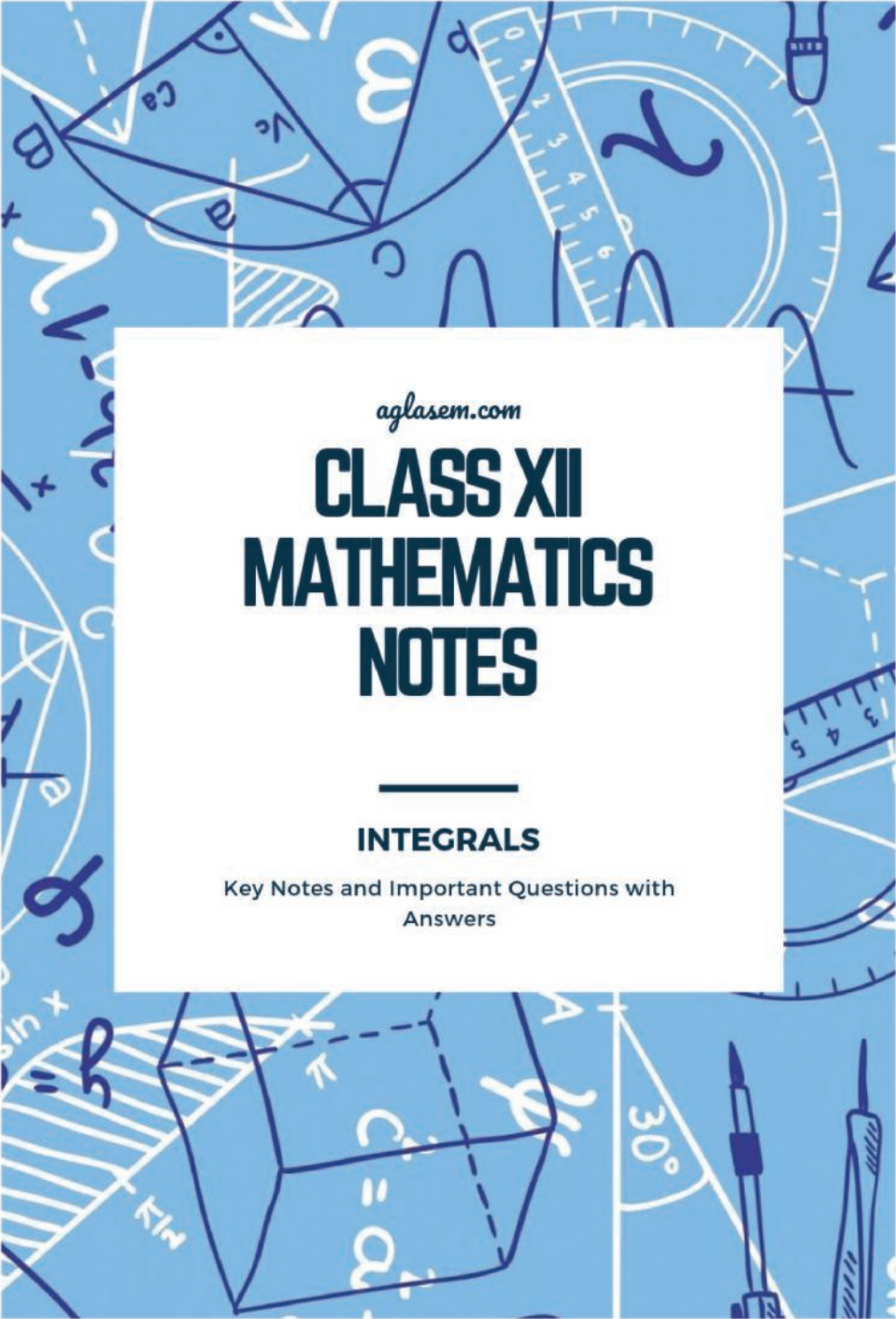


The background is a light blue field filled with various white mathematical sketches. These include geometric shapes like triangles with labeled sides (a, b, c), circles with radii, a 3D cube with dashed lines, a sector of a circle with a 30-degree angle, and various mathematical symbols such as π , ∞ , λ , and μ . There are also drawings of a ruler and a compass.

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CLASS XII MATHEMATICS NOTES

INTEGRALS

Key Notes and Important Questions with
Answers

CHAPTER 7

INTEGRALS

POINTS TO REMEMBER

- Integration or anti derivative is the reverse process of Differentiation.
- Let $\frac{d}{dx} F(x) = f(x)$ then we write $\int f(x) dx = F(x) + c$.
- These integrals are called indefinite integrals and c is called constant of integration.
- From geometrical point of view, an indefinite integral is the collection of family of curves each of which is obtained by translating one of the curves parallel to itself upwards or downwards along y -axis.

