{dt} = KA\frac{d\theta_A}{dt} - KA\frac{d\theta_B}{dt} \qquad \Rightarrow ms\frac{d\theta}{dt} = KA\left(\frac{d\theta_A}{dt} - \frac{d\theta_B}{dt}\right)$$

$$\Rightarrow 0.4 \times \frac{d\theta}{dt} = 200 \times 1 \times 10^{-4}\,(5 - 2.5)\ °C/cm$$

$$\Rightarrow 0.4 \times \frac{d\theta}{dt} = 200 \times 10^{-4} \times 2.5$$

$$\Rightarrow \frac{d\theta}{dt} = \frac{200 \times 2.5 \times 10^{-4}}{0.4 \times 10^{-2}}\ °C/m = 1250 \times 10^{-2} = 12.5\ °C/m$$

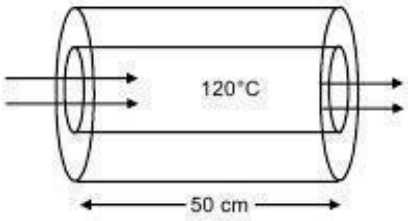
19. Given

$K_{rubber} = 0.15$ J/m-s-°C $\qquad T_2 - T_1 = 90°C$

We know for radial conduction in a Cylinder

$$\frac{Q}{t} = \frac{2\pi Kl(T_2 - T_1)}{\ln(R_2/R_1)}$$

$$= \frac{2 \times 3.14 \times 15 \times 10^{-2} \times 50 \times 10^{-1} \times 90}{\ln(1.2/1)} = 232.5 \approx 233\ j/s.$$



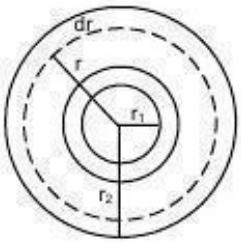
20. $\frac{dQ}{dt}$ = Rate of flow of heat

Let us consider a strip at a distance r from the center of thickness dr.

$$\frac{dQ}{dt} = \frac{K \times 2\pi rd \times d\theta}{dr} \qquad [d\theta = \text{Temperature diff across the thickness dr}]$$

$$\Rightarrow C = \frac{K \times 2\pi rd \times d\theta}{dr} \qquad \left[c = \frac{d\theta}{dr}\right]$$

$$\Rightarrow C\frac{dr}{r} = K2\pi d\ d\theta$$



Integrating

$$\Rightarrow C\int_{r_1}^{r_2}\frac{dr}{r} = K2\pi d\int_{\theta_1}^{\theta_2} d\theta \qquad \Rightarrow C[\log r]_{r_1}^{r_2} = K2\pi d\,(\theta_2 - \theta_1)$$

$$\Rightarrow C(\log r_2 - \log r_1) = K2\pi d\,(\theta_2 - \theta_1) \Rightarrow C\log\left(\frac{r_2}{r_1}\right) = K2\pi d\,(\theta_2 - \theta_1)$$

$$\Rightarrow C = \frac{K2\pi d(\theta_2 - \theta_1)}{\log(r_2/r_1)}$$

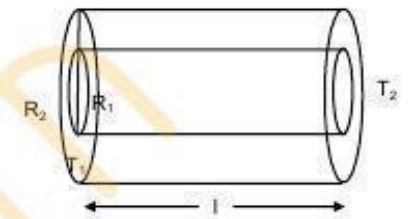
21. $T_1 > T_2$

$A = \pi(R_2^2 - R_1^2)$

$$\text{So, } Q = \frac{KA(T_2 - T_1)}{l} = \frac{KA(R_2^2 - R_1^2)(T_2 - T_1)}{l}$$

Considering a concentric cylindrical shell of radius 'r' and thickness 'dr'. The radial heat flow through the shell

$$H = \frac{dQ}{dt} = -KA\frac{d\theta}{dt}$$ [(-)ve because as r – increases θ decreases]



$A = 2\pi rl \qquad H = -2\pi rl\,K\frac{d\theta}{}$

$$= \frac{dQ}{dt}$$

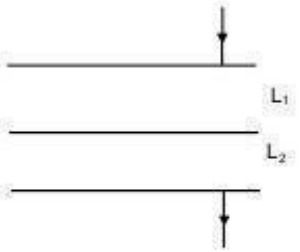
$$\text{or } \int_{R_1}^{R_2}\frac{dr}{r} = -\frac{2\pi LK}{H}\int_{T_1}^{T_2} d\theta$$

Integrating and simplifying we get

$$H = \frac{dQ}{dt} = \frac{2\pi KL(T_2 - T_1)}{\text{Loge}(R_2/R_1)} = \frac{2\pi KL(T_2 - T_1)}{\ln(R_2/R_1)}$$

22. Here the thermal conductivities are in series,

$$\therefore \frac{\frac{K_1A(\theta_1-\theta_2)}{l_1}\times\frac{K_2A(\theta_1-\theta_2)}{l_2}}{\frac{K_1A(\theta_1-\theta_2)}{l_1}+\frac{K_2A(\theta_1-\theta_2)}{l_2}} = \frac{KA(\theta_1-\theta_2)}{l_1+l_2}$$



$$\Rightarrow \frac{\frac{K_1}{l_1}\times\frac{K_2}{l_2}}{\frac{K_1}{l_1}+\frac{K_2}{l_2}} = \frac{K}{l_1+l_2}$$

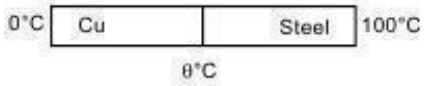
$$\Rightarrow \frac{K_1K_2}{K_1l_2+K_2l_1} = \frac{K}{l_1+l_2} \Rightarrow K = \frac{(K_1K_2)(l_1+l_2)}{K_1l_2+K_2l_1}$$

23. $K_{Cu} = 390$ w/m-°C $K_{St} = 46$ w/m-°C

Now, Since they are in series connection,

So, the heat passed through the crossections in the same.

