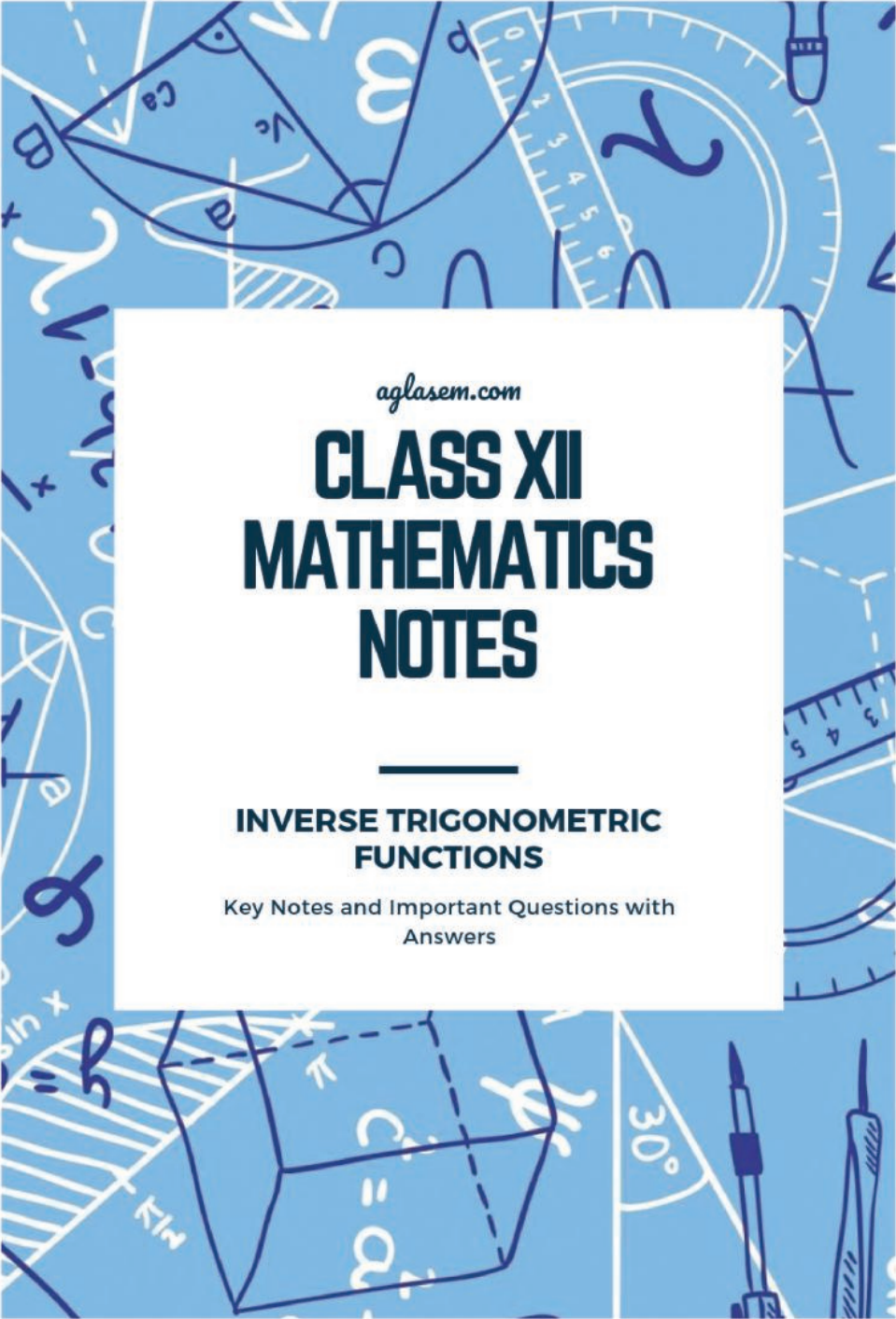


The background is a light blue field filled with various white mathematical sketches. These include geometric shapes like triangles with labeled sides (a, b, c), circles with radii, a 3D cube, a 30-degree angle, a ruler with markings, and various mathematical symbols such as π , λ , ϕ , and ψ .