STANDARD FORMULAE

1. $\int x^n dx = \begin{cases} \frac{x^{n+1}}{n+1} + c & n \neq -1 \\ \log_e|x| + c & n = -1 \end{cases}$
2. $\int (ax + b)^n dx = \begin{cases} \frac{(ax+b)^{n+1}}{(n+1)a} + c & n \neq -1 \\ \frac{1}{a} \log|ax + b| + c & n = -1 \end{cases}$
3. $\int \sin x dx = -\cos x + c$.
4. $\int \cos x dx = \sin x + c$
5. $\int \tan x. dx = -\log|\cos x| + c = \log|\sec x| + c$.
6. $\int \cot x dx = \log|\sin x| + c$.
7. $\int \sec^2 x dx = \tan x + c$

$$8. \quad \int \operatorname{cosec}^2 x \, dx = -\cot x + c$$

$$9. \quad \int \sec x \tan x \, dx = \sec x + c$$

$$10. \quad \int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + c$$

$$11. \quad \int \sec x \, dx = \log|\sec x + \tan x| + c \\ = \log\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| + c$$

$$12. \quad \int \operatorname{cosec} x \, dx = \log|\operatorname{cosec} x - \cot x| + c \\ = \log\left|\tan\frac{x}{2}\right| + c$$

$$13. \quad \int e^x \, dx = e^x + c$$

$$14. \quad \int a^x \, dx = \frac{a^x}{\log a} + c$$

$$15. \quad \int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + c, |x| < 1 \\ = -\cos^{-1} x + c$$

$$16. \quad \int \frac{1}{1+x^2} \, dx = \tan^{-1} x + c \\ = -\cot^{-1} x + c$$

$$17. \quad \int \frac{1}{|x|\sqrt{x^2-1}} \, dx = \sec^{-1} x + c, |x| > 1 \\ = -\operatorname{cosec}^{-1} x + c$$

$$18. \quad \int \frac{1}{a^2-x^2} \, dx = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + c$$

19. $\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c$
20. $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$
21. $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} + c = -\cos^{-1} \frac{x}{a} + c$
22. $\int \frac{1}{\sqrt{a^2 + x^2}} dx = \log |x + \sqrt{a^2 + x^2}| + c$
23. $\int \frac{1}{\sqrt{x^2 - a^2}} dx = \log |x + \sqrt{x^2 - a^2}| + c$
24. $\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + c$
25. $\int \sqrt{a^2 + x^2} dx = \frac{x}{2} \sqrt{a^2 + x^2} + \frac{a^2}{2} \log |x + \sqrt{a^2 + x^2}| + c$
26. $\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log |x + \sqrt{x^2 - a^2}| + c$

RULES OF INTEGRATION

1. $\int [(f_1(x) \pm f_2(x) \pm \dots \pm f_n(x))] dx = \int f_1(x) dx \pm \int f_2(x) dx \pm \dots \pm \int f_n(x) dx$
2. $\int k \cdot f(x) dx = k \int f(x) dx.$
3. $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + c$

INTEGRATION BY SUBSTITUTION

1. $\int \frac{f'(x)}{f(x)} dx = \log |f(x)| + c$

$$2. \int [f(x)]^n f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + c$$

$$3. \int \frac{f'(x)}{[f(x)]^n} dx = \frac{(f(x))^{-n+1}}{-n+1} + c$$

INTEGRATION BY PARTS

$$\int f(x) g(x) dx = f(x) \int g(x) dx - \int [f'(x) \int g(x) dx]$$

DEFINITE INTEGRALS

$$\int_a^b f(x) dx = F(b) - F(a), \text{ where } F(x) = \int f(x) dx$$

DEFINITE INTEGRAL AS A LIMIT OF SUMS.

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+n-1h)]$$

$$\text{Where } h = \frac{b-a}{n} \text{ or } \int_a^b f(x) dx = \lim_{h \rightarrow 0} [h \sum_{r=1}^n f(a+rh)]$$

PROPERTIES OF DEFINITE INTEGRAL

$$1. \int_a^b f(x) dx = - \int_b^a f(x) dx$$

$$2. \int_a^b f(x) dx = \int_a^b f(t) dt.$$

$$3. \int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx.$$

$$4. (i) \int_a^b f(x) dx = \int_a^b f(a+b-x) dx.$$

$$(ii) \int_0^a f(x) dx = \int_0^a f(a-x) dx$$

$$5. \int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx, \quad \text{if } f(x) \text{ is even function}$$

$$6. \int_{-a}^a f(x) dx = 0 \text{ if } f(x) \text{ is an odd function}$$

$$7. \int_0^{2a} f(x) dx = \begin{cases} 2 \int_0^a f(x) dx, & \text{if } f(2a-x) = f(x) \\ 0 & \text{if } f(2a-x) = -f(x) \end{cases}$$