So, $Q_1 = Q_2$



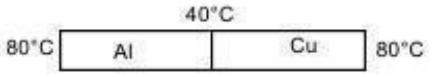
$$\text{Or } \frac{K_{Cu}\times A\times(\theta-0)}{l} = \frac{K_{St}\times A\times(100-\theta)}{l}$$

$$\Rightarrow 390(\theta-0) = 46\times 100 - 46\,\theta \Rightarrow 436\,\theta = 4600$$

$$\Rightarrow \theta = \frac{4600}{436} = 10.55 \approx 10.6°C$$

24. As the Aluminum rod and Copper rod joined are in parallel

$$\frac{Q}{t} = \left(\frac{Q}{t_1}\right)_{Al} + \left(\frac{Q}{t}\right)_{Cu}$$



$$\Rightarrow \frac{KA(\theta_1-\theta_2)}{l} = \frac{K_1A(\theta_1-\theta_2)}{l} + \frac{K_2A(\theta_1-\theta_2)}{l}$$

$$\Rightarrow K = K_1 + K_2 = (390 + 200) = 590$$

$$\frac{Q}{t} = \frac{KA(\theta_1-\theta_2)}{l} = \frac{590\times 1\times 10^{-4}\times(60-20)}{1} = 590\times 10^{-4}\times 40 = 2.36 \text{ Watt}$$

25. $K_{Al} = 200$ w/m-°C $K_{Cu} = 400$ w/m-°C

$A = 0.2\ cm^2 = 2\times 10^{-5}\ m^2$

$l = 20\ cm = 2\times 10^{-1}\ m$

Heat drawn per second

$$= Q_{Al} + Q_{Cu} = \frac{K_{Al}\times A(80-40)}{l} + \frac{K_{Cu}\times A(80-40)}{l} = \frac{2\times 10^{-5}\times 40}{2\times 10^{-1}}[200+400] = 2.4\ J$$

Heat drawn per min $= 2.4\times 60 = 144$ J

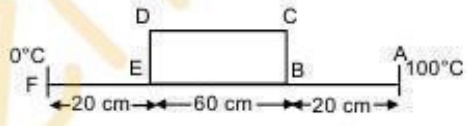
26. $(Q/t)_{AB} = (Q/t)_{BE\ bent} + (Q/t)_{BE}$

$$(Q/t)_{BE\ bent} = \frac{KA(\theta_1-\theta_2)}{70} \qquad (Q/t)_{BE} = \frac{KA(\theta_1-\theta_2)}{60}$$

$$\frac{(Q/t)_{BE\,bent}}{(Q/t)_{BE}} = \frac{60}{70} = \frac{6}{7}$$

$(Q/t)_{BE\ bent} + (Q/t)_{BE} = 130$

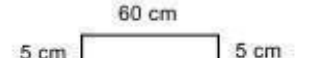
$\Rightarrow (Q/t)_{BE\ bent} + (Q/t)_{BE}\ 7/6 = 130$



$$\Rightarrow \left(\frac{7}{6}+1\right)(Q/t)_{BE\ bent} = 130 \qquad \Rightarrow (Q/t)_{BE\ bent} = \frac{130\times 6}{13} = 60$$

27. $$\frac{Q}{t}\text{bent} = \frac{780\times A\times 100}{70}$$

$$\frac{Q}{t}\text{str} = \frac{390\times A\times 100}{}$$

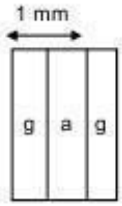


t 60

$$\frac{(Q/t)\text{bent}}{(Q/t)\text{ str}} = \frac{780\times A\times 100}{70}\times\frac{60}{390\times A\times 100} = \frac{12}{7}$$



28. (a) $\frac{Q}{t} = \frac{KA(\theta_1-\theta_2)}{\ell} = \frac{1\times 2\times 1(40-32)}{2\times 10^{-3}} = 8000$ J/sec.