aglasem.com

CLASS XII MATHEMATICS NOTES

INVERSE TRIGONOMETRIC FUNCTIONS

Key Notes and Important Questions with
Answers

CHAPTER 2

INVERSE TRIGONOMETRIC FUNCTIONS

IMPORTANT POINTS TO REMEMBER

<i>Function</i>	<i>Domain</i>	<i>Range</i> <i>(Principal Value Branch)</i>
$\sin^{-1}x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1}x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\cot^{-1}x$	R	$(0, \pi)$
$\sec^{-1}x$	$R - (-1, 1)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$\operatorname{cosec}^{-1}x$	$R - (-1, 1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

- If $\sin \theta = x$, $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, then $\theta = \sin^{-1}x$

If $\cos \theta = x$, $\theta \in [0, \pi]$, then $\theta = \cos^{-1}x$

If $\tan \theta = x$, $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then $\theta = \tan^{-1}x$

If $\cot \theta = x$, $\theta \in (0, \pi)$, then $\theta = \cot^{-1}x$

If $\sec \theta = x$, $\theta \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$, then $\theta = \sec^{-1}x$

If $\operatorname{cosec} \theta = x$, $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$, then find $\theta = \operatorname{cosec}^{-1} x$

- $\sin^{-1}(\sin x) = x \quad \forall x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\cos^{-1}(\cos x) = x \quad \forall x \in [0, \pi]$$

$$\tan^{-1}(\tan x) = x \quad \forall x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\cot^{-1}(\cot x) = x \quad \forall x \in (0, \pi)$$

$$\sec^{-1}(\sec x) = x \quad \forall x \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$$

$$\operatorname{cosec}^{-1}(\operatorname{cosec} x) = x \quad \forall x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$$

- $\sin(\sin^{-1} x) = x \quad \forall x \in [-1, 1]$

$$\cos(\cos^{-1} x) = x \quad \forall x \in [-1, 1]$$

$$\tan(\tan^{-1} x) = x \quad \forall x \in \mathbb{R}$$

$$\cot(\cot^{-1} x) = x \quad \forall x \in \mathbb{R}$$

$$\sec(\sec^{-1} x) = x \quad \forall x \in \mathbb{R} - (-1, 1)$$

$$\operatorname{cosec}(\operatorname{cosec}^{-1} x) = x \quad \forall x \in \mathbb{R} - (-1, 1)$$

- $\sin^{-1} x = \operatorname{cosec}^{-1}\left(\frac{1}{x}\right) \quad \forall x \in [-1, 1]$

$$\tan^{-1} x = \cot^{-1}(1/x) \quad \forall x > 0$$

$$\sec^{-1} x = \cos^{-1}(1/x), \quad \forall |x| \geq 1$$

- $\sin^{-1}(-x) = -\sin^{-1} x \quad \forall x \in [-1, 1]$

$$\tan^{-1}(-x) = -\tan^{-1} x \quad \forall x \in \mathbb{R}$$

$$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x \quad \forall |x| \geq 1$$

- $\cos^{-1}(-x) = \pi - \cos^{-1}x \quad \forall x \in [-1, 1]$

$$\cot^{-1}(-x) = \pi - \cot^{-1}x \quad \forall x \in \mathbb{R}$$

$$\sec^{-1}(-x) = \pi - \sec^{-1}x \quad \forall |x| \geq 1$$

- $\sin^{-1}X + \cos^{-1}X = \frac{\pi}{2}, X \in [-1, 1]$

$$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2} \quad \forall x \in \mathbb{R}$$

$$\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2} \quad \forall |x| \geq 1$$