Very Short Answer Type Questions (1 Mark)

Evaluate the following integrals:

$$1. \int (\sin^{-1} \sqrt{x} + \cos^{-1} \sqrt{x}) dx$$

$$2. \int_{-1}^1 e^{|x|} dx$$

$$3. \int \frac{dx}{1-\sin^2 x}$$

$$4. \int_{-1}^1 x^{99} \cos^4 x dx$$

$$5. \int \frac{1}{x \log x \log(\log x)} dx$$

$$6. \int_{\frac{1}{2}}^1 \cos x \log \left(\frac{1+x}{1-x} \right) dx$$

$$7. \int (e^{a \log x} + e^{x \log a}) dx$$

$$8. \int \frac{\cos 2x + 2 \sin^2 x}{\cos^2 x} dx$$

9. $\int_{-\pi/2}^{\pi/2} \sin^7 x \, dx$
10. $\int \sqrt{10 - 4x + x^2} \, dx$
11. $\int_{-1}^1 x^3 |x| \, dx$
12. $\int \frac{1}{\sin^2 x \cos^2 x} \, dx$
13. $\int_{-2}^2 \frac{dx}{1+|x-1|}$
14. $\int e^{-\log x} \, dx$
15. $\int \frac{e^x}{a^x} \, dx$
16. $\int \frac{x}{\sqrt{x+1}} \, dx$
17. $\int \frac{x}{(x+1)^2} \, dx$
18. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx$
19. $\int \cos^2 \alpha \, dx$
20. $\int \frac{1}{x \cos \alpha + 1} \, dx$
21. $\int \sec x \log(\sec x + \tan x) \, dx$
22. $\int \frac{1}{\cos \alpha + x \sin \alpha} \, dx$
23. $\int \frac{\sec^2(\log x)}{x} \, dx$
24. $\int \frac{e^x}{\sqrt{4+e^{2x}}} \, dx$
25. $\int \frac{1}{x(2+3 \log x)} \, dx$
26. $\int \frac{1-\sin x}{x+\cos x} \, dx$

27. $\int \frac{1-\cos x}{\cos x} dx$
28. $\int \frac{x^{e-1}+e^{x-1}}{x^e+e^x} dx$
29. $\int \frac{(x+1)}{x} (x + \log x) dx$
30. $\int_0^\pi |\cos x| dx$
31. $\int_0^2 [x] dx$ where $[x]$ is greatest integers function.
32. $\int \frac{1}{\sqrt{9-4x^2}} dx$
33. $\int_a^b \frac{f(x)}{f(x)+f(a+b-x)} dx$
34. $\int_{-2}^1 \frac{|x|}{x} dx$
35. $\int_{-1}^1 x |x| dx$
36. $\int x \sqrt{x+2} dx$
37. $\int_a^b f(x) dx + \int_b^a f(x) dx$
38. $\int \frac{\sin x}{\sin 2x} dx$
39. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} |\sin x| dx$
40. $\int \frac{1}{\sec x + \tan x} dx$
41. $\int \frac{\sin^2 x}{1+\cos x} dx$
42. $\int \frac{1-\tan x}{1+\tan x} dx$

VERY SHORT ANSWER TYPE QUESTIONS [2 MARKS]

Evaluate :

43. $\int_e [\log(x+1) - \log x] dx$

44. $\int \frac{1}{\sqrt{x+1} + \sqrt{x+2}} dx$

45. $\int \sin x \sin 2x dx$

46. $\int \left[\frac{x}{a} + \frac{a}{x} + x^a + a^x \right] dx$

47. $\int_0^{\pi/2} \left[\frac{5+3\cos x}{5+3\sin x} \right] dx$

48. $\int \frac{a^x + b^x}{c^x} dx$

49. $\int \left(\sqrt{ax} - \frac{1}{\sqrt{ax}} \right)^2 dx$

50. $\int e^x 2^x dx$

Short Answer Type Questions (4 Marks)

Evaluate :

51. (I) $\int \frac{x \operatorname{cosec}(\tan^{-1} x^2)}{1+x^4} dx$

(II) $\int \frac{\sqrt{x+1} - \sqrt{x-1}}{\sqrt{x+1} + \sqrt{x-1}} dx$

(III) $\int \frac{1}{\sin(x-a) \sin(x-b)} dx$

$$(IV) \int \frac{\cos(x+a)}{\cos(x-a)} dx$$

$$(V) \int \cos 2x \cos 4x \cos 6x dx$$

$$(VI) \int \tan 2x \tan 3x \tan 5x dx$$

$$(VII) \int \sin^2 x \cos^4 x dx$$

$$(VIII) \int \cot^3 x \operatorname{cosec}^4 x dx$$

$$(IX) \int \frac{\sin x \cos x}{\sqrt{a^2 \sin^2 x + b^2 \cos^2 x}} dx \quad [\text{Hint: Put } a^2 \sin^2 x + b^2 \cos^2 x = t \text{ or } t^2]$$