(b) Resistance of glass $= \frac{\ell}{ak_g}+\frac{\ell}{ak_g}$

Resistance of air $= \frac{\ell}{ak_a}$

Net resistance $= \frac{\ell}{ak_g}+\frac{\ell}{ak_g}+\frac{\ell}{ak_a}$

$$= \frac{\ell}{a}\left(\frac{2}{k_g}+\frac{1}{k_a}\right) = \frac{\ell}{a}\left(\frac{2k_a+k_g}{K_gk_a}\right)$$

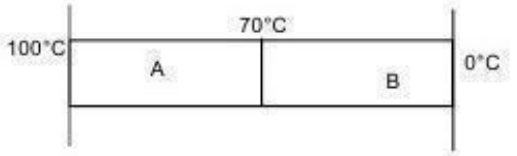
$$= \frac{1\times 10^{-3}}{2}\left(\frac{2\times 0.025+1}{0.025}\right)$$

$$= \frac{1\times 10^{-3}\times 1.05}{0.05}$$

$$\frac{Q}{t} = \frac{\theta_1-\theta_2}{R} = \frac{8\times 0.05}{1\times 10^{-3}\times 1.05} = 380.9 \approx 381 \text{ W}$$

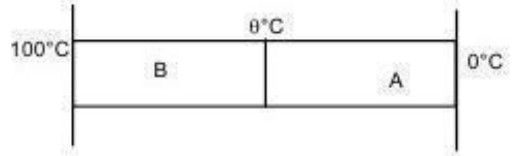
29. Now; Q/t remains same in both cases

In Case I : $\frac{K_A\times A\times(100-70)}{\ell} = \frac{K_B\times A\times(70-0)}{\ell}$



$\Rightarrow 30\,K_A = 70\,K_B$

In Case II : $\frac{K_B\times A\times(100-\theta)}{\ell} = \frac{K_A\times A\times(\theta-0)}{\ell}$



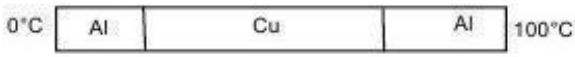
$\Rightarrow 100K_B - K_B\theta = K_A\theta$

$\Rightarrow 100K_B - K_B\theta = \frac{70}{30}K_B\theta$

$\Rightarrow 100 = \frac{7}{3}\theta+\theta \qquad \Rightarrow \theta = \frac{300}{10} = 30°C$

30. $\theta_1-\theta_2 = 100$

$\frac{Q}{t} = \frac{\theta_1-\theta_2}{R}$



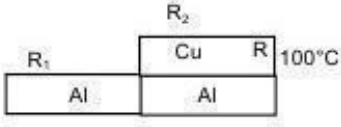
$$R = R_1+R_2+R_3 = \frac{\ell}{aK_{Al}}+\frac{\ell}{aK_{Cu}}+\frac{\ell}{aK_{Al}} = \frac{\ell}{a}\left(\frac{2}{200}+\frac{1}{400}\right) = \frac{\ell}{a}\left(\frac{4+1}{400}\right) = \frac{\ell}{a}\frac{1}{80}$$

$$\frac{Q}{t} = \frac{100}{(\ell/a)(1/80)} \Rightarrow 40 = 80\times 100\times\frac{a}{\ell}$$

$$\Rightarrow \frac{a}{\ell} = \frac{1}{200}$$

For (b)

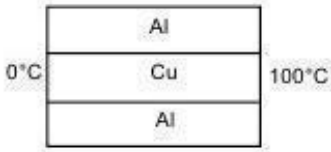
$$R = R_1+R_2 = R_1+\frac{R_{Cu}R_{Al}}{R_{Cu}+R_{Al}} = R_{Al}+\frac{R_{Cu}R_{Al}}{R_{Cu}+R_{Al}} = \frac{\frac{l}{AK_{Al}}+\frac{l}{AK_{Cu}}+\frac{l}{AK_{Al}}}{\frac{l}{A_{Cu}}+\frac{l}{A_{Al}}}$$



$$= \frac{l}{AK_{Al}}+\frac{l}{A}+\frac{l}{K_{Al}+K_{Cu}} = \frac{l}{A}\left(\frac{1}{200}+\frac{1}{200+400}\right) = \frac{l}{A}\times\frac{4}{600}$$

$$\frac{Q}{t} = \frac{\theta_1-\theta_2}{R} = \frac{100}{(l/A)(4/600)} = \frac{100\times 600}{4}\frac{A}{l} = \frac{100\times 600}{4}\times\frac{1}{200} = 75$$

For (c)



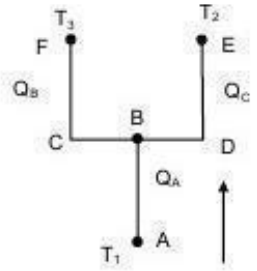
$$\frac{1}{R} = \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} = \frac{1}{\frac{l}{aK_{Al}}}+\frac{1}{\frac{l}{aK_{Cu}}}+\frac{1}{\frac{l}{aK_{Al}}}$$

$$= \frac{a}{l}(K_{Al} + K_{Cu} + K_{Al}) = \frac{a}{l}(2\times 200 + 400) = \frac{a}{l}(800)$$

$$\Rightarrow R = \frac{l}{a}\times\frac{1}{800}$$

$$\Rightarrow \frac{Q}{t} = \frac{\theta_1 - \theta_2}{R} = \frac{100\times 800\times a}{l}$$

$$= \frac{100\times 800}{200} = 400\ W$$

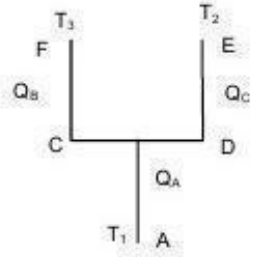


31. Let the temp. at B be T

$$\frac{Q_A}{t} = \frac{Q_B}{t} + \frac{Q_C}{t} \qquad \Rightarrow \frac{KA(T_1 - T)}{l} = \frac{KA(T - T_3)}{l + (l/2)} + \frac{KA(T - T_2)}{l + (l/2)}$$

$$\Rightarrow \frac{T_1 - T}{l} = \frac{T - T_3}{3l/2} + \frac{T - T_2}{3l/2} \qquad \Rightarrow 3T_1 - 3T = 4T - 2(T_2 + T_3)$$

$$\Rightarrow -7T = -3T_1 - 2(T_2 + T_3) \qquad \Rightarrow T = \frac{3T_1 + 2(T_2 + T_3)}{7}$$