- $$\tan^{-1}x + \tan^{-1}y = \begin{cases} \tan^{-1}\frac{x+y}{1-xy} & \text{if } xy < 1 \\ \pi + \tan^{-1}\frac{x+y}{1-xy} & \text{if } xy > 1; x > 0 \\ & y > 0 \\ -\pi + \tan^{-1}\frac{x+y}{1-xy} & \text{if } xy > 1; x < 0 \\ & y < 0 \end{cases}$$

- $$\tan^{-1}x - \tan^{-1}y = \begin{cases} \tan^{-1}\frac{x-y}{1+xy} & \text{if } xy > -1 \\ \pi + \tan^{-1}\frac{x-y}{1+xy} & \text{if } xy < -1; x > 0 \\ & y < 0 \\ -\pi + \tan^{-1}\frac{x-y}{1+xy} & \text{if } xy < -1; x < 0 \\ & y > 0 \end{cases}$$

- $2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right), |x| < 1$

$$2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right), |x| \leq 1,$$

$$2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), x \geq 0.$$

$$\bullet \quad \sin^{-1}x \pm \sin^{-1}y = \sin^{-1}(x\sqrt{1-y^2} \pm y\sqrt{1-x^2})$$

$$\cos^{-1}x \pm \cos^{-1}y = \cos^{-1}(xy \mp \sqrt{1-x^2}\sqrt{1-y^2})$$

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Write the principal value of

(i) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

(ii) $\cos^{-1}(\sqrt{3}/2)$

(iii) $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

(iv) $\operatorname{cosec}^{-1}(-2)$

(v) $\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(vi) $\sec^{-1}(-2)$.

2. What is the value of the following functions (using principal value)

(i) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) - \sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

(ii) $\sin^{-1}\left(-\frac{1}{2}\right) - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(iii) $\tan^{-1}(1) - \cot^{-1}(-1)$

(iv) $\operatorname{cosec}^{-1}(\sqrt{2}) + \sec^{-1}(\sqrt{2})$

(v) $\tan^{-1}(1) + \cot^{-1}(1) + \sin^{-1}(1)$.

(vi) $\sin^{-1}\left(\sin \frac{4\pi}{5}\right)$

(vii) $\tan^{-1}\left(\tan \frac{5\pi}{6}\right)$

(viii) $\operatorname{cosec}^{-1}\left(\operatorname{cosec} \frac{3\pi}{4}\right)$

3. If $\tan^{-1}x + \tan^{-1}y = \frac{4\pi}{5}$, find $\cot^{-1}x + \cot^{-1}y$.

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

4. Find the value of the following

$$(i) \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right) + \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$$

$$(ii) \sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\left(\cos\frac{4\pi}{3}\right)$$

$$(iii) \sin\left\{\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right\}$$

$$(iv) \tan^{-1}\left(\tan\frac{7\pi}{6}\right) + \cos^{-1}\left(\cos\frac{7\pi}{6}\right)$$

5. Simplify

$$(i) \tan^{-1}\left(\frac{\sin x}{1 + \cos x}\right)$$

$$(ii) \cot^{-1}\left(\frac{1}{\sqrt{x^2 - 1}}\right), x < -1$$

$$(iii) \cos\left\{\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \frac{\pi}{6}\right\}$$

$$(iv) \tan\left[\frac{1}{2}\cos^{-1}\left(\frac{3}{\sqrt{11}}\right)\right]$$

$$6. \text{ Simplify : } \sin^{-1}\left\{\frac{\sin x + \cos x}{\sqrt{2}}\right\}, -\frac{\pi}{4} < x < \frac{\pi}{4}$$

$$7. \text{ Prove that : } \tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}.$$

$$8. \text{ Prove that : } \tan^{-1}\frac{m}{n} - \tan^{-1}\left(\frac{m-n}{m+n}\right) = \frac{\pi}{4}, m, n > 0$$

$$9. \text{ Prove that : } \tan^{-1}\left\{\frac{a \cos x - b \sin x}{b \cos x + a \sin x}\right\} = \tan^{-1}\left(\frac{a}{b}\right) - x$$

$$10. \text{ Evaluate : } \tan^{-1}\left[2 \cos\left(2 \sin^{-1}\frac{1}{2}\right)\right]$$