$$(X) \int \frac{1}{\sqrt{\cos^3 x \cos(x+a)}} dx$$

$$(XI) \int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x} dx$$

$$(XII) \int \frac{\sin x + \cos x}{\sqrt{\sin 2}} dx$$

Evaluate :

$$52. (i) \int \frac{x}{x^4 + x^2 + 1} dx$$

$$(ii) \int \frac{1}{x[6(\log x)^2 + 7 \log x + 2]} dx$$

$$(iii) \int \frac{1}{\sqrt{\sin^3 x \cos^5 x}} dx$$

$$(iv) \int \frac{x^2 + 1}{x^4 + 1} dx$$

$$(v) \int \frac{1}{\sqrt{(x-a)(x-b)}} dx$$

$$(vi) \int \frac{5x-2}{3x^2+2x+1} dx$$

$$(vii) \int \frac{x^2}{x^2+6x+1} dx$$

$$(viii) \int \frac{x+2}{\sqrt{4x-x^2}} dx$$

$$(ix) \int x \sqrt{1+x-x^2} dx$$

$$(x) \int \frac{\sin^4 x}{\cos^8 x} dx$$

$$(xi) \int \sqrt{\sec x - 1} dx \text{ [Hint: Multiply and divided by } \sqrt{\sec x + 1}]$$

Evaluate :

$$53. (I) \int \frac{dx}{x(x^7+1)}$$

$$(II) \int \frac{3x+5}{x^3-x^2-x+1} dx$$

$$(III) \int \frac{\sin \theta \cos \theta}{\cos^2 \theta - \cos \theta - 2} d\theta$$

$$(iv) \int \frac{dx}{(2-x)(x^2+3)}$$

$$(v) \int \frac{x^2+x+2}{(x-2)(x-1)} dx$$

$$(vi) \int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} dx$$

$$(vii) \int \frac{dx}{(2x+1)(x^2+4)}$$

$$(viii) \int \frac{x^2-1}{x^4+x^2+1} dx$$

$$(ix) \int \sqrt{\tan x} dx$$

$$(x) \int \frac{dx}{\sin x - \sin 2x}$$

54. Evaluate:

$$(I) \int x^5 \sin x^3 dx$$

$$(II) \int \sec^3 x dx$$

$$(III) \int e^{ax} \cos(bx + c) dx$$

$$(IV) \int \sin^{-1} \left(\frac{6x}{1+9x^2} \right) dx \quad [\text{Hint: Put } 3x = \tan \theta]$$

$$(V) \int \cos \sqrt{x} dx$$

$$(VI) \int x^3 \tan^{-1} x dx$$

$$(VII) \int e^{2x} \left(\frac{1+\sin 2x}{1+\cos 2x} \right) dx$$

$$(VIII) \int \left[\frac{1}{\log x} - \frac{1}{(\log x)^2} \right] dx$$

$$(IX) \int \sqrt{2ax - x^2} dx$$

$$(X) \int e^x \frac{(x^2+1)}{(x+1)^2} dx$$

$$(XI) \int x^3 \sin^{-1} \left(\frac{1}{x} \right) dx$$

$$(XII) \int \left\{ \log(\log x) + \frac{1}{(\log x)^2} \right\} dx \quad [\text{Hint: Put } \log x = t \\ x = e^t]$$

$$(XIII) \int (6x + 5) \sqrt{6 + x - x^2} dx$$

$$(XIV) \int \frac{1}{x^3 + 1} dx$$

$$(XV) \int \tan^{-1} \left(\frac{x-5}{1+5x} \right) dx$$

$$(XVI) \int \frac{dx}{5+4 \cos x}$$

55. Evaluate the following definite integrals:

$$(i) \int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$$

$$(ii) \int_0^{\pi/2} \cos 2x \log \sin x dx$$

$$(iii) \int_0^1 x \sqrt{\frac{1-x^2}{1+x^2}} dx$$

$$(iv) \int_0^{1/\sqrt{2}} \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx$$

$$(v) \int_0^{\pi/2} \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$$

$$(vi) \int_0^1 \sin \left(2 \tan^{-1} \sqrt{\frac{1+x}{1-x}} \right) dx$$

$$(vii) \int_0^{\pi/2} \frac{x + \sin x}{1 + \cos x} dx$$

$$(viii) \int_0^1 x \log \left(1 + \frac{x}{2} \right) dx$$

$$(ix) \int_{-1}^{1/2} |x \cos \pi x| dx$$

$$(x) \int_{-\pi}^{\pi} (\cos a x - \sin b x)^2 dx$$

56. Evaluate:

$$(i) \int_2^5 [|x - 2| + |x - 3| + |x - 4|] dx$$

$$(ii) \int_0^{\pi} \frac{x}{1 + \sin x} dx$$

$$(iii) \int_{-1}^1 e^{\tan^{-1} x} \left[\frac{1+x+x^2}{1+x^2} \right] dx$$

$$(iv) \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$$

$$(v) \int_0^2 [x^2] dx$$

$$(vi) \int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$$

$$(vii) \int_0^{\pi} \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x} dx \text{ [Hint: use } \int_0^a f(x) dx = \int_0^a f(a-x) dx]$$