32. The temp at the both ends of bar F is same

Rate of Heat flow to right = Rate of heat flow through left

$$\Rightarrow (Q/t)_A + (Q/t)_C = (Q/t)_B + (Q/t)_D$$

$$\Rightarrow \frac{K_A(T_1 - T)A}{l} + \frac{K_C(T_1 - T)A}{l} = \frac{K_B(T - T_2)A}{l} + \frac{K_D(T - T_2)A}{l}$$

$$\Rightarrow 2K_0(T_1 - T) = 2\times 2K_0(T - T_2)$$

$$\Rightarrow T_1 - T = 2T - 2T_2$$

$$\Rightarrow T = \frac{T_1 + 2T_2}{3}$$

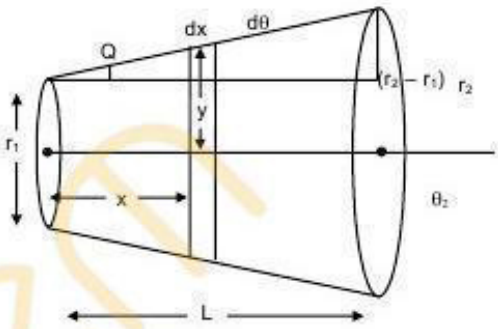
33. $\text{Tan}\ \phi = \frac{r_2 - r_1}{L} = \frac{(y - r_1)}{x}$

$$\Rightarrow xr_2 - xr_1 = yL - r_1L$$

Differentiating wr to 'x'

$$\Rightarrow r_2 - r_1 = \frac{Ldy}{dx} - 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{r_2 - r_1}{L} \Rightarrow dx = \frac{dyL}{(r_2 - r_1)} \qquad ...(1)$$



Now $\frac{Q}{T} = \frac{K\pi y^2 d\theta}{dx} \Rightarrow \frac{\theta dx}{T} = k\pi y^2 d\theta$

$$\Rightarrow \frac{\theta L dy}{r_2 r_1} = K\pi y^2 d\theta \qquad \text{from(1)}$$

$$\Rightarrow d\theta\ \frac{QLdy}{(r_2 - r_1)K\pi y^2}$$

Integrating both side

$$\Rightarrow \int_{\theta_1}^{\theta_2} d\theta = \frac{QL}{(r_2 - r_1)K\pi}\int_{r_1}^{r_2}\frac{dy}{y}$$

$$\Rightarrow (\theta_2 - \theta_1) = \frac{QL}{(r_2 - r_1)K\pi}\times\left[\frac{-1}{y}\right]_{r_1}^{r_2}$$

$$\Rightarrow (\theta_2 - \theta_1) = \frac{QL}{(r_2 - r_1)K\pi}\times\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$\Rightarrow (\theta_2 - \theta_1) = \frac{QL}{(r_2 - r_1)K\pi}\times\left[\frac{r_2 - r_1}{r_1 + r_2}\right]$$

$$\Rightarrow Q = \frac{K\pi r_1 r_2(\theta_2 - \theta_1)}{L}$$

34. $\frac{d\theta}{dt} = \frac{60}{10\times 60} = 0.1°C/sec$

$$\frac{dQ}{dt} = \frac{KA}{d}(\theta_1 - \theta_2)$$

$$= \frac{KA \times 0.1}{d} + \frac{KA \times 0.2}{d} + ... + \frac{KA \times 60}{d}$$

$$= \frac{KA}{d}(0.1 + 0.2 + ... + 60) = \frac{KA}{d} \times \frac{600}{2} \times (2 \times 0.1 + 599 \times 0.1)$$

[$\therefore$ a + 2a +.....+ na = n/2{2a + (n – 1)a}]

$$= \frac{200 \times 1 \times 10^{-4}}{20 \times 10^{-2}} \times 300 \times (0.2 + 59.9) = \frac{200 \times 10^{-2} \times 300 \times 60.1}{20}$$

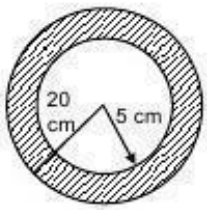
$= 3 \times 10 \times 60.1 = 1803$ w ≈ 1800 w

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35. $a = r_1 = 5$ cm $= 0.05$ m

$b = r_2 = 20$ cm $= 0.2$ m

$\theta_1 = T_1 = 50°C$ $\theta_2 = T_2 = 10°C$

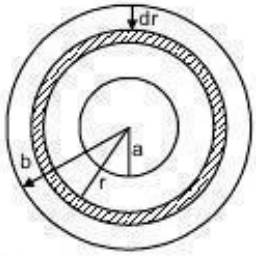


Now, considering a small strip of thickness 'dr' at a distance 'r'.