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

11. Show that : $\tan^{-1} \left[\frac{\sqrt{1+\cos x} + \sqrt{1-\cos x}}{\sqrt{1+\cos x} - \sqrt{1-\cos x}} \right] = \frac{\pi}{4} + \frac{x}{2}; x \in [0, \pi]$

12. Prove that :

$$\tan^{-1} \left(\frac{\cos x}{1-\sin x} \right) - \cot^{-1} \left(\sqrt{\frac{1+\cos x}{1-\cos x}} \right) = \frac{\pi}{4} \quad x \in (0, \pi/2).$$

13. Prove that $\tan^{-1} \left(\frac{x}{\sqrt{a^2-x^2}} \right) = \sin^{-1} \frac{x}{a} = \cos^{-1} \left(\frac{\sqrt{a^2-x^2}}{a} \right).$

14. prove that:

$$\cot^{-1} \left[2 \tan \left(\cos^{-1} \frac{8}{17} \right) \right] + \tan^{-1} \left[2 \tan \left(\sin^{-1} \frac{8}{17} \right) \right] = \tan^{-1} \left(\frac{300}{161} \right)$$

15. Prove that:

$$\tan^{-1} \left(\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right) = \frac{\pi}{4} + \frac{1}{2} \cos^{-1} x^2$$

16. Solve:

$$\cot^{-1} 2x + \cot^{-1} 3x = \frac{\pi}{4}$$

17. Prove that:

$$\tan \left[\frac{\pi}{4} + \frac{1}{2} \tan^{-1} \left(\frac{a}{b} \right) \right] + \tan \left[\frac{\pi}{4} - \frac{1}{2} \tan^{-1} \left(\frac{a}{b} \right) \right] = \frac{2\sqrt{a^2+b^2}}{b}$$

18. Solve for x, $\cos^{-1} \left(\frac{x^2-1}{x^2+1} \right) + \tan^{-1} \left(\frac{-2x}{1-x^2} \right) = \frac{2\pi}{3}$

19. Prove that: $\tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{7} + \tan^{-1} \frac{1}{8} = \frac{\pi}{4}$

20. Solve for x, $\tan(\cos^{-1} x) = \sin(\tan^{-1} 2); x > 0$

21. If $y = \cot^{-1}(\sqrt{\cos x}) - \tan^{-1}(\sqrt{\cos x})$, then prove that $\sin y = \tan^2\left(\frac{x}{2}\right)$

22. Prove that:

$$\cot\left\{\tan^{-1}x + \tan^{-1}\left(\frac{1}{x}\right)\right\} + \cos^{-1}(1 - 2x^2) + \cos^{-1}(2x^2 - 1) = \pi, x > 0$$

23. Prove that:

$$\tan^{-1}\left(\frac{a-b}{1+ab}\right) + \tan^{-1}\left(\frac{b-c}{1+bc}\right) + \tan^{-1}\left(\frac{c-a}{1+ca}\right) = 0 \text{ where } a, b, c > 0$$

24. If $\tan^{-1}a + \tan^{-1}b + \tan^{-1}c = \pi$, then

prove that $a + b + c = abc$

25. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, prove that $x^2 + y^2 + z^2 + 2xyz = 1$

[Hint: Let $\cos^{-1}x = A$, $\cos^{-1}y = B$, $\cos^{-1}z = C$ then $A + B + C = \pi$ or $A + B = \pi - C$ take $\cos$ on both the sides].

26. If $\tan^{-1}\left(\frac{1}{1+1.2}\right) + \tan^{-1}\left(\frac{1}{1+2.3}\right) + \dots + \tan^{-1}\left(\frac{1}{1+n.(n+1)}\right) = \tan^{-1}\emptyset$
then find the value of $\emptyset$.