57. Evaluate the following integrals:

$$(i) \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$$

$$(ii) \int_{-\pi/2}^{\pi/2} (\sin |x| + \cos |x|) dx$$

$$(iii) \int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx$$

$$(iv) \int_0^{\pi} \frac{x \tan x}{\sec x + \operatorname{cosec} x} dx$$

$$(v) \int_{-a}^a \sqrt{\frac{a-x}{a+x}} dx$$

58. Evaluate

$$(i) \int \frac{\sin^{-1} \sqrt{x} - \cos^{-1} \sqrt{x}}{\sin^{-1} \sqrt{x} + \cos^{-1} \sqrt{x}} dx \quad x \in [0, 1]$$

$$(ii) \int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx$$

$$(iii) \int \frac{x^2 e^x}{(x+z)^2} dx$$

$$(iv) \int \frac{x^2}{(x \sin x + \cos x)^2} dx$$

$$(v) \int \sin^{-1} \sqrt{\frac{x}{a+x}} dx$$

$$(vi) \int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$$

$$(vii) \int \frac{\sin x}{\sin 4x} dx$$

$$(viii) \int_{-1}^{3/2} |x \sin \pi x| \, dx$$

$$(ix) \int \frac{\sin(x-a)}{\sin(x+a)} \, dx$$

$$(x) \int \frac{x^2}{(x^2+4)(x^2+9)} \, dx$$

$$(xi) \int \frac{\cos 5x + \cos 4x}{1 - 2 \cos 3x} \, dx$$

Long Answer Type Questions (6 Marks)

59. Evaluate the following integrals:

$$(i) \int \frac{x^5 + 4}{x^5 - x} \, dx$$

$$(ii) \int \frac{2e^t}{e^{3t} - 6e^{2t} + 11e^t - 6} \, dt$$

$$(iii) \int \frac{2x^3}{(x+1)(x-3)^2} \, dx$$

$$(iv) \int \frac{1 + \sin x}{\sin x (1 + \cos x)} \, dx$$

$$(v) \int_0^{\pi/2} (\sqrt{\tan x} + \sqrt{\cot x}) \, dx$$

$$(vi) \int_0^1 x \sqrt{\frac{1-x^2}{1+x^2}} \, dx$$

$$(vii) \int_0^{\pi/2} \frac{\cos x}{1 + \cos x + \sin x} dx$$

60. Evaluate the following integrals as limit of sums:

$$(i) \int_2^4 (2x + 1) dx$$

$$(ii) \int_0^2 (x^2 + 3) dx$$

$$(iii) \int_1^3 (3x^2 - 2x + 4) dx$$

$$(iv) \int_0^4 (3x^2 + e^{2x}) dx$$

$$(v) \int_0^1 e^{2-3x} dx$$

$$(vi) \int_0^1 (3x^2 + 2x + 1) dx$$

61. Evaluate:

$$(i) \int \frac{dx}{(\sin x - 2 \cos x)(2 \sin x + \cos x)}$$

$$(ii) \int_0^1 \frac{\log(1+x)}{1+x^2} dx$$

$$(iii) \int_0^{\pi/2} (2 \log \sin x - \log \sin 2x) dx$$

$$62. \int_0^1 x(\tan^{-1} x)^2 dx$$

$$63. \int_0^{\pi/2} \log \sin x dx$$

$$64. \text{ Prove that } \int_0^1 \tan^{-1} \left(\frac{1}{1-x+x^2} \right) dx = 2 \int_0^1 \tan^{-1} x dx$$

Hence or otherwise evaluate the integral $\int \tan^{-1}(1-x+x^2) dx$.

$$65. \text{ Evaluate } \int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx.$$