$A = 4\pi r^2$

$H = -4\pi r^2 K \frac{d\theta}{dr}$ [(–)ve because with increase of r, θ decreases]

$$= \int_a^b \frac{dr}{r^2} = \frac{-4\pi K}{H} \int_{\theta_1}^{\theta_2} d\theta$$ On integration,



$$H = \frac{dQ}{dt} = K\frac{4\pi ab(\theta_1 - \theta_2)}{(b-a)}$$

Putting the values we get

$$\frac{K \times 4 \times 3.14 \times 5 \times 20 \times 40 \times 10^{-3}}{15 \times 10^{-2}} = 100$$

$$\Rightarrow K = \frac{15}{4 \times 3.14 \times 4 \times 10^{-1}} = 2.985 \approx 3 \text{ w/m-°C}$$

36. $\frac{Q}{t} = \frac{KA(T_1 - T_2)}{L}$ Rise in Temp. in $T_2 \Rightarrow \frac{KA(T_1 - T_2)}{Lms}$

Fall in Temp in $T_1 = \frac{KA(T_1 - T_2)}{Lms}$ Final Temp. $T_1 \Rightarrow T_1 - \frac{KA(T_1 - T_2)}{Lms}$

Final Temp. $T_2 = T_2 + \frac{KA(T_1 - T_2)}{Lms}$

Final $\frac{\Delta T}{dt} = T_1 - \frac{KA(T_1 - T_2)}{Lms} - T_2 - \frac{KA(T_1 - T_2)}{Lms}$

$$= (T_1 - T_2) - \frac{2KA(T_1 - T_2)}{Lms} = \frac{dT}{dt} = -\frac{2KA(T_1 - T_2)}{Lms} \Rightarrow \int_{(T_1 - T_2)}^{(T_1 - T_2)} \frac{dt}{(T_1 - T_2)} = \frac{-2KA}{Lms} dt$$

$$\Rightarrow \text{Ln}\frac{(T_1 - T_2)/2}{(T_1 - T_2)} = \frac{-2KAt}{Lms} \quad \Rightarrow \ln(1/2) = \frac{-2KAt}{Lms} \quad \Rightarrow \ln_2 = \frac{2KAt}{Lms} \quad \Rightarrow t = \ln_2 \frac{Lms}{2KA}$$

37. $\frac{Q}{t} = \frac{KA(T_1 - T_2)}{L}$ Rise in Temp. in $T_2 \Rightarrow \frac{KA(T_1 - T_2)}{Lm_1s_1}$

Fall in Temp in $T_1 \Rightarrow \frac{KA(T_1 - T_2)}{Lm_2s_2}$ Final Temp. $T_1 = T_1 - \frac{KA(T_1 - T_2)}{Lm_1s_1}$

Final Temp. $T_2 = T_2 + \frac{KA(T_1 - T_2)}{Lm_1s_1}$

$$\frac{\Delta T}{dt} = T_1 - \frac{KA(T_1 - T_2)}{Lm_1s_1} - T_2 - \frac{KA(T_1 - T_2)}{Lm_2s_2} = (T_1 - T_2) - \left[\frac{KA(T_1 - T_2)}{Lm_1s_1} + \frac{KA(T_1 - T_2)}{Lm_2s_2}\right]$$

$$\Rightarrow \frac{dT}{dt} = -\frac{KA(T_1 - T_2)}{L}\left(\frac{1}{m_1s_1} + \frac{1}{m_2s_2}\right) \quad \Rightarrow \frac{dT}{(T_1 - T_2)} = -\frac{KA}{L}\left(\frac{m_2s_2 + m_1s_1}{m_1s_1m_2s_2}\right)dt$$

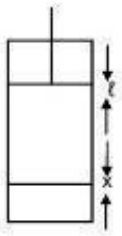
$$\Rightarrow \ln\Delta t = -\frac{KA}{L}\left(\frac{m_2s_2 + m_1s_1}{m_1s_1m_2s_2}\right)t + C$$

At time $t = 0$, $T = T_0$, $\Delta T = \Delta T_0$ $\Rightarrow C = \ln\Delta T_0$

$$\Rightarrow \ln\frac{\Delta T}{\Delta T_0} = -\frac{KA}{L}\left(\frac{m_2s_2 + m_1s_1}{m_1s_1m_2s_2}\right)t \Rightarrow \frac{\Delta T}{\Delta T_0} = e^{-\frac{KA}{L}\left(\frac{m_1s_1+m_2s_2}{m_1s_1m_2s_2}\right)t}$$

$$\Rightarrow \Delta T = \Delta T_0\, e^{-\frac{KA}{L}\left(\frac{m_1s_1+m_2s_2}{m_1s_1m_2s_2}\right)t} = (T_2 - T_1)\, e^{-\frac{KA}{L}\left(\frac{m_1s_1+m_2s_2}{m_1s_1m_2s_2}\right)t}$$