27. If $(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$ then find x .

28. If $\sin[\cot^{-1}(x+1)] = \cos(\tan^{-1}x)$, then find x .

29. Solve the following for x

(i) $\sin^{-1}(6x) + \sin^{-1}(6\sqrt{3}x) = -\frac{\pi}{2}$

(ii) $\sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}x$

(iii) $\sin^{-1}\left(\frac{5}{x}\right) + \sin^{-1}\left(\frac{12}{x}\right) = \frac{\pi}{2}$

(iv) $\sin^{-1}\left(\frac{x}{2}\right) + \cos^{-1}x = \frac{\pi}{6}$

30. If $\cos^{-1} \frac{x}{2} + \cos^{-1} \frac{y}{3} = \alpha$, then prove that
 $9x^2 - 12xy \cos \alpha + 4y^2 = 36 \sin^2 \alpha$
31. Prove that: $\tan^{-1} \left[\frac{3 \sin 2\theta}{5 + 3 \cos 2\theta} \right] + \tan^{-1} \left[\frac{1}{4} \tan \theta \right] = \theta$
32. Prove that: $\cot^{-1} \left[\cot \left(\sin^{-1} \sqrt{\frac{2-\sqrt{3}}{4}} + \cos^{-1} \frac{\sqrt{3}}{2} + \cos^{-1} \frac{1}{\sqrt{2}} \right) \right] = \frac{\pi}{2}$
33. Prove that: $2 \tan^{-1} \left(\sqrt{\frac{a-b}{a+b}} \tan \frac{x}{2} \right) = \cos^{-1} \left(\frac{a \cos x + b}{a + b \cos x} \right)$
34. Prove that: $2 \tan^{-1} [\tan \alpha/2 \tan \beta/2] = \cos^{-1} \left[\frac{\cos \alpha + \cos \beta}{1 + \cos \alpha \cos \beta} \right]$

ANSWERS

1. (i) $-\frac{\pi}{3}$ (ii) $\frac{\pi}{6}$ (iii) $-\frac{\pi}{6}$ (iv) $-\frac{\pi}{6}$ (v) $\frac{\pi}{3}$ (vi) $\frac{2\pi}{3}$
2. (i) 0 (ii) $-\frac{\pi}{3}$ (iii) $-\frac{\pi}{2}$ (iv) $\frac{\pi}{2}$
 (v) π (vi) $\frac{\pi}{5}$ (vii) $-\frac{\pi}{6}$ (viii) $\frac{\pi}{4}$
3. $\pi/5$
4. (i) $\frac{\pi}{6}$ (ii) π (iii) 1 (iv) π
5. (i) $\frac{x}{2}$ (ii) $\pi - \sec^{-1} x$ (iii) -1 (iv) $\frac{\sqrt{11}-3}{\sqrt{2}}$
6. $x + \frac{\pi}{4}$

9. 1 10. $\frac{\pi}{4}$ 12. $\tan \frac{\pi}{12} = 2 - \sqrt{3}$

14. $\frac{\sqrt{5}}{3}$ 16. 1 18. $\tan \frac{\pi}{12} = 2 - \sqrt{3}$

20. $\frac{\sqrt{5}}{3}$ 24. **Hint:** Let $\tan^{-1} a = \alpha$

$$\tan^{-1} b = \beta$$

$$\tan^{-1} c = \gamma$$

Then given, $\alpha + \beta + \gamma = \pi$

$$\alpha + \beta = \pi - \gamma$$

Take tangent on both sides

$$\tan(\alpha + \beta) = \tan(\pi - \gamma)$$

26. $\emptyset = \frac{n}{n+2}$

27. $X = -1$

28. $x = -\frac{1}{2}$

29. (i) $x = -\frac{1}{12}$

(ii) $x = 0, \frac{1}{2}$

(iii) $x = 13$

(iv) $x = 1$