Answers

$$1. \quad \frac{\pi}{2}x + c$$

$$2. \quad 2e - 2$$

$$3. \quad \tan x + c$$

$$4. \quad 0$$

$$5. \quad \log|\log|\log x|| + c$$

$$6. \quad 0$$

$$7. \quad \frac{x^{a+1}}{a+1} + \frac{a^x}{\log a} + c$$

$$8. \quad \tan x + c$$

$$9. \quad 0$$

$$10. \quad \frac{(x-2)\sqrt{x^2-4x+10}}{2} + 3 \log|(x-2) + \sqrt{x^2-4x+10}| + c$$

$$11. \quad 0$$

$$12. \quad \tan x - \cot x + c$$

$$13. \quad 3 \log_e 2$$

$$14. \quad \log|x| + c$$

- | | |
|---|---|
| 15. $\frac{\left(\frac{e}{a}\right)^x}{\log\left(\frac{e}{a}\right)} + c$ | 29. $\frac{(x+\log)^2}{2} + c$ |
| 16. $\frac{2}{3}(x+1)^{3/2} - 2(x+1)^{1/2} + c$ | 30. 2 |
| 17. $\log x+1 + \frac{1}{x+1} + c$ | 31. 1 |
| 18. $2e^{\sqrt{x}} + c$ | 32. $\frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right) + c$ |
| 19. $x\cos^2\alpha + c$ | 33. $\frac{b-a}{2}$ |
| 20. $\frac{\log x\cos\alpha+1 }{\cos\alpha} + c$ | 34. -1 |
| 21. $\frac{(\log \sec x + \tan x)^2}{2} + c$ | 35. 0 |
| 22. $\frac{\log \cos\alpha + x\sin\alpha }{\sin\alpha} + c$ | 36. $\frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + c$ |
| 23. $\tan \log x + c$ | 37. 0 |
| 24. $\log e^x + \sqrt{4 + e^{2x}} + c$ | 38. $\frac{1}{2}\log \sec x + \tan x + c$ |
| 25. $\frac{1}{3}\log 2 + 3\log x + c$ | 39. $2-\sqrt{2}$ |
| 26. $\log x + \cos x + c$ | 40. $\log 1 + \sin x + c$ |
| 27. $2\log\left \sec\frac{x}{2}\right + c$ | 41. $x - \sin x + c$ |
| 28. $\frac{1}{e}\log x^e + e^x + c$ | 42. $\log \cos x + \sin x + c$ |
| | 43. $x + \log x + c$ |

$$44. \frac{2}{3} \left[(x+2)^{3/2} - (x+1)^{3/2} \right] + c$$

$$45. \frac{-1}{2} \left[\frac{\sin 3x}{3} - \sin x \right] + c$$

$$46. \frac{1}{a} \frac{x^2}{2} + a \log |x| + \frac{x^{a+1}}{a+1} + \frac{a^x}{\log a} + c$$

$$47. 0$$

$$48. \frac{\left(\frac{a}{c}\right)^x}{\log \left| \frac{a}{c} \right|} + \frac{\left(\frac{b}{c}\right)^x}{\log \left| \frac{b}{c} \right|} + c$$

$$49. \frac{ax^2}{2} + \frac{\log |x|}{a} - 2x + c$$

$$50. \frac{2^x e^x}{\log(2e)} + c$$

$$51.$$

$$(I) \quad \frac{1}{2} \log \left[\operatorname{cosec}(\tan^{-1} x^2) - \frac{1}{x^2} \right] + c$$

$$(II) \quad \frac{1}{2} (x^2 - x\sqrt{x^2 - 1}) + \frac{1}{2} \log |x + \sqrt{x^2 - 1}| + c$$

$$(III) \quad \frac{1}{\sin(a-b)} \log \left| \frac{\sin(x-a)}{\sin(x-b)} \right| + c$$

$$(IV) \quad x \cos 2a - \sin 2a \log |\sec(x-a)| + c$$

$$(V) \quad \frac{3}{8}x - \frac{1}{4} \sin 2x + \frac{1}{32} \sin 4x + c$$

$$(VI) \quad \frac{1}{5} \log |\sec 5x| - \frac{1}{2} \log |\sec 2x| - \frac{1}{3} \log |\sec 3x| + c$$

$$(VII) \quad \frac{1}{32} \left[2x + \frac{1}{2} \sin 2x - \frac{1}{2} \sin 4x - \frac{1}{6} \sin 6x \right] + c$$

$$(VIII) \quad - \left(\frac{\cot^6 x}{6} + \frac{\cot^4 x}{4} \right) + c$$

$$(IX) \quad \frac{1}{a^2 - b^2} \sqrt{a^2 \sin^2 x + b^2 \cos^2 x} + c$$

$$(X) \quad -2 \operatorname{cosec} a \sqrt{\cos a - \tan x \sin a} + c$$

$$(XI) \quad \tan x - \cot x - 3x + c$$

$$(XII) \quad \sin^{-1} [\sin x - \cos x] + c$$

$$52. \quad (I) \quad \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{2x^2 + 1}{\sqrt{3}} \right) + c$$

$$(II) \quad \log \left| \frac{2 \log x}{3 \log x} \right| + c$$

$$(III) \quad \frac{-2}{\sqrt{\tan x}} + \frac{2}{3} \tan^{3/2} x + c$$

$$(IV) \quad \frac{1}{\sqrt{2}} \tan^{-1} \left\{ \frac{1}{\sqrt{2}} \left(x - \frac{1}{x} \right) \right\} + c$$