38. $\frac{Q}{t} = \frac{KA(T_s - T_0)}{x} \Rightarrow \frac{nC_P dT}{dt} = \frac{KA(T_s - T_0)}{x}$



$$\Rightarrow \frac{n(5/2)RdT}{dt} = \frac{KA(T_s - T_0)}{x} \Rightarrow \frac{dT}{dt} = \frac{-2LA}{5nRx}(T_S - T_0)$$

$$\Rightarrow \frac{dT}{(T_S - T_0)} = -\frac{2KAdt}{5nRx} \Rightarrow \ln(T_S - T_0)_{T_0}^{T} = -\frac{2KAdt}{5nRx}$$

$$\Rightarrow \ln\frac{T_S - T}{T_S - T_0} = -\frac{2KAdt}{5nRx} \Rightarrow T_S - T = (T_S - T_0)e^{-\frac{2KAt}{5nRx}}$$

$$\Rightarrow T = T_S - (T_S - T_0)e^{-\frac{2KAt}{5nRx}} = T_S + (T_S + T_0)e^{+\frac{2KAt}{5nRx}}$$

$$\Rightarrow \Delta T = T - T_0 = (T_S - T_0) + (T_S - T_0)e^{+\frac{2KAt}{5nRx}} = (T_S - T_0) + \left(1 + e^{+\frac{2KAt}{5nRx}}\right)$$

$$\Rightarrow \frac{P_aAL}{nR} = (T_S - T_0) + \left(1 + e^{+\frac{2KAt}{5nRx}}\right) \qquad [p_a dv = nRdt \quad P_aAl = nRdt \quad dT = \frac{P_aAL}{nR}]$$

$$\Rightarrow L = \frac{nR}{P_aA}(T_S - T_0) + \left(1 - e^{-\frac{2KAt}{5nRx}}\right)$$

39. $A = 1.6\ m^2$, $T = 37°C = 310\ K$, $\sigma = 6.0 \times 10^{-8}\ w/m^2\text{-}K^4$

Energy radiated per second

$= A\sigma T^4 = 1.6 \times 6 \times 10^{-8} \times (310)^4 = 8865801 \times 10^{-4} = 886.58 \approx 887\ J$

40. $A = 12\ cm^2 = 12 \times 10^{-4}\ m^2$ $T = 20°C = 293\ K$

$e = 0.8$ $\sigma = 6 \times 10^{-8}\ w/m^2\text{-}k^4$

$$\frac{Q}{t} = Ae\,\sigma T^4 = 12 \times 10^{-4}\ 0.8 \times 6 \times 10^{-8}\ (293)^4 = 4.245 \times 10^{12} \times 10^{-13} = 0.4245 \approx 0.42$$

41. $E \rightarrow$ Energy radiated per unit area per unit time

Rate of heat flow $\rightarrow$ Energy radiated

(a) Per time = E × A

$$\text{So, } E_{Al} = \frac{e\sigma T^4 \times A}{e\sigma T^4 \times A} = \frac{4\pi r^2}{4\pi(2r)^2} = \frac{1}{4} \qquad \therefore 1:4$$

(b) Emissivity of both are same

$$= \frac{m_1S_1dT_1}{m_2S_2dT_2} = 1$$

$$\Rightarrow \frac{dT_1}{dT_2} = \frac{m_2S_2}{m_1S_1} = \frac{s_1 4\pi r_1^3 \times S_2}{s_2 4\pi r_2^3 \times S_1} = \frac{1 \times \pi \times 900}{3.4 \times 8\pi \times 390} = 1:2:9$$

42. $\frac{Q}{t} = Ae\,\sigma T^4$

$$\Rightarrow T^4 = \frac{\theta}{teA\sigma} \Rightarrow T^4 = \frac{100}{0.8 \times 2 \times 3.14 \times 4 \times 10^{-5} \times 1 \times 6 \times 10^{-8}}$$

$\Rightarrow T = 1697.0 \approx 1700\ K$

43. (a) $A = 20\ cm^2 = 20 \times 10^{-4}\ m^2$, $T = 57°C = 330\ K$

$E = A\,\sigma T^4 = 20 \times 10^{-4} \times 6 \times 10^{-8} \times (330)^4 \times 10^4 = 1.42\ J$

(b) $\frac{E}{t} = A\sigma e(T_1^4 - T_2^4)$, $A = 20\ cm^2 = 20 \times 10^{-4}\ m^2$

$\sigma = 6 \times 10^{-8}$ $T_1 = 473\ K$, $T_2 = 330\ K$

$= 20 \times 10^{-4} \times 6 \times 10^{-8} \times 1[(473)^4 - (330)^4]$

$= 20 \times 6 \times [5.005 \times 10^{10} - 1.185 \times 10^{10}]$

$= 20 \times 6 \times 3.82 \times 10^{-2} = 4.58\ w$ from the ball.

44. $r = 1\ cm = 1 \times 10^{-3}\ m$

$A = 4\pi(10^{-2})^2 = 4\pi \times 10^{-4}\ m^2$

$E = 0.3, \qquad \sigma = 6 \times 10^{-8}$

$\frac{E}{t} = A\sigma e(T_1^4 - T_2^4)$

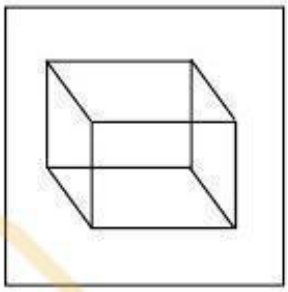
$= 0.3 \times 6 \times 10^{-8} \times 4\pi \times 10^{-4} \times [(100)^4 - (300)^4]$