$$(V) \quad 2 \log |\sqrt{x-a} + \sqrt{x-b}| + c$$

$$(VI) \quad \frac{5}{6} \log |3x^2 + 2x + 1| + \frac{-11}{3\sqrt{2}} \tan^{-1} \left(\frac{3x+1}{\sqrt{2}} \right) + c$$

$$(VII) \quad x - 3 \log |x^2 + 6x + 12| + 2\sqrt{3} \tan^{-1} \left(\frac{x+3}{\sqrt{3}} \right) + c$$

$$(VIII) \quad -\sqrt{4x-x^2} + 4 \sin^{-1} \left(\frac{x-2}{2} \right) + c$$

$$(IX) \quad -\frac{1}{3}(1+x-x^2)^{3/2} + \frac{1}{8}(2x-1)\sqrt{1+x-x^2} + \frac{5}{16} \sin^{-1} \left(\frac{2x-1}{\sqrt{5}} \right) + c$$

$$(X) \quad \frac{\tan^5 x}{5} + \frac{\tan^7 x}{7} + c$$

$$(XI) \quad -\log \left| \cos x + \frac{1}{2} + \sqrt{\cos^2 x + \cos x} \right| + c$$

$$53. \quad (I) \quad \frac{1}{7} \log \left| \frac{x^7}{x^7+1} \right| + c$$

$$(II) \quad \frac{1}{2} \log \left| \frac{x+1}{x-1} \right| - \frac{4}{x-1} + c$$

$$(III) \quad -\frac{2}{3} \log |\cos \theta - 2| - \frac{1}{3} \log |1 + \cos \theta| + c$$

$$(IV) \quad \frac{1}{14} \log \left| \frac{x^2+3}{(2-x)^2} \right| + \frac{2}{7\sqrt{3}} \tan^{-1} \left(\frac{x}{\sqrt{3}} \right) + c$$

$$(V) \quad x + 4 \log \left| \frac{(x-2)^2}{x-1} \right| + c$$

$$(VI) \quad x + \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{x}{\sqrt{3}} \right) - 3 \tan^{-1} \left(\frac{x}{2} \right) + c$$

$$(VII) \quad \frac{2}{17} \log |2x+1| - \frac{1}{17} \log |x^2+4| + \frac{1}{34} \tan^{-1} \frac{x}{2} + c$$

$$(VIII) \quad \frac{1}{2} \log \left| \frac{x^2-x+1}{x^2+x+1} \right| + c$$

$$(IX) \quad \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{\tan x - 1}{\sqrt{2} \tan x} \right) + \frac{1}{2\sqrt{2}} \log \left| \frac{\tan x - \sqrt{2} \tan x + 1}{\tan x + \sqrt{2} \tan x + 1} \right| + c$$

$$(X) -\frac{1}{2}\log|\cos x - 1| - \frac{1}{6}\log|\cos x + 1| + \frac{2}{3}\log|1 - 2\cos x| + c$$

$$54. (i) \quad \frac{1}{3}[-x^3 \cos x^3 + \sin x^3] + c$$

$$(ii) \quad \frac{1}{2}[\sec x \tan x + \log|\sec x + \tan x|] + c$$

$$(iii) \quad \frac{e^{ax}}{a^2+b^2}[a \cos(bx+c) + b \sin(bx+c)] + c$$

$$(iv) \quad 2x \tan^{-1} 3x - \frac{1}{3}\log|1+9x^2| + c$$

$$(v) \quad 2[\sqrt{x} \sin \sqrt{x} + \cos \sqrt{x}] + c$$

$$(vi) \quad \left(\frac{x^4-1}{4}\right) \tan^{-1} x - \frac{x^3}{12} + \frac{x}{4} + c$$

$$(vii) \quad \frac{1}{2}e^{2x} \tan x + c$$

$$(viii) \quad \frac{x}{\log x} + c$$

$$(ix) \quad \left(\frac{x-a}{2}\right) \sqrt{2ax-x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x-a}{a}\right) + c$$

$$(x) \quad e^x \left(\frac{x-1}{x+1}\right) + c$$

$$(xi) \quad \frac{x^4}{4} \sin^{-1} \left(\frac{1}{x}\right) + \frac{x^2+2}{12} \sqrt{x^2-1} + c$$

$$(xii) \quad x \log|\log x| - \frac{x}{\log x} + c$$

$$(xiii) \quad -2(6+x-x^2)^{\frac{3}{2}} + 8 \left[\frac{2x-1}{4} \sqrt{6+x-x^2} + \frac{25}{8} \sin^{-1} \left(\frac{2x-1}{5}\right) \right] + c$$

$$(xiv) \quad \frac{1}{3}\log|x+1| - \frac{1}{6}\log|x^2-x+1| + \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{2x-1}{\sqrt{3}}\right) + c$$

$$(xv) \quad x \tan^{-1} x - \frac{1}{2}\log|1+x^2| - x \tan^{-1} 5 + c$$

$$(xvi) \quad \frac{2}{3} \tan^{-1} \left(\frac{1}{3} \tan \frac{x}{2}\right) + c$$