$= 0.3 \times 6 \times 4\pi \times 10^{-12} \times [1 - 0.0081] \times 10^{12}$

$= 0.3 \times 6 \times 4 \times 3.14 \times 9919 \times 10^{-4}$

$= 4 \times 18 \times 3.14 \times 9919 \times 10^{-5} = 22.4 \approx 22\ W$

45. Since the Cube can be assumed as black body



$e = \ell$

$\sigma = 6 \times 10^{-8}\ w/m^2\text{-}k^4$

$A = 6 \times 25 \times 10^{-4}\ m^2$

$m = 1\ kg$

$s = 400\ J/kg\text{-}°K$

$T_1 = 227°C = 500\ K$

$T_2 = 27°C = 300\ K$

$\Rightarrow ms\frac{d\theta}{dt} = e\sigma A(T_1^4 - T_2^4)$

$\Rightarrow \frac{d\theta}{dt} = \frac{e\sigma A(T_1^4 - T_2^4)}{ms}$

$= \frac{1 \times 6 \times 10^{-8} \times 6 \times 25 \times 10^{-4} \times [(500)^4 - (300)^4]}{1 \times 400}$

$= \frac{36 \times 25 \times 544}{400} \times 10^{-4} = 1224 \times 10^{-4} = 0.1224°C/s \approx 0.12°C/s.$

46. $Q = e\sigma A(T_2^4 - T_1^4)$

For any body, $210 = eA\sigma[(500)^4 - (300)^4]$

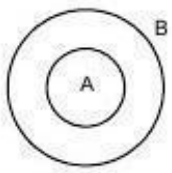
For black body, $700 = 1 \times A\sigma[(500)^4 - (300)^4]$

Dividing $\frac{210}{700} = \frac{e}{1} \Rightarrow e = 0.3$

47. $A_A = 20\ cm^2, \qquad A_B = 80\ cm^2$

$(mS)_A = 42\ J/°C, \qquad (mS)_B = 82\ J/°C,$

$T_A = 100°C, \qquad T_B = 20°C$



K_B is low thus it is a poor conducter and K_A is high.

Thus A will absorb no heat and conduct all

$\left(\frac{E}{t}\right)_A = \sigma A_A[(373)^4 - (293)^4] \qquad \Rightarrow (mS)_A\left(\frac{d\theta}{dt}\right)_A = \sigma A_A[(373)^4 - (293)^4]$

$\Rightarrow \left(\frac{d\theta}{dt}\right)_A = \frac{\sigma A_a[(373)^4 - (293)^4]}{(mS)_A} = \frac{6 \times 10^{-8}[(373)^4 - (293)^4]}{42} = 0.03\ °C/S$

Similarly $\left(\frac{d\theta}{dt}\right)_B = 0.043\ °C/S$

48. $\frac{Q}{t} = eAe(T_2^4 - T_1^4)$

$\Rightarrow \frac{Q}{At} = 1 \times 6 \times 10^{-8}[(300)^4 - (290)^4] \qquad = 6 \times 10^{-8}(81 \times 10^8 - 70.7 \times 10^8) = 6 \times 10.3$

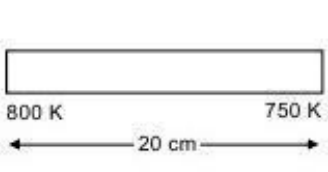
$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l}$

$\Rightarrow \frac{Q}{tA} = \frac{K(\theta_1 - \theta_2)}{l} = \frac{K \times 17}{0.5} = 6 \times 10.3 = \frac{K \times 17}{0.5} \Rightarrow K = \frac{6 \times 10.3 \times 0.5}{17} = 1.8$

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49. $\sigma = 6 \times 10^{-8}\ w/m^2\text{-}k^4$

$L = 20\ cm = 0.2\ m, \qquad K = ?$



$\Rightarrow E = \frac{KA(\theta_1 - \theta_2)}{d} = A\sigma(T_1^4 - T_2^4)$

$\Rightarrow K = \frac{s(T_1 - T_2) \times d}{\theta_1 - \theta_2} = \frac{6 \times 10^{-8} \times (750^4 - 300^4) \times 2 \times 10^{-1}}{50}$

$\Rightarrow K = 73.993 \approx 74.$

50. $v = 100$ cc

$\Delta\theta = 5°C$

$t = 5$ min

For water

$$\frac{mS\Delta\theta}{dt} = \frac{KA}{l}\Delta\theta$$

$$\Rightarrow \frac{100\times10^{-3}\times1000\times4200}{5} = \frac{KA}{l}$$

For Kerosene

$$\frac{ms}{at} = \frac{KA}{l}$$

$$\Rightarrow \frac{100\times10^{-3}\times800\times2100}{t} = \frac{KA}{l}$$

$$\Rightarrow \frac{100\times10^{-3}\times800\times2100}{t} = \frac{100\times10^{-3}\times1000\times4200}{5}$$

$$\Rightarrow T = \frac{5\times800\times2100}{1000\times4200} = 2 \text{ min}$$

51. 50°C 45°C 40°C

Let the surrounding temperature be 'T'°C

$$\text{Avg. } t = \frac{50+45}{2} = 47.5$$

Avg. temp. diff. from surrounding

$T = 47.5 - T$

$$\text{Rate of fall of temp} = \frac{50-45}{5} = 1\ °C/mm$$

From Newton's Law

$1°C/mm = bA \times t$

$$\Rightarrow bA = \frac{1}{t} = \frac{1}{47.5-T} \quad ...(1)$$

In second case,

$$\text{Avg, temp} = \frac{40+45}{2} = 42.5$$

Avg. temp. diff. from surrounding

$t' = 42.5 - t$

$$\text{Rate of fall of temp} = \frac{45-40}{8} = \frac{5}{8}\ °C/mm$$

From Newton's Law

$$\frac{5}{B} = bAt'$$

$$\Rightarrow \frac{5}{8} = \frac{1}{(47.5-T)}\times(42.5-T)$$

By C & D [Componendo & Dividendo method]

We find, $T = 34.1°C$

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52. Let the water eq. of calorimeter = m

$$\frac{(m+50\times10^{-3})\times4200\times5}{10} = \text{Rate of heat flow}$$

$$\frac{(m+100\times10^{-3})\times4200\times5}{18} = \text{Rate of flow}$$

$$\Rightarrow \frac{(m+50\times10^{-3})\times4200\times5}{10} = \frac{(m+100\times10^{-3})\times4200\times5}{18}$$

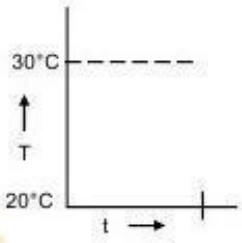
$\Rightarrow (m + 50 \times 10^{-3})18 = 10m + 1000 \times 10^{-3}$

$\Rightarrow 18m + 18 \times 50 \times 10^{-3} = 10m + 1000 \times 10^{-3}$

$\Rightarrow 8m = 100 \times 10^{-3}$ kg

$\Rightarrow m = 12.5 \times 10^{-3}$ kg $= 12.5$ g

53. In steady state condition as no heat is absorbed, the rate of loss of heat by conduction is equal to that of the supplied.



i.e. H = P

m = 1Kg, Power of Heater = 20 W, Room Temp. = 20°C

(a) $H = \frac{d\theta}{dt} = P = 20$ watt

(b) by Newton's law of cooling

$\frac{-d\theta}{dt} = K(\theta - \theta_0)$

$-20 = K(50 - 20) \Rightarrow K = 2/3$

Again, $\frac{-d\theta}{dt} = K(\theta - \theta_0) = \frac{2}{3} \times (30 - 20) = \frac{20}{3}$ w

(c) $\left(\frac{dQ}{dt}\right)_{20} = 0, \quad \left(\frac{dQ}{dt}\right)_{30} = \frac{20}{3} \quad \left(\frac{dQ}{dt}\right)_{avg} = \frac{10}{3}$

T = 5 min = 300 ′

Heat liberated = $\frac{10}{3} \times 300 = 1000$ J

Net Heat absorbed = Heat supplied – Heat Radiated = 6000 – 1000 = 5000 J

Now, $m\Delta\theta' = 5000$

$\Rightarrow S = \frac{5000}{m\Delta\theta} = \frac{5000}{1 \times 10} = 500$ J $Kg^{-1}°C^{-1}$

54. Given:

Heat capacity = m × s = 80 J/°C

$\left(\frac{d\theta}{dt}\right)_{increase} = 2$ °C/s

$\left(\frac{d\theta}{dt}\right)_{decrease} = 0.2$ °C/s

(a) Power of heater = $mS\left(\frac{d\theta}{dt}\right)_{increasing} = 80 \times 2 = 160$ W

(b) Power radiated = $mS\left(\frac{d\theta}{dt}\right)_{decreasing} = 80 \times 0.2 = 16$ W

(c) Now $mS\left(\frac{d\theta}{dt}\right)_{decreasing} = K(T - T_0)$

$\Rightarrow 16 = K(30 - 20) \qquad \Rightarrow K = \frac{16}{10} = 1.6$

Now, $\frac{d\theta}{dt} = K(T - T_0) = 1.6 \times (30 - 25) = 1.6 \times 5 = 8$ W

(d) $P.t = H \Rightarrow 8 \times t$

55. $\frac{d\theta}{dt} = -K(T - T_0)$

Temp. at t = 0 is θ_1

(a) Max. Heat that the body can loose = $\Delta Q_m = ms(\theta_1 - \theta_0)$

($\therefore$ as, $\Delta t = \theta_1 - \theta_0$)

(b) if the body loses 90% of the max heat the decrease in its temp. will be

$\frac{\Delta Q_m \times 9}{10ms} = \frac{(\theta_1 - \theta_0) \times 9}{10}$

If it takes time t_1, for this process, the temp. at t_1

$= \theta_1 - (\theta_1 - \theta_0)\frac{9}{10} = \frac{10\theta_1 - 9\theta_1 - 9\theta_0}{10} = \frac{\theta_1 - 9\theta_0}{10} \times 1$

Now, $\frac{d\theta}{dt} = -K(\theta - \theta_1)$

Let $\theta = \theta_1$ at t = 0; & θ be temp. at time t

$\int_{\theta}^{\theta} \frac{d\theta}{\theta - \theta_o} = -K\int_0^t dt$

or, $\ln \frac{\theta - \theta_0}{\theta_1 - \theta_0} = -Kt$

or, $\theta - \theta_0 = (\theta_1 - \theta_0)\, e^{-kt}$ ...(2)

Putting value in the Eq (1) and Eq (2)

$$\frac{\theta_1 - 9\theta_0}{10} - \theta_0 = (\theta_1 - \theta_0)\, e^{-kt}$$

$$\Rightarrow t_1 = \frac{\ln 10}{k}$$