55. (I) $\frac{1}{20} \log 3$
 (II) $-\pi/4$
 (III) $\frac{\pi}{4} - \frac{1}{2}$
 (IV) $\frac{\pi}{4} - \frac{1}{2} \log 2$
 (V) $\frac{\pi}{2}$
 (VI) $\pi/4$
 (VII) $\pi/2$
 (VIII) $\frac{3}{4} + \frac{3}{2} \log \frac{2}{3}$
 (IX) $\frac{3}{2\pi} - \frac{1}{\pi^2}$
 (X) $2\pi + \frac{1}{2a} \sin 2a\pi - \frac{1}{2b} \sin 2b\pi$
56. (I) $\frac{1}{2}$
 (II) π
 (III) $e^{\pi/4} + e^{-\pi/4}$
 (IV) $\frac{1}{4} \pi^2$
 (V) $5 - \sqrt{3} - \sqrt{2}$
 (VI) $\frac{\pi^2}{16}$
 (VII) $\frac{\pi^2}{2a}$

57. (I) $\frac{\pi}{12}$

(II) 2

(III) $\frac{\pi}{2}$

(IV) $\frac{\pi^2}{4}$

(V) $a\pi$

58. (I) $\frac{2(2x-1)}{\pi} \sin^{-1} \sqrt{x} + \frac{2\sqrt{x-x^2}}{\pi} - x + c$

(II) $-2\sqrt{1-x} + \cos^{-1} \sqrt{x} + \sqrt{x-x^2} + c$

(III) $\frac{x-2}{x+2} e^x + c$

(IV) $\frac{\sin x - x \cos x}{x \sin x + \cos x} + c$

(V) $(x+a) \tan^{-1} \sqrt{\frac{x}{a}} - \sqrt{ax} + c$

(VI) $2 \sin^{-1} \frac{\sqrt{3}-1}{2}$

(VII) $\frac{1}{8} \log \left| \frac{1-\sin x}{1+\sin x} \right| - \frac{1}{4\sqrt{2}} \log \left| \frac{1+\sqrt{2} \sin x}{1-\sqrt{2} \sin x} \right| + c$

(VIII) $\frac{3}{\pi} + \frac{1}{\pi^2}$

(IX) $(\cos 2a)(x+a) - (\sin 2a) \log |\sin(x+a)| + c$

(X) $-\frac{4}{5} \log |x^2+4| + \frac{9}{5} \log |x^2+9| + c$

(XI) $-\left(\frac{1}{2} \sin 2x + \sin x\right) + c$

$$59. \quad (I) \quad x - 4 \log|x| + \frac{5}{4} \log|x-1| + \frac{3}{4} \log|x+1| + \log|x^2 + 1| - \frac{1}{2} \tan^{-1} x + c$$

$$(II) \quad \log \left| \frac{(e^t-1)(e^t-3)}{(e^t-2)^2} \right| + c$$

$$(III) \quad 2x - \frac{1}{8} \log|x+1| + \frac{81}{8} \log|x-3| - \frac{27}{2(x-3)} + c$$

$$(IV) \quad \frac{1}{4} \log \left| \frac{1-\cos x}{1+\cos x} \right| + \frac{1}{2(1+\cos x)} + \tan \frac{x}{2} + c$$

$$(V) \quad \frac{\pi}{\sqrt{2}}$$

$$(VI) \quad \frac{\pi-2}{4}$$

$$(I) \quad \frac{\pi}{4} - \frac{1}{2} \log 2$$

$$60. \quad (I) \quad 14$$

$$(II) \quad \frac{26}{3}$$

$$(III) \quad 26$$

$$(IV) \quad \frac{1}{2} (127 + e^8)$$

$$(V) \quad \frac{1}{3} \left(e^2 - \frac{1}{e} \right)$$

$$(VI) \quad 3$$

$$61. \quad (I) \quad \frac{1}{5} \log \left| \frac{\tan x - 2}{2 \tan x + 1} \right| + c$$

$$(II) \quad \frac{\pi}{8} \log 2$$

$$(III) \quad \frac{\pi}{2} \log \frac{1}{2}$$

62. $\frac{\pi^2}{16} - \frac{\pi}{4} + \frac{1}{2} \log 2$

63. $\frac{-\pi}{2} \log 2$

64. $\log 2$

65. $\frac{1}{\sqrt{2}} \log |\sqrt{2} + 1